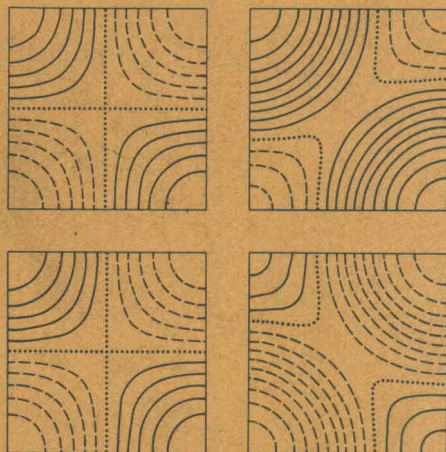


# Fluid Mechanics in Energy Conversion

Proceedings of a SIMS Conference

• Alta, Utah • June 25-29, 1979



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SIAM INSTITUTE FOR MATHEMATICS AND SOCIETY (SIMS)

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# Fluid Mechanics in Energy Conversion

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John David Buckmaster, editor

Proceedings of a Conference sponsored by  
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**siam** Philadelphia  
1980

**Fluid Mechanics  
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# SIMS

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## SIAM-SIMS CONFERENCE SERIES

1. ECOSYSTEM ANALYSIS AND PREDICTION. Simon A. Levin, editor. Proceedings of SIMS Conference, Alta, Utah, July 1-5, 1974.
2. EPIDEMIOLOGY. Donald Ludwig and Kenneth L. Cooke, editors. Proceedings of SIMS Conference, Alta, Utah, July 8-12, 1974.
3. ENERGY: *Mathematics and Models*. Fred S. Roberts, editor. Proceedings of SIMS Conference, Alta, Utah, July, 1975.
4. ENVIRONMENTAL HEALTH: *Quantitative Methods*. Alice Whittemore, editor. Proceedings of SIMS Conference, Alta, Utah, July, 1976.
5. TIME SERIES AND ECOLOGICAL PROCESSES. H. H. Shugart, Jr., editor. Proceedings of SIAM-SIMS Conference, Alta, Utah, June-July, 1977.
6. ENERGY AND HEALTH. Norman E. Breslow and Alice S. Whittemore, editors. Proceedings of SIAM-SIMS Conference, Alta, Utah, June 26-30, 1978.
7. FLUID MECHANICS IN ENERGY CONVERSION. John David Buckmaster, editor. Proceedings of SIAM-SIMS Conference, Alta, Utah, June 25-29, 1979.

## PREFACE

In 1974 SIMS embarked on a series of five-day Research Application Conferences (RAC's) at Alta, Utah, for the purpose of probing in depth selected societal fields in light of their susceptibility to mathematical analysis and their concern to society. The first six conferences dealt with ecosystems, epidemiology, energy, environmental health, time series and ecological processes, and energy and health.

These proceedings are a result of the seventh conference, on energy conversion and fluid mechanics, which was held in 1979. The twenty-four speakers and observers contributed their expertise in such disciplines as mechanics, engineering, fluid dynamics, and mathematics. All were concerned with the complex interactions between fluid mechanics and energy producing processes; the conference focused on energy from the oceans, energy from the ground, nuclear reactor cooling, and combustion. John Buckmaster of the University of Illinois served as Conference Chairman. Donald R. Snow of Brigham Young University was Local Coordinator.

The conference was supported jointly by the Argonne National Laboratory (Department of Energy) and by SIMS.

D. L. Thomsen, Jr.  
President, SIMS

## INTRODUCTION

High in the mountains above Salt Lake City, applied mathematicians from academe, industry, and government gathered for a week in June 1979 to describe and discuss the role of their discipline in energy conversion techniques, both new and old. The seventh SIMS Research Application Conference sought to identify those fluid mechanical aspects of energy conversion in which classical applied mathematics can and must play a significant role. All of the papers presented at that conference are contained in this volume. They describe such diverse topics as wave power, tidal power, ocean thermal energy conversion, windmills, geothermal energy, oil recovery, nuclear reactor cooling, turbulent combustion, and shale and coal combustion.

For the participants, both authors and observers, it was an intellectually stimulating week. It is regrettable that we cannot carry in these pages the content of the vigorous debates that followed each presentation.

As far as the topics covered by these Proceedings are concerned, there are some obvious omissions. None of my enquiries, or those of colleagues I prevailed upon, identified anyone willing to claim that there is significant mathematical work to be done in direct solar energy conversion. The subject is, therefore, not represented.

Efforts to have a mathematician from the oil industry describe the impingement of mathematics on the practical problems of oil recovery failed at a late date. Indeed, it is a disadvantage of a small, week-long conference that, while academics are generally delighted to contemplate a week in the mountains, it is difficult for scientists

and mathematicians from industry to make such a commitment of time. Perhaps, as one of the participants suggested, program managers and industrialists are not convinced of the value of academic mathematics. We all should work hard to overcome such attitudes where they exist.

The Proceedings are divided into four sections, the first of which is concerned with the oceans (water and air). The British government is supporting a substantial program in wave power, a field surveyed in the paper by Count, who is intimately involved in this program. Newman's paper complements this survey by describing recent work on three-dimensional wave energy absorbers, an area in which much work remains to be done. Greenberg, Jirka, and Wilson survey, respectively, the problems of tidal power, the exploitation of the temperature difference between deep water and shallow water in tropical and semi-tropical latitudes (OTEC), and windmills.

Section Two is concerned with energy in the ground, geothermal and oleic. A common link here is the need to understand the way in which fluids move through porous media. It is unfortunate that none of the papers provides a general survey of oil recovery technology, but apparently such an article is scheduled to appear in the near future in a well-known review series. Kassoy's paper surveys the field of geothermal energy. The papers by Straus, Albright and Concus, and Ford are concerned with certain numerical and mathematical questions and techniques that arise in porous media studies.

