MATHEMATICS
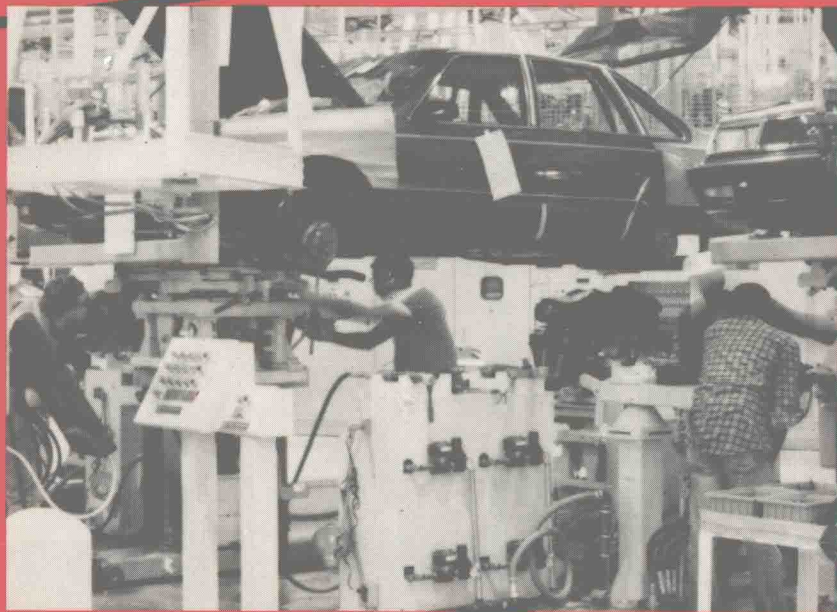## FOR THE
AUTOMOTIVE
TRADES

David L. Goetsch

Deborah M. Goetsch

Raymond L. Rickman

MATHEMATICS
FOR
THE AUTOMOTIVE TRADES

DAVID L. GOETSCH

DEBORAH M. GOETSCH

RAYMOND L. RICKMAN

Editorial/production supervision and
  interior design: *Ellen Denning*
Manufacturing buyer: *Lorraine Fumoso*
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  Chrysler Corporation.

Dedicated to Shellie, Kellie, Karmen, and Karla—my family,
with love. . .

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PREFACE

WHO IS THIS BOOK FOR?

This book is for three groups of people: (1) students who wish to become automotive technicians, (2) practicing automotive technicians, and (3) persons working in other occupations who would like or who need to make a career change. *Mathematics for the Automotive Trades* is designed for use in both a traditional classroom setting and in nontraditional settings such as workshops, seminars, or on-the-job learning situations. Any person in any setting who needs to learn the fundamentals of math as well as how to apply those fundamentals in a "real world" setting will find this book helpful.

WHAT TYPES OF SCHOOLS AND AGENCIES MIGHT USE THIS BOOK?

Any institution or agency providing technical training will find *Mathematics for the Automotive Trades* to be appropriate for use. Such institutions and agencies include community colleges, technical colleges, vocational and technical schools, private educational agencies, government training agencies, military training agencies, and private-sector companies that offer on-the-job training.

PREREQUISITES

This book has been especially structured to require no prerequisites. Part 1 is a special, self-paced arithmetic review for students who graduated from high school without a solid grasp of arithmetic or for practitioners who have not used arithmetic for a prolonged period of time. It may be used as a separate course that is part of a training program, as an integrated part of a self-paced training program, or as a learning tool in workshops, seminars, or on-the-job training situations.

NEED FOR THE BOOK

For the past 10 years, the authors have been involved in preparing technical students for the world of work. During this time it became apparent that math skills needed close attention. It also became apparent that the "technical math" books already on the market were not exactly what was needed.

The consensus among practitioners we have dealt with during the past 10 years is that the hands-on technician needs to have a solid foundation of knowledge and skills in three critical math areas: algebra, geometry, and trigonometry.

The consensus among educators we have dealt with during the past 10 years is that many students come into their programs ill-prepared to undertake a study of algebra, geometry, and trigonometry. First, they need to develop basic arithmetic skills. Numerous nationally circulated reports on America's public schools echo their thoughts.

Practitioners also feel that too many beginning technicians do not know how to apply their math skills in a real-world setting. If they have studied math, they can work the types of problems found at the end of the chapter, but seem to have trouble working math problems that are part of actual work projects.

Educators also feel that most technical math books on the market try to cover too much territory. They attempt to cover, in one book, arithmetic through calculus. The result is that the book overpowers or frightens students. Additionally, most technical math books attempt to include problems for several different technical programs, such as drafting, electronics, automotive, heating and cooling, machining, and so forth. In so doing, they lose the specificity that promotes understanding, acceptance on the part of students, and learning.

