NONLINEAR LIAPUNOV DYNAMICS

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PREFACE

Realistic models of the physical world are nonlinear, involving large amplitudes of motion and thus usually several equilibria of the system concerned. Hence in the majority of applications nonlinearities may not be truncated without seriously effecting the adequacy of the model. In spite of this, nonlinear dynamics has remained without much interface with applications for a long time. The reason for this seems to be at least two-fold. First, there was lack of practical methods implementing the nonlinear results in everyday applied dynamics. Second, the massive investment of time and resources in the linearized techniques generated a potent disincentive for the change, even at the expense of accurate modelling. The first aspect lost its ground with the arrival of fast computers, but the second still persists. Indeed, changes in the attitudes of people are always slower than changes in technology. There is, however, good news. Not only is nonlinear dynamics stimulated by computers, but the converse occurs as well. The nonlinear techniques speed up computation by identifying the types of dynamic trajectories concerned and, perhaps even more importantly, by helping to check the results. Furthermore, the Liapunov formalism, fundamental for nonlinear dynamics, becomes an essential part of the theory of parallel computing and neural networks. The latter are presently successfully replacing artificial intelligence and expert systems in most progressive applications, and thus generating considerable demand for Liapunov type algorithms and for nonlinear dynamics in general.

The book gives the Liapunov background for the analysis and synthesis (design) of dynamic behaviour of general networks which represent a large class of nonlinear systems, predominantly physical, and in particular mechanical. It is meant to be a self-learning and thought provoking reference text. It has grown from junior graduate level lectures in Engineering Science and Applied Mathematics over a number of years at several universities (Notre Dame, Ottawa, Queensland, Alberta, Southern California) and circulated for a while in the format of lecture notes. The first, introductory chapter refers to the basic concepts of static characteristics and dynamic processes. The second and third describe various formats of dynamic models and give a reference frame for their behaviour. The fourth chapter introduces basic energy relations, fundamental to the dynamic use of the Liapunov method. The method itself is described in chapter five, with implementations in chapters six and seven. The methods of Liapunov Design (synthesis) and Control in chapter eight close the text.

The prerequisite background is not above elementary analytic dynamics (mechanics) and differential equations. Readers not interested in the systems interpretation of dynamics may leave out the first chapter without consequence to later reading. On the other hand chapters four and five are fundamental to the whole text. The material is slightly longer than for a semester course, giving the instructor the option of cutting off some sections which are less useful to his particular group of students.

The author is indebted to his wife Elzbieta and children, Joanna and Michal for their patience and help while writing this text.

Los Angeles, January 1990

J.M. Skowronski

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CHAPTER 1

Structures in System Dynamics

The purpose of this chapter is to describe what one may call the philosophy of "change in time" and thus to introduce some fundamental structures, due to the role of System Dynamics as a rather universal tool in Science. In fact, the task requires as much rigour as abstractness, but we must not include too many formal technicalities for the sake of applied science readers, or too many physical details for the sake of mathematicians. This dictates a rather discursive approach. Readers familiar with the subject may pass on to further chapters.

Let us begin with a simple example offering nomenclature which is suggestive of the strict definitions that will come later.

§1.1 An Oscillator.

Consider a cube-spring system, Fig. 1.1(a), consisting of a cube with mass M suspended from a rigid frame on some massless spring that extends only vertically. Dynamics investigates motion of the cube in time subject to forces acting on or within the system. If one can, it is convenient to precede such an investigation by a study of the motion and the forces separately.

To the first end, let us mention the obvious geometric possibilities of the cube moving in time *without any regard to what causes the motion.* The extension of the spring, if any, may be measured along a real axis Q. This axis is the geometric locus for all possible time-instantaneous positions of the system while in motion. This is why Q is a particular case of what is called the system Configuration Space.