Section Three contains two papers tenuously linked by their relevance to nuclear power. Drew's paper describes some of the deficiencies in the modeling of water/steam coolant flow. With the Three Mile Island incident fresh in the public's mind at the time of the conference, these deficiencies elicited a great deal of comment.

Walker describes some attractive asymptotic techniques in the magnetohydrodynamics of lithium blankets for fusion reactors. MHD



power generation is not represented in these Proceedings; a well-developed discipline with complicated physics, it is, for the most part, a subject for the specialist. However, the somewhat different MHD problems discussed by Walker are accessible to the nonspecialist, and important work remains to be done, especially in the understanding of inertial effects.

Section Four is concerned with combustion, which in the past decade has seen some important and interesting developments. Libby's paper surveys the problem of turbulent reacting flows. Chemical reaction compounds the difficulties common to turbulent flows; the paper by Ludford and Buckmaster complements that of Libby by describing an asymptotic technique (activation-energy asymptotics) peculiar to the mathematics of combustion reactions that promises to be a valuable tool. George, Harris, and Thomas discuss oil shale combustion. The last paper, by Georgakis, is concerned with coal combustion in fluidized beds. It is in the last field that the conference made an immediate payoff, for it became clear that activation-energy asymptotics, not used in the subject up to that time, can play a useful role.

Many people worked hard to make the conference a success. D. Thomsen was the prime mover; D. Snow was responsible for coordinating the arrangements in Utah; J. Keller played an important early role in defining the nature of the conference; and D. Kassoy helped me assemble a body of speakers. It is a pleasure to thank all of them.

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**SECTION ONE**  
**ENERGY FROM THE OCEANS**



# **Power Generation from Ocean Waves— A Theoretical Review**

Brian M. Count\*

Abstract. For the past five years a substantial research programme has been undertaken in the United Kingdom to examine the prospects of large scale power generation from Ocean Waves. Many novel devices have been invented which led to an early optimism in the future potential in Wave Power, but more recent engineering studies have highlighted many difficult problems and consequently cost estimates have proved very disappointing.

In this paper the major devices being investigated in the United Kingdom are reviewed and theoretical methods used to analyse the characteristics of wave energy absorption are described in detail. The theory has been most successful to date in assisting in the understanding of the underlying physics and has not only described device behaviour previously determined experimentally, but has indicated where improvements could be made to produce superior systems, although the demands on technological improvement are considerable.

1. Introduction. With the rapid increase in energy consumption over the past two decades, there is a growing concern both that non-renewable resources of energy are being used too rapidly and that, too often, insufficient consideration is given to the environment. Although this is a debatable point of view in itself, much attention is now given to the argument that the renewable sources - solar, wind, waves, tidal, geothermal and ocean thermal power - could provide a viable alternative to a significant proportion of the power which would otherwise be generated from more conventional methods and with a minimum risk to the environment. This paper is concerned with one of the more promising of these, Ocean Wave Power.

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*Other volumes in the SIAM-SIMS Conference Series:*

**ECOSYSTEM: ANALYSIS AND PREDICTION, edited by Simon A. Levin**

Thirty-two articles assess the application of mathematics to the prediction of the response of ecosystems to perturbations. General theory and applications to specific systems are discussed. The articles range from the descriptive to the empirical to the theoretical, from the natural historical to the mathematical, and over a broad expanse of mathematical formalisms. The approach is at a multiplicity of levels, including the individual population, the population-community and population-ecosystem interfaces, and the ecosystem. Questions of basic philosophy and management are also considered.

1975, 337 pp., \$16.25/list, \$13.50/SIAM members.

**EPIDEMIOLOGY, edited by Donald Ludwig and Kenneth L. Cooke**

Prominent mathematicians, statisticians, medical researchers, and public health workers present 17 papers on epidemiology. Some of the areas discussed are environmental pollution and public health, control of venereal diseases, community vaccinations programs for childhood diseases, surveillance of family units, strategies for control of mosquito population, chemotherapy for cancer patients, and general models for transmission of communicable diseases.

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**ENERGY: MATHEMATICS & MODELS, edited by Fred S. Roberts**

The role of mathematics in attacking the many aspects of the energy problem is assessed in 20 articles. Mathematicians and energy researchers delineate circumstances under which application of mathematical analysis to the complex problems of energy are useful and meaningful. Among mathematical tools discussed are linear and geometric programming, statistical decision theory, graph theory, econometrics, computer simulation, and more classical techniques.

1976, 276 pp., \$18.50/list, \$14.80/SIAM members.

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1976, 259 pp., \$19.50/list, \$15.60/SIAM members.

**TIME SERIES AND ECOLOGICAL PROCESSES, edited by H. H. Shugart**

Practitioners of time series analysis inspect and interpret ecosystem data. They apply time series analysis to ecosystems ranging from entire hemispheres, to oceans, to the habitat of small montane plants—over time scales ranging from eons to a few minutes. Contents include topics of fundamental importance in time series analysis—e.g. Box-Jenkins forecasting models, dissecting techniques such as spectral analysis, and ecological considerations involved in time and space series.

1978, 328 pp., \$18.50/list, \$14.80/SIAM members.

**ENERGY AND HEALTH, edited by Alice S. Whittemore and Norman E. Breslow**

Mathematicians and scientists analyze and report on current uses of quantitative theories and techniques for identifying and evaluating risks to health engendered in energy production. Topics include: carcinogenic and mutagenic effects of fuel byproducts and of radiation; statistical models for health investigation and risk assessment; energy production and susceptible population subgroups; energy production and occupational exposures; and policy issues in energy planning.

1979, 352 pp., \$16.50/list, \$12.20/SIAM members.