Mathematics for the Automotive Trades was written in response to the stated needs of both practitioners and educators. It was especially designed to include those characteristics they felt were positive and to exclude those they felt were negative.

INNOVATIONS

This book, because it was designed to meet specific needs, has a number of important built-in innovations that set it apart from other books:

1. The format is structured to allow the book to be used in a traditional classroom, a self-paced classroom, a workshop setting, or an on-the-job training situation.
2. A special three-part teaching and learning approach is utilized in which students first learn or review basic arithmetic; then develop the essential skills from the critical math areas of algebra, geometry, and trigonometry without the additional burden of trying to apply those skills to actual real-world projects; and finally, once all the necessary skills have been developed, apply them to a wide range of actual work-world projects that include math problems which mirror those that will be confronted on the job.
3. The book is divided into three distinct parts, each especially designed to meet specific needs. Consequently, each part has its own format and special features.

With minor adaptions, portions of Parts 1 and 2 and the Appendices are taken from *Math for Computer-Aided Drafting* by David L. Goetsch and Deborah M. Goetsch, Prentice-Hall, Inc., © 1987. This material is adapted by permission of Prentice-Hall, Inc.

MAJOR FEATURES

The major features of the book must be presented in three different parts which match the three parts of the book. The major features of Part 1 include:

1. Pretests and posttests for each chapter so that students who already have the skills covered can skip that which is known if the instructor wishes them to.
2. Practical application problems within each chapter for each new concept presented. This allows for immediate application of the concept that is being learned.
3. A comprehensive set of review problems at the end of each chapter that provides learners with opportunities to apply their learning for all of the concepts presented in a given chapter.

The major features of Part 2 include:

1. Practical application problems within each chapter for each new concept presented. This allows for immediate application of the concept that is being learned.
2. A comprehensive set of review problems at the end of each chapter that provides learners with opportunities to apply their learning for all of the concepts presented in a given chapter.

The major features of Part 3 include:

1. Each chapter is concerned with nothing but application and contains real-world projects.
2. The projects are to be accomplished as if in an actual work setting. To do so, learners will have to apply all of the skills learned in Parts 1 and 2.

Other major features of the book include:

1. An extensive appendix for ready reference in solving problems.
2. Answers to all pretest and posttest problems from Part 1.
3. Answers to the odd-numbered practical application problems for Parts 1 and 2.
4. Answers to the odd-numbered review problems for Parts 1 and 2.

DAVID L. GOETSCH
DEBORAH M. GOETSCH
RAYMOND L. RICKMAN

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INTRODUCTION

The automotive trades are among the most important of the various transportation-technology-related occupations. Every automobile produced requires periodic maintenance and repair of its various systems. Such tasks as diagnosis, troubleshooting, and repair still form the nucleus of the automotive technician's job.

However, the ways in which these tasks are accomplished are continually changing and becoming more and more technical in nature. The automotive trades are in a state of transition from the traditional manual and mechanical processes to high-technology processes relying on electronics and computers. Words such as "computer," "circuit board," and "microchip" have become part of the language of the automotive trades.

This transition is having an effect on the knowledge and skills needed by automotive technicians. Modern automotive technicians must know as much about electronics as they do about mechanics. This means that cognitive capabilities such as math skills are more important than ever.

Before beginning a study of math for the automotive trades, students should know the types of math skills they will need on the job for today and tomorrow and some of the more common ways these skills will be used on the job.

MATH SKILLS NEEDED IN THE MODERN AUTOMOTIVE TRADES

There are several categories or levels of math. The categories that are most important to students in the automotive trades are arithmetic, algebra, geometry, and trigonometry. Within each of these categories there are a number of specific skill areas that are important. Those that represent the minimum skills required of automotive technicians are listed in the subsections that follow.

Minimum Arithmetic Skills Needed by Automotive Technicians

At a minimum, students in the automotive trades should be able to do the following:

1. Add, subtract, multiply, and divide whole numbers
2. Add, subtract, multiply, and divide common fractions
3. Add, subtract, multiply, and divide mixed numbers
4. Add, subtract, multiply, and divide decimal fractions
5. Convert from common fractions to decimal fractions
6. Convert from decimal fractions to common fractions
7. Raise numbers to powers
8. Take roots of numbers
9. Calculate percentages

Minimum Algebra Skills Needed by Automotive Technicians

At a minimum, students in the automotive trades should be able to do the following:

1. Apply symbolism in solving problems
2. Manipulate signed numbers
3. Perform basic algebraic operations
4. Solve equations
5. Apply standard shop formulas

Minimum Geometry Skills Needed by Automotive Technicians

At a minimum, students in the automotive trades should be able to do the following:

