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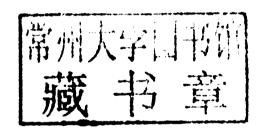
Robust Portfolio Construction in the Presence of Fat Tails

MALCOLM KEMP

Extreme Events

# Robust Portfolio Construction in the Presence of Fat Tails

# Malcolm H. D. Kemp





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 	 Preface	 	

There are two reasons for writing this book. The first is timing. This book has been written during a worldwide credit and economic crisis. The crisis has reminded us that extreme events can occur more frequently than we might like. It followed an extended period of relative stability and economic growth that, with hindsight, was the calm before a storm. Throughout this book, this crisis is referred to as the '2007–09 credit crisis'.

During the preceding relatively stable times, people had, maybe, paid less attention than they should have to the possibility of extreme events occurring. Perhaps you were one of them. Perhaps I was too. Adversity is always a good spur to careful articulation of underlying reality.

Extreme events were evident even right at the start of the 2007–09 credit crisis, at the end of July 2007 and in the first two weeks of August 2007. This marked the time when previously ruling relationships between interbank money market rates for different deposit terms first started to unravel. It also coincided with some sudden and unexpectedly large losses being incurred by some high profile (quantitatively run) hedge funds that unwound some of their positions when banks cut funding and liquidity lines to them. Other investors suffered unusually large losses on positions they held that had similar economic sensitivities to the ones that these hedge funds were liquidating.

During 2007 investors were also becoming more wary of US sub-prime debt. But, even as late as April 2008, central banks were still expecting markets gradually to regain their poise. It was not to be. In late 2008 the credit crisis erupted into a full-blown global banking crisis with the collapse of Lehman Brothers, the need for Western governments to shore up their banking systems and economies across the globe entering recessions. Market volatility during these troubled times was exceptional, reaching levels not seen since the Great Depression in the early 1930s. Extreme events had well and truly returned to the financial landscape.

The second reason for writing this book is that it naturally follows on from my earlier book on *Market Consistency*; see Kemp (2009). Market consistency was defined there as the activity of taking account of 'what the market has to say' in financial practice. In *Market Consistency* I argued that portfolio construction should be viewed as the third strand in a single overarching branch of financial theory and practice, the other two strands being valuation methodology and risk management processes. However, I also noted that the application of market consistency to portfolio construction was in some sense simultaneously both core and peripheral. It was 'core', in the sense that ultimately there is little point in merely valuing and measuring positions or risk; the focus should also be on managing them. It was also 'peripheral', because portfolio management generally also involves taking views about when the market is right and when it

is wrong, and then acting accordingly, i.e., it necessarily does not always agree with 'what the market has to say'.

These different drivers meant that *Market Consistency* naturally focused more on valuation methodology and risk management and less on portfolio construction. So, this book seeks to balance the exposition in *Market Consistency* by exploring in more depth the portfolio construction problem. It is particularly aimed at those practitioners, students and others who, like me, find portfolio construction a fascinating topic to study or a useful discipline to apply in practice.

The style I have adopted when writing this book is similar to the one used for *Market Consistency*. This involves aiming simultaneously to write both a discursive text and a reference book. Readers will no doubt want a reference book to help them navigate through the many techniques that can be applied to portfolio construction. However, undue focus on the 'how to' rather than also on the 'why should I want to' or 'what are the strengths and weaknesses of doing so' is typically not the right way to help people in practice. Instead, readers ideally also expect authors to express opinions about the intrinsic usefulness of different techniques.

As in *Market Consistency* I have tried to find a suitable balance between mathematical depth and readability, to avoid some readers being overly daunted by unduly complicated mathematics. The book focuses on core principles and on illuminating them where appropriate with suitably pitched mathematics. Readers wanting a more detailed articulation of the underlying mathematics are directed towards the portfolio construction pages of the www.nematrian.com website, referred to throughout this book as Kemp (2010). Kemp (2010) also makes available a wide range of online tools that, inter alia, can be applied to the portfolio construction problem. Most of the charts in this book and the analyses on which they are based use these tools, and where copyrighted are reproduced with kind permission from Nematrian Limited. A few of the charts and principles quoted in this book are copied from ones already contained in *Market Consistency* and are reproduced with permission from John Wiley & Sons, Ltd.

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However, while I am very grateful for the support I have received from various sources when writing this book, I still take sole responsibility for any errors and omissions that it contains.

