

ADVANCED TOPICS IN SCIENCE AND TECHNOLOGY IN CHINA

Maohong Yu
Guowei Ma
Jianchun Li

Structural Plasticity

Limit, Shakedown and Dynamic
Plastic Analyses of Structures



ZHEJIANG UNIVERSITY PRESS
浙江大学出版社



Springer

Maohong Yu
Guowei Ma
Jianchun Li

Structural Plasticity Limit, Shakedown and Dynamic Plastic Analyses of Structures

With 238 figures

 ZHEJIANG UNIVERSITY PRESS
浙江大学出版社

 Springer

图书在版编目 (CIP) 数据

结构塑性力学=Structural Plasticity:英文 / 俞茂宏, 马国伟, 李建春著. —杭州:浙江大学出版社, 2009.3

ISBN 978-7-308-06168-1

I. 结… II. ①俞…②马…③李… III. 结构力学—英文②塑性力学—英文 IV. 034

中国版本图书馆 CIP 数据核字 (2008) 第 212849 号

Not for sale outside Mainland of China

此书仅限中国大陆地区销售

结构塑性力学

俞茂宏 马国伟 李建春 著

责任编辑	尤建忠
封面设计	Frido Steinen-Broo
出版发行	浙江大学出版社 网址: http://www.zjupress.com Springer-Verlag GmbH 网址: http://www.springer.com
排版	林智广告公司
印刷	杭州富春印务有限公司
开本	787mm×960mm 1/16
印张	25.25
字数	834 千
版印次	2009 年 3 月第 1 版 2009 年 3 月第 1 次印刷
书号	ISBN 978-7-308-06168-1 (浙江大学出版社) ISBN 978-3-540-88151-3 (Springer-Verlag GmbH)
定价	150.00 元

版权所有 翻印必究 印装差错 负责调换

浙江大学出版社发行部邮购电话 (0571)88925591

Preface

Structural Plasticity: Limit, Shakedown and Dynamic Plastic Analyses of Structures is the second monograph on plasticity. The others are *Generalized Plasticity* (Springer Berlin Heidelberg, 2006) and *Computational Plasticity* (forthcoming Zhejiang University Press Hangzhou and Springer Berlin Heidelberg, 2009) with emphasis on the application of the unified strength theory.

Generalized Plasticity, the first monograph on plasticity in this series, covers both traditional plasticity for metals (non-SD materials) and plasticity for geomaterials (SD materials). It describes the unified slip line theory for plane strain problems and characteristics theory for plane stress and axisymmetric problems, as well as the unified fracture criterion for mixed cracks. *Generalized Plasticity* can be used for either non-SD materials or SD materials. The second one is *Structural Plasticity: Limit, Shakedown and Dynamic Plastic Analyses of Structures*, which deals with limit analysis, shakedown analysis and dynamic plastic analyses of structures using the analytical method. The third one is *Computational Plasticity*, in which numerical methods are applied. The advances in strength theories of materials under complex stress are summarized in the book *Unified Strength Theory and Its Applications* (Springer Berlin Heidelberg, 2004).

The elastic and plastic limit analysis and shakedown analysis for structures can provide a very useful tool for the design of engineering structures. Conventionally, the Tresca yield criterion, the Huber-von Mises yield criterion, the maximum principal stress criterion and the Mohr-Coulomb criterion are applied in elastic-plastic limit analysis and shakedown analysis of structures. However, the result from each of the criteria above is a single solution suitable only for one kind of material. Only one or two principal stresses are taken into account in the maximum principal stress criterion, the Tresca criterion and the Mohr-Coulomb criterion. In addition, the Huber-von Mises criterion is inconvenient to use because of its nonlinear mathematical expression.

In the last decade more general solutions of plastic limit analysis and shakedown analysis for structures with a new unified strength theory have been presented. A series of unified solutions using the unified strength theory have been given. Unified plastic limit solutions of structures were presented in the literature, including unified solutions for circular plates, annular plates, oblique plates, rhombus plates, rectangular plates and square plates, orthogonal circular plates, thin plates with a hole, rotating discs and cylinders. So did unified solutions for the shakedown limit of pressure vessels, circular plates and rotating discs and for the dynamic plastic behavior of circular plates under soft impact. These unified solutions encompass not only the Tresca solution and the Mohr-Coulomb solution as special cases, but also a series of new solutions. The Huber-von Mises solution can also be approximated by the unified solution. The unified solution is a systematical one covering all results from a lower result to an upper result. These results can be suitable for a wide range of materials and engineering structures.

As an example, the unified solution of the limit load for oblique plates ($\theta = \pi/3$, $l_1 = 2l_2$) is illustrated in Fig.0.1. It can be seen from the figure that the limit load q can be obtained for various oblique plates with different angles and length and for various materials with a different strength ratio in tension and in compression and for various failure criteria with different parameter b .

