ADVANCED TOPICS IN SCIENCE AND TECHNOLOGY IN CHINA

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Structural Plasticity

Limit, Shakedown and Dynamic Plastic Analyses of Structures

ZHEJIANG UNIVERSITY PRESS
浙江大学出版社
Springer

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With 238 figures

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图书在版编目（CIP）数据

结构塑性力学＝Structural Plasticity：英文 / 俞茂宏，马国伟，李建春著. —杭州：浙江大学出版社，2009.3

ISBN 978-7-308-06168-1

Ⅰ.结… Ⅱ.①俞…②马…③李… Ⅲ.结构力学-英文②塑性力学—英文 Ⅳ.O34

中国版本图书馆 CIP 数据核字（2008）第 212849 号

结构塑性力学

俞茂宏　马国伟　李建春　著

责任编辑　尤建忠
封面设计　Frido Steinen-Broo
出版发行　浙江大学出版社
网址：http://www.zjupress.com
Springer-Verlag GmbH
网址：http://www.springer.com
排　　版　林智广告公司
印　　刷　杭州富春印务有限公司
开　　本　787mm×960mm　1/16
印　　张　25.25
字　　数　834 千
版 印 次　2009 年 3 月第 1 版　2009 年 3 月第 1 次印刷
书　　号　ISBN 978-7-308-06168-1（浙江大学出版社）
ISBN 978-3-540-88151-3（Springer-Verlag GmbH）
定　　价　150.00 元

Preface

Structural Plasticity: Limit, Shakedown and Dynamic Plastic Analyses of Structures is the second monograph on plasticity. The others are *Generalized Plasticity*(Springer Berlin Heidelberg, 2006) and *Computational Plasticity*(forthcoming Zhejiang University Press Hangzhou and Springer Berlin Heidelberg, 2009) with emphasis on the application of the unified strength theory.

Generalized Plasticity, the first monograph on plasticity in this series, covers both traditional plasticity for metals (non-SD materials) and plasticity for geomaterials (SD materials). It describes the unified slip line theory for plane strain problems and characteristics theory for plane stress and axisymmetric problems, as well as the unified fracture criterion for mixed cracks. *Generalized Plasticity* can be used for either non-SD materials or SD materials. The second one is *Structural Plasticity: Limit, Shakedown and Dynamic Plastic Analyses of Structures*, which deals with limit analysis, shakedown analysis and dynamic plastic analyses of structures using the analytical method. The third one is *Computational Plasticity*, in which numerical methods are applied. The advances in strength theories of materials under complex stress are summarized in the book *Unified Strength Theory and Its Applications* (Springer Berlin Heidelberg, 2004).

The elastic and plastic limit analysis and shakedown analysis for structures can provide a very useful tool for the design of engineering structures. Conventionally, the Tresca yield criterion, the Huber-von Mises yield criterion, the maximum principal stress criterion and the Mohr-Coulomb criterion are applied in elastic-plastic limit analysis and shakedown analysis of structures. However, the result from each of the criteria above is a single solution suitable only for one kind of material. Only one or two principal stresses are taken into account in the maximum principal stress criterion, the Tresca criterion and the Mohr-Coulomb criterion. In addition, the Huber-von Mises criterion is inconvenient to use because of its nonlinear mathematical expression.

In the last decade more general solutions of plastic limit analysis and shakedown analysis for structures with a new unified strength theory have been presented. A series of unified solutions using the unified strength theory have been given. Unified plastic limit solutions of structures were presented in the literature, including unified solutions for circular plates, annular plates, oblique plates, rhombus plates, rectangular plates and square plates, orthogonal circular plates, thin plates with a hole, rotating discs and cylinders. So did unified solutions for the shakedown limit of pressure vessels, circular plates and rotating discs and for the dynamic plastic behavior of circular plates under soft impact. These unified solutions encompass not only the Tresca solution and the Mohr-Coulomb solution as special cases, but also a series of new solutions. The Huber-von Mises solution can also be approximated by the unified solution. The unified solution is a systematical one covering all results from a lower result to an upper result. These results can be suitable for a wide range of materials and engineering structures.