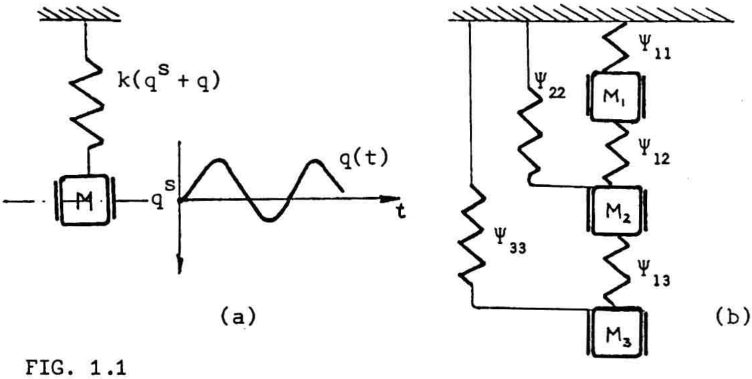

FIG. 1.1

Elementary mechanics, however, teaches that the knowledge of positions does not suffice for a full information upon the state of the system. We need to know velocities as well. Assuming the time-instantaneous values of velocity measured along a real axis P perpendicular to Q, we obtain the states of the system as points in the plane Q × P varying in time. The Cartesian product $X \doteq Q \times P$ is called the state space of the system, or traditionally the phase space. We justify the use of this latter name in Chapter 3.

The state space, or some admissible subset of it, represents the geometric locus of states varying in time during the motion of the system, i.e. the kinematic capacity of the system. It will be pertinent for the notion of kinematic structure which we introduce later in this text.

It is then up to the forces acting to use this capacity and select specific paths for the system in the state space.

In turn, we look at the forces above, i.e. the system itself, to see what actually is moving, without reference to the motion. For this purpose, we let the system rest in an equilibrium - that is, the acting forces, the weight of the cube W = Mg (g - gravity coefficient) and the restoring force F_s in the spring balance to zero: $Mg - F_s = 0$. We call such a system *static*.

The spring extends under load. A certain static extension q^s measured in Q corresponds to the weight Mg. Let k be the load necessary to produce a unit extension. Then $F_s = Mg = kq^s$ and $q^s = {}^{Mg}/_{k}$. The static extension is also the equilibrium position of the cube. Suppose the weight W changes, producing a deflection $\pm\ \delta q$ from the equilibrium position. It makes the total extension of the spring equal to some $q^s \pm \delta q = q \in Q$. Then the corresponding restoring force in the spring is $-\ kq$. Without narrowing generality, we may conveniently place the origin of Q at the equilibrium position, $q^s = 0$, yielding $q = \pm\ \delta q$. Obviously the values kq represent the capacity of the spring to bear various loads W, i.e. the capacity to do the job. Thus the function $\Psi : y = -\ kq,\ \ q \in Q$ is called the *static characteristic* of the spring. Note that considering y versus q is conventional. For some purposes, just the opposite may be more suitable, both y and q being ex-equo what will be later called system variables. As the spring adjoints the cube to its frame of suspension (environment) the static characteristic of the spring relates W through the restoring force to the frame of reference - the origin of Q. Analogously, we may say that the equality $Mg = \text{constant}$ works as the static characteristic of the cube, relating the weight to itself, i.e. it is the *eigen-characteristic.* These two characteristics together belong to the *static organization* of the system. In our case, the organization takes the shape of the resultant force. The restoring force is $-\ kq$, where in general $q = q^s \pm \delta q$. The force adds to the weight Mg yielding the resultant

$$F = Mg - k(q^s \pm \delta q) = Mg - F_s \pm k\delta q = \pm\ k\delta q\ .$$

For $q = \pm \delta q$ the resultant is identical with the spring-characteristic. A suitable class of static organizations will be later defined to be a *static structure* of the system. Incidentally, the organization is obviously a relation on the space of the system variables. It ranges in the *sub-space of force-values*, or more generally, space of organization values.

We shall now go a step further in our static study of the cube-spring system. The load used to produce a unit extension may not be constant. In fact, it may depend upon the extension. For instance, the suspension of our cube may be required to be less stiff (softer) or more stiff (harder) for longer extensions of the spring. In general, this yields a *non-linear static characteristic* of the spring: $\Psi(q) = k(q)q$, say specifically $y = q - q^3$. The same argument relates Ψ to the resultant. The equilibrium requires as before $\Psi(q) = q - q^3 = 0$, granted that we stick to a neighbourhood of the equilibrium discussed previously: $q = 0$ with $q^S = 0$. Presently there will be two other equilibria $q = \pm 1$. By the above, we have rearranged the organization of the system. The spring is now different, say conical instead of cylindric, even if the wire maintains its previous cross-section.