# Abbreviations

ALM asset-liability management APT Arbitrage Pricing Theory

AR autoregressive

ARMA autoregressive moving average

BL Black-Litterman bp basis point

CAPM Capital Asset Pricing Model
CDO collateralised debt obligation
cdf cumulative distribution function

CDS credit default swap CML capital market line

COSO Committee of Sponsoring Organisations of the Treadway Commission

CPPI constant proportional portfolio insurance

CRO chief risk officer

CRRA constant relative risk aversion

CVaR conditional Value-at-Risk (aka TVaR and Expected Shortfall)

EM expectation-maximisation algorithm

EMH efficient markets hypothesis
ERM enterprise risk management
ESG economic scenario generator

EU European Union

EVD extreme value distribution
EVT extreme value theory
FOC first order conditions

FSA Financial Services Authority (UK)

GAAP Generally Accepted Accounting Principles

GMM Gaussian (i.e., multivariate Normal) mixture models GRS *F*-test Gibbons, Ross and Shanken (*Econometrica* 1989) *F*-test

HHI Herfindahl Hirshman Index

i.i.d. independent and identically distributedIAS International Accounting StandardsIASB International Accounting Standards Board

ICA independent components analysis

IT information technology lpm lower partial moment

LTCM Long Term Capital Management

MA moving average

MCMC Markov chain Monte Carlo MDA maximum domain of attraction

MRP minimum risk portfolio
OLS ordinary least squares

ORSA Own Risk and Solvency Assessment

OSLL out-of-sample log-likelihood
PCA principal components analysis
pdf probability density function

QQ quantile—quantile
RE resampled efficiency
RS regime switching

S&Ls (US) savings and loans associations SCR Solvency Capital Requirement

SETAR self-exciting threshold autoregressive

SIV structured investment vehicle

SYSC Senior Management Arrangements, Systems and Controls

TAR threshold autoregressive

TVaR tail Value-at-Risk (aka CVaR and Expected Shortfall)
UK United Kingdom of Great Britain and Northern Ireland

US and USA United States of America

VaR Value-at-Risk

## Notation \_ \_

```
0 = \text{vector of zeros}
```

1 = vector of ones

a,  $\mathbf{a}$  = active positions, (linear combination) signal mixing coefficients

 $\hat{\alpha}$  = tail index parameter estimate

b,  $\mathbf{b}$  = benchmark, minimum risk portfolio, (distributional mixture) mixing coefficients

 $\beta$  = portfolio or individual security beta

 $\gamma$ ,  $\gamma_i$  = cumulant of a distribution (if i = 1 then skew and if i = 2 then 'excess' kurtosis)

 $\Gamma_0$  = option gamma

 $\Delta$  = amount to invest in the underlying in a hedging algorithm

 $\Delta_0$  = option delta

E = analogue of energy in simulated annealing algorithm

E(X) =expected value of X

 $E(X | \theta)$  = expected value of X given  $\theta$ 

 $\varepsilon_{i,t}$  = error terms in a regression analysis

f(x) =probability density function

F(x),  $F^{-1}(x)$  = cumulative distribution function (or more generally some specified distributional form) and its inverse function

 $\langle f \rangle$  = Monte Carlo estimate of the average of a function f

 $\langle \langle f \rangle \rangle$  = true average of a function f

h =time period length

i, j, k = counting indexes

I, I = Identity matrix

 $I_t = index series$ 

 $I_{\alpha}(t)$  = 'hit' series when backtesting

K =option strike price

L = lottery, Lagrange multiplier

 $\lambda$  = risk-reward trade-off parameter, also eigenvalue

m = number of assets (or liabilities, or both) in the portfolio optimisation problem

 $\mu$ ,  $\hat{\mu}$  = mean of a univariate distribution, vector of population means of a multivariate distribution, vector of sample or estimated means (likewise use of 'hat' symbol for other variables)

n = number of time periods, observations or simulations

N(z),  $N^{-1}(z)$  = cumulative distribution function for the Normal distribution, and its inverse function

 $o(x) = \text{tends to zero more rapidly than } x \text{ as } x \to 0$ 

O(n) =of order (magnitude) a constant times n (in analysis of algorithm run times)

p(X) = probability of X occurring

 $P(X \le x)$  = probability that a random variable X is less than some value x

 $p(X|\theta), P(X|\theta) = \text{probability of } X \text{ given } \theta$ 

r = return on an asset, liability or index

 $\bar{r} = \text{mean of } r$ 

 $r_{rf}$ ,  $r_b = \text{risk-free}$  rate of return, benchmark return

 $\rho = \text{correlation coefficient, risk measure}$ 

s =stress test, regime

S = spectrum of an autoregressive time series

 $S_t = \text{stock index series}$ 

**S**  $(\alpha, \beta, \gamma, \delta_k; k)$  = Stable distribution with parameters  $(\alpha, \beta, \gamma, \delta_k)$  using parameter definition k (= 0 or 1)

 $\sigma$  = standard deviation

T = matrix transpose (if used as a superscript), also (when not confusing) time at end of analysis/time horizon, also analogue of temperature in simulated annealing algorithm

T(z) = transfer function

V = a volume in a multi-dimensional space

V = covariance matrix

 $w_i, w_t, \mathbf{w}_t, w_{i,j}$  = weight in asset *i*, innovation at time *t* (possibly expressed in vector form for vector autoregressive series), elements of mixing matrix *W* 

W = mixing matrix, terminal wealth

 $X_n \stackrel{\triangle}{=} X = \text{equality in distributional form}$ 

 $Z_n \stackrel{D}{\longrightarrow} F$  = the sequence of random variables,  $Z_n$ , converges in distribution to F as  $n \to \infty$ , i.e., in the limit takes the distributional form characterised by F

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