As an example, the unified solution of the limit load for oblique plates ($\theta = \pi/3$, $l_1 = 2l_2$) is illustrated in Fig.0.1. It can be seen from the figure that the limit load q can be obtained for various oblique plates with different angles and length and for various materials with a different strength ratio in tension and in compression and for various failure criteria with different parameter b.

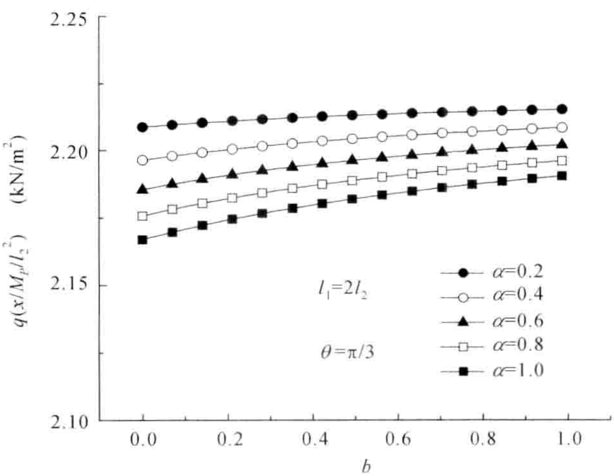

Fig. 0.1. Limit loads of oblique plate ($\theta = \pi/3, l_1 = 2l_2$) for different materials

The solution with $b = 0$ is the same as the solution of the Mohr-Coulomb material, and the solution with $b = 0$ and $\alpha = 1.0$ is the same for the Tresca material. The solution with $b = 1.0$ is for the generalized twin-shear criterion and the solution with $b = \alpha = 1.0$ is the solution of the twin-shear stress criterion. Other serial solutions between the single-shear theory (Tresca-Mohr-Coulomb theory) and the twin-shear theory are new solutions for different materials. Therefore the unified solution can be adopted for more materials and structures. It can be noted that all the solutions for the bearing capacity of structures with $b > 0$ are higher than those with the Tresca or Mohr-Coulomb criterion. The application of the unified solution is economical in the use of materials and energy. The other example is the determination of the limit pressure and thickness of pressure vessels in design. The relationship between limit pressure and wall thickness of a thin-walled vessel with the unified strength theory parameter b is shown in Fig.0.2 and Fig.0.3 respectively.