1. Solve problems involving the principles of lines
2. Solve problems involving the principles of angles
3. Solve problems involving the principles of triangles
4. Solve problems involving the principles of polygons
5. Solve problems involving the principles of circles
6. Solve problems involving the principles of arcs

Minimum Trigonometry Skills Needed by Automotive Technicians

At a minimum, students in the automotive trades should be able to do the following:

1. Solve right triangles
2. Solve oblique triangles
3. Calculate the area of triangles

APPLYING MATH SKILLS IN THE AUTOMOTIVE TRADES

It is important to develop the math skills listed in the preceding section, but just having the skills is not enough. You must also be able to apply these skills in a work setting. There are many ways that math skills are applied on the job. Some of the most frequent applications in the automotive trades are in:

1. Taking inventory
2. Performing quantity calculations
3. Performing mileage calculations
4. Performing fuel economy calculations
5. Performing capacity calculations
6. Performing weight calculations
7. Performing length calculations
8. Performing bore diameter calculations
9. Performing expansion and contraction calculations
10. Performing clearance calculations
11. Performing cost calculations
12. Writing invoices
13. Paying shop bills
14. Estimating repair costs
15. Performing percentage calculations
16. Calculating discounts
17. Calculating sales tax
18. Performing insurance work calculations
19. Calculating overhead and operating expenses
20. Performing commission calculations
21. Performing wage and earnings calculations
22. Performing efficiency calculations
23. Performing temperature calculations
24. Performing horsepower calculations
25. Performing revolutions per minute (rpm) calculations
26. Performing turning radius calculations
27. Performing piston displacement calculations
28. Performing Ohm's law calculations
29. Performing percent-of-grade calculations

PART 1
Arithmetic Review

1

WHOLE NUMBERS

INTRODUCTION TO PART 1

Part I is a comprehensive review of arithmetic. It consists of five chapters and has been especially designed to serve several purposes: (1) a comprehensive review for learners who have already developed arithmetic skills but need to review them before pursuing a study of algebra, geometry, and trigonometry; (2) an opportunity to develop arithmetic skills for those learners who either graduated from or left high school without them; and (3) a quick reference for learners who might need to refer back to certain arithmetic concepts on occasion.

Each chapter has been especially structured to allow learners to skip over material they already know if they do not wish to review it. The Pretest* for each chapter in Part 1 gives learners an opportunity to measure their knowledge of the material covered in that chapter. If a learner scores 90% or better on a Pretest, the chapter can be skipped (with the instructor's permission). A score of less than 90% indicates that a review of the material covered in that chapter is in order.

Each chapter in Part 1 also contains a Posttest. All learners studying a given chapter in Part 1 should complete the Posttest for that chapter before proceeding to the next chapter. A socre of less than 90% on the Posttest indicates that additional work is in order before proceeding. Learners may find it helpful to work the Practical Application problems within each chapter and the Review Problems at the end of each chapter before attempting the Posttest.

Learners who are able to work the problems set forth in each chapter and at the end of each chapter are then ready to proceed to Part 2 for a study of the essentials of algebra, geometry, and trigonometry. Chapter 1 covers whole numbers and the various mathematical operations that technicians need to be able to perform on whole numbers. Chapter 2 covers common fractions and the operations technicians need to be able to perform using common fractions. Chapter 3 covers mixed numbers and operations with

*Pretests and Posttests for Chapters 1 through 5 appear in Appendix B.

mixed numbers. Chapter 4 deals with decimal fractions and operations with decimal fractions. Chapter 5 concludes Part 1 with a discussion of powers, roots, percentages, and associated operations.

Technicians must be proficient in mathematics. Such proficiency begins with an understanding of whole numbers. Whole numbers are those we know as the *counting numbers* (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, . . .). Whole numbers form the basis of all other numbers used in mathematics. In fact, all other numbers can be defined in terms of whole numbers.

To assist you in recalling your understanding of whole numbers, this chapter covers the following major concepts: expressing place values of whole numbers, expanding whole numbers, reading and writing whole numbers, adding whole numbers, subtracting whole numbers, multiplying whole numbers, and dividing whole numbers.

EXPANDING PLACE VALUES OF WHOLE NUMBERS

In the decimal system, all numbers can be formed using 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9). Each digit represents a certain value depending on its location or "place" in the number. This value is known as the *place value* of the digit. Place values begin with units and go up infinitely. The place values for whole numbers representing billions or less are: units, tens, hundreds, thousands, ten thousands, hundred thousands, millions, ten millions, hundred millions, and billions.