Now let us move the static system in time. By an impulse or a sudden application and removal of an external force, vibrations of the system about the equilibrium can be obtained. Beginning at some time instant $t_0 \in R$ for each $t \in [t_0,\infty)$ the cube deflects from the equilibrium by some quantity $q(t)$, thus measuring the time-instantaneous *dynamic extension* of the spring. In deriving the differential equation of motion, we use the Newton's second law stating that the product of mass and its acceleration is equal to the resultant in direction of the acceleration

$$(1.1.1) \qquad M\ddot{q}(t) + \Psi(q(t)) = 0 ,$$

where dots mean differentiation with respect to time. Equivalently, we have the system of two equations

$$(1.1.2) \qquad \begin{cases} \dot{q} = p , \\ \dot{p} = -\psi(q) , \end{cases}$$

where $\psi(q) = \Psi(q)/M$, accepting the common abbreviation $q = q(t)$, $p = p(t)$. The values $q(t)$, $p(t)$ describe now the mentioned state of the system or a point $x(t) = (q(t),p(t))$ in the two dimensional state space $X = Q \times P$. The variables q, p are thus called state variables. In particular, for (1.1.2), owing to its shape (symplectic), Q of q's is the Configuration Space and P of p's is the Space of Velocities.

A t_0- family of solutions to (1.1.2) called motions takes the geometric shape of a curve $x : [t_0,\infty) \to X$ which passes through some initial point $x^0 = (q^0,p^0) \in X$ and represents the state-path of the system called the trajectory. We designate it by $x(x^0,[t_0,\infty))$ meaning $\{x(x^0,t) \in X \mid t \in [t_0,\infty)\}$. The motions may be obtained individually as curves in the space of events $X \times R$ starting at some initial event $(x^0,t_0) \in X \times R$. For details, the reader is referred to Sections 2.2.1 and 2.3.1. Since $X = Q \times P$, we can map a trajectory from X into Q obtaining the corresponding configuration-trajectory. We can also map a motion from $X \times R$ into $Q \times R$ obtaining a configuration-motion.

Consider again $y = \Psi(q)$ but let the argument q be now a function of time. Since Ψ is stationary (not dependent explicitly upon t), the static characteristic $\Psi(q)$ lifted along the t- axis, cf. Fig. 1.2, produces a cylindric surface S with identical t- sections, on which the values $\Psi(q(t)) = \Psi(t)$ are located. The surface may be seen as a t-family of static characteristics. It is a part of what we shall introduce later as the space of system values. Now, envisage a configuration-motion underneath S in $Q \times R$. Lifted up into S it produces a line joining the values $\Psi(t)$ along the motion. The line is called a *dynamic characteristic* of the spring. We obviously have as many of them as there are motions. Mapping the t-family S, together with all the dynamic characteristics on it, into the plane 0qy one obtains in our stationary case the static characteristic.

Similar reasoning may be made for any other type of forces appearing in the system - including explicitly time dependent external forces, cf. Fig. 1.2(b), but then the surface S ceases to be cylindrical - will live on its own, independently of the statics.

Further from the above, the reader may now expect the conclusion that the *kinetic* system of the moving cube under forces is time instantaneously static, cf. Fig. 1.2(c). Moreover, that the dynamic characteristic along its selected (kinematic) motion actually uses only some of the possibilities offered by S, i.e. the t-family of static characteristics. These possibilities represent the kinetic structure of the system.

The pattern changes only slightly if three cubes instead of one are considered, cf. Fig. 1.1(b). Three configuration axes Q_1, Q_2, Q_3 compassing the 3-dimensional configuration space Q together with three velocity axes P_1, P_2, P_3 compassing the 3-dimensional velocity space P make up the state space $X = Q_1 \times Q_2 \times Q_3 \times P_1 \times P_2 \times P_3$ which represents the kinematic capacity of the system. As before, we let the origin of X be located at one of the equilibria with the dynamic