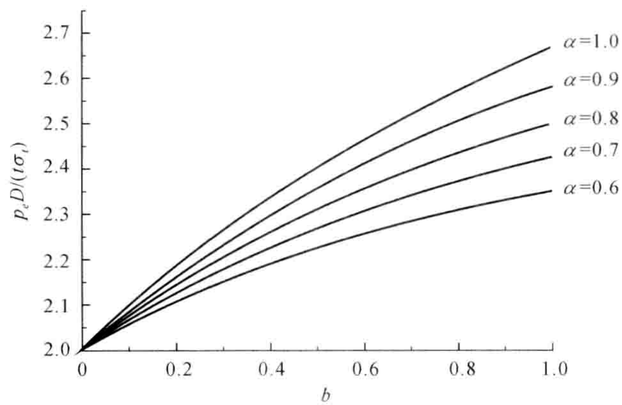

Fig. 0.2. Limit pressure versus unified strength theory parameter b

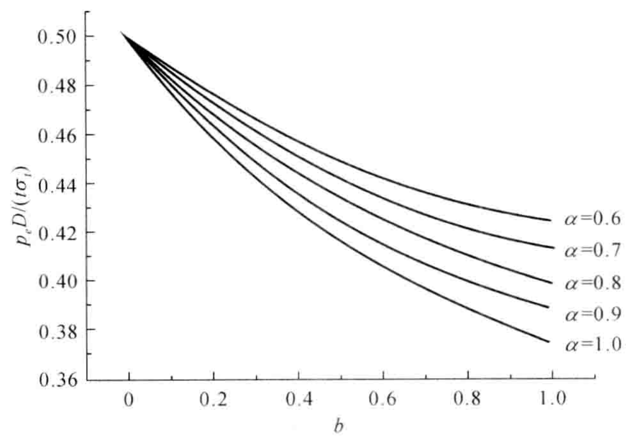

Fig. 0.3. Wall thickness versus unified strength theory parameter b

It can be seen that

- The conventional solution is a single solution ($b = 0$ in Fig.0.2 and Fig.0.3), which can be adopted only for one kind of material. The new solution is a unified solution, a serial solution, which can be adopted for more materials and structures.
- The solution for Tresca material ($b = 0$ and $\alpha = 1.0$) is identical to the solution for the Mohr-Coulomb material ($b = 0$ and $\alpha \neq 1.0$). It appears that the SD effect of materials ($\alpha \neq 1.0$ material) cannot be considered by the Mohr-Coulomb strength theory in this case.
- All the solutions for the bearing capacity of structures with $b > 0$ are higher than those for the Tresca-Mohr-Coulomb criterion. All the solutions for the wall thickness of a pressure vessel with $b > 0$ are lower than the solution using the conventional Tresca criterion or Mohr-Coulomb criterion.
- The applications of the unified strength theory and the unified solutions are more economical in the use of materials and the use of energy, leading to a reduction in environmental pollution.
- The wider application of the enhancement-factor concept on a global scale is, on the one hand, going to bring tremendous energy saving and pollution mitigation. It calls, on the other hand, for a theoretical support on which the concept can be based. Engineering practice in general has a desire to have a new strength theory, which should be more rational and more consistent with the experimental data than what can be achieved by using the Tresca-Mohr-Coulomb single-shear strength theory.

A series of the unified solutions for various structures are described in this book. It is organized as:

Chapters 2~4 give a brief introduction to the fundamental stress state, yield function and limit analysis theorem.

Chapters 5~9 deal with plastic limit analyses for circular plates, annular plates, oblique plates, rhombus plates, rectangular plates, square plates and cylinders by using the unified strength theory.

In Chapters 10, 11, the unified solutions of the dynamic plastic analysis of plates, and the limit velocity of rotating discs and cylinders are emphasized.

Penetration, wellbore analyses and orthogonal circular plates are presented in Chapters 12~14.

Chapters 15~17 are devoted to the shakedown theorem and shakedown analysis of pressure vessels, simply supported and clamped circular plates and rotating discs, using the unified strength theory. Brief summaries and references are given at the end of each chapter.

The unified strength theory and unified solutions provide a fundamental theory for the application of strength design of engineering structures. They can also be used for increasing the admissible loads or decreasing the cross-sections and the weight of structures. This results in a reduction in materials and energy consumption and a reduction in environmental pollution and the cost of structures.

The applications of the unified strength theory in plastic limit analysis and shakedown analysis for different structures are still developing. This book summarizes the research results obtained up to now. It is expected that the unified strength theory will have more and more applications in the future in addition to the plate and cylindrical structures discussed in this book. The applications of the unified strength theory in computational analysis are still growing. We hope that the Chinese idiom "Throwing out a brick to attract a piece of jade" becomes real and this book can serve as a solid brick.

The results of bearing capacity and shakedown loads obtained by using various yield criteria are very different. The results are influenced strongly by the selection of the yield criterion. We need to use a new efficient criterion. The straight-line segments on the unified strength theory make it convenient for analytical treatment of plasticity problems. The unified strength theory provides us with a very effective approach to studying the effect of yield criterion for various engineering problems. The serial results can be appropriate for most materials, from metallic materials to geomaterials.