The chart in Figure 1-1 shows how place values in numbers can be determined. To determine the place values of digits within a number, begin with the right-hand-most digit in the number. This digit represents units or ones. The next digit to the left represents tens. The next digit to the left represents hundreds. Notice that each digit in a number represents a place value 10 times the value of the digit to its immediate right. In Figure 1-1 the place value of the 5 is billions, of the 6 is millions, of the 8 is thousands, and of the 3 is units or ones.

5	BILLIONS
4	HUNDRED MILLIONS
7	TEN MILLIONS
6	MILLIONS
2	HUNDRED THOUSANDS
0	TEN THOUSANDS
8	THOUSANDS
9	HUNDREDS
1	TENS
3	UNITS

Figure 1-1

SAMPLE PROBLEM AND SOLUTION

Express the place value of the underlined digit in the following number:

572,852   (hundreds)

SAMPLE PROBLEM AND SOLUTION

Express the place value of the underlined digit in the following number:

678,561,211   (hundred-thousands)

SAMPLE PROBLEM AND SOLUTION

Express the place value of the underlined digit in the following number:

1,051   (thousands)

PRACTICE PROBLEM SET 1-1

Express the place value of the digit underlined in each number.

1. 16,847
2. 32
3. 179
4. 156,305
5. 1,712,869
6. 5,082
7. 10
8. 15,973
9. 56,823,970
10. 3,564

EXPANDING WHOLE NUMBERS

We use the digits 2 and 8, written as 28, to represent the number twenty-eight. This (28) is actually a short way of writing 2 tens plus 8 units or ones. Whole numbers can be expanded by using a three-step procedure:

1. Determining the place value of each digit
2. Writing the digit times its place value in parentheses
3. Separating each set of parentheses with a plus sign (+)

Using this procedure, the number 762 can be expanded as follows:

$$(7 \times 100) + (6 \times 10) + (2 \times 1)$$

SAMPLE PROBLEM AND SOLUTION

Expand the number 981.

$$(9 \times 100) + (8 \times 10) + (1 \times 1)$$

SAMPLE PROBLEM AND SOLUTION

Expand the number 6,854.

$$(6 \times 1,000) + (8 \times 100) + (5 \times 10) + (4 \times 1)$$

SAMPLE PROBLEM AND SOLUTION

Expand the number 79,845.

$$(7 \times 10,000) + (9 \times 1,000) + (8 \times 100) + (4 \times 10) + (5 \times 1)$$

PRACTICE PROBLEM SET 1-2

Expand each whole number.

| 1. 172 | 2. 3 | 3. 16 | 4. 2,283 | 5. 99 |
| 6. 54,333 | 7. 7,524 | 8. 4 | 9. 62 | 10. 413 |

READING AND WRITING WHOLE NUMBERS

Once you know how to express the place values of whole numbers and to expand whole numbers, they may be easily read and written. Notice in the preceding sections that whole numbers with more than three digits were grouped using commas (3,266,759). This is the first step in reading or writing whole numbers. Whole numbers are read and written as word statements. For example, the number 1,617 is written one thousand, six hundred seventeen.

Before attempting to read or write a whole number containing more than three digits, group the digits using commas. To do this, begin with the right-hand-most digit and work your way to the left making groups of three digits each, as in the following example:

$$4,629,812$$

To read or write this number as a word statement, work from left to right using the words that represent each number according to its grouping. The number in the example above is read or written: four million, six hundred twenty-nine thousand, eight hundred twelve. Notice that the commas in the word statement match the commas in the number.

SAMPLE PROBLEM AND SOLUTION

Write the word statement for 819.

eight hundred nineteen

SAMPLE PROBLEM AND SOLUTION

Write the word statement for 4,132.

four thousand, one hundred thirty-two

PRACTICE PROBLEM SET 1-3

Write each whole number as a word statement.

1. 719	2. 64	3. 10,419	4. 215,333
5. 7,326,424	6. 201	7. 8,888	8. 97,615
9. 112,537	10. 5,233,648		

ADDING WHOLE NUMBERS

Technicians must be able to add whole numbers. The addition of whole numbers involves three elements: the addends, the plus sign, and the sum. The *addends* are the numbers to be added. The *plus sign* (+) indicates that

the process of addition is to be performed. The *sum* is the result of the process. These three elements are illustrated in the following example:

$$
\begin{array}{rl}
& 123 \quad \text{addend} \\
\text{plus sign} \quad + & \underline{411} \quad \text{addend} \\
& 534 \quad \text{sum}
\end{array}
$$

To add whole numbers, you must know the complete set of sums of all pairs of single-digit numbers. These sums are illustrated in the chart in Figure 1-2.

Whole numbers are prepared for addition by stacking them in columns. Make sure that digits in the units, tens, hundreds, and so on, places line up, as in the following example:

```
hundreds
   tens
    units

7  6  2
5  1  3
+ 4  5  5
```

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Figure 1-2