Appreciation must be expressed for the support of the China Academy of Launch Vehicle Technology in Beijing, the Aircraft Strength Research Institute of China in Xi'an, the MOE Key Lab for Strength and Vibration at Xi'an Jiaotong University, Nanyang Technological University in Singapore, Springer and Zhejiang University Press.

I am pleased to acknowledge Dr. Guowei Ma, Dr. Jianchun Li, Dr. Fang Wang, Dr. Junhai Zhao, Dr. Xueying Wei, Dr. Yanping Wang, Dr. Shuanqiang Xu and others for their research in plastic analysis and design using the unified yield criterion and the unified strength theory. A series of unified solutions for various structures were presented. The pioneering work on the unified solution of the static and dynamic limit-bearing capacity of structures for non-SD materials using the unified yield criterion was performed by Dr. Ma during the period 1993 to 1999. The work on the unified solution of static and dynamic limit analysis of structures for SD materials was conducted by Drs. Li, Wei, Wang, Zhao, Fan and others. The work of unified solutions of shakedown for cylinders, rotating discs and circular plates was carried out by Prof. Xu Shuanqiang and Dr. Li.

I am also indebted to thank many other researchers for their research on the unified solution of structures in the fields of soil mechanics, rock mechanics, concrete mechanics and computational mechanics. I would like to express sincere thanks to Academician Shen ZJ, Academician Yang XM, Academician Chen SY, Academician Shen ZY, Academician Chen HQ, Prof. Fan SC at Nanyang Technological University, Singapore, Prof. Jiang MJ, Prof. Zhao JH, Prof. Fan W, Prof. Fung XD, Prof. Zhou XP, Prof. Yang XL, Prof. Liao HJ, Prof. Zhu XR, Prof. Chen CF, Dr. Zhang LY, Dr. Zhang SQ, et al. In addition, acknowledgement is made to Prof. Bingfeng Yu, Vice President of the Science and Technology Council, IIR and Director of Xi'an Jiaotong University Library and Dr. Maozheng Yu, Professor of the School of Energy and Power Engineering, Xi'an Jiaotong University for their kind discussions and suggestions. I would like to express my sincere thanks to the editors of Springer and Zhejiang University Press for their excellent editorial work on our manuscript.

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Contents

1

Introduction

1.1 Background

Plasticity is one of the underlying principles in the design of structures, especially metal and reinforced concrete structures. Numerous textbooks and monographs on structural plasticity and plastic design have been published since the 1950s (Baker et al., 1956; Baker, 1956; Neal, 1956; Heyman, 1958; 1971; Hodge, 1959; 1963; Horne, 1964; 1978; Baker and Heyman, 1969; Save and Massonnet, 1972; Chen, 1975; 1982; Morris and Randall, 1979; Horne and Morris, 1981; Zyczkowski, 1981; König and Maier, 1981; König, 1987; Mrazik et al., 1987; Save et al., 1997; Nielsen, 1999). The European Recommendations for the design of steelwork and reinforced concrete structures apply widely the plastic behavior of materials (Horne and Morris, 1981).

The advantages of the plastic method for structural analysis were discussed by Massonnet, Beedle, Heyman and Chen as follows: "The method of plastic design represents reality better than the conventional elastic method; it must lead to better proportioned and more economical structures" (Heyman, 1960). "For plastic design to represent reality means that the collapse load computed from plastic theory can be closely observed in practice" (Heyman, 1960).

"Engineers and research workers have been stimulated to study the plastic strength of steel structures and its application to design for three principal reasons: (a) it has a more logical design basis; (b) it is more economical in the use of steel, and (c) it represents a substantial saving of time in the design" (Beedle, 1960).

"The calculation of load-carrying capacity by use of the limit theorems is much easier than the calculation of stress. Answers obtained are not only physically more meaningful but also simpler. The simplicity of limit analysis opens the way to limit design, to direct design as contrasted with the trial-and-error procedure normally followed in conventional design." "The estimation of the collapse load is of great value, not only as a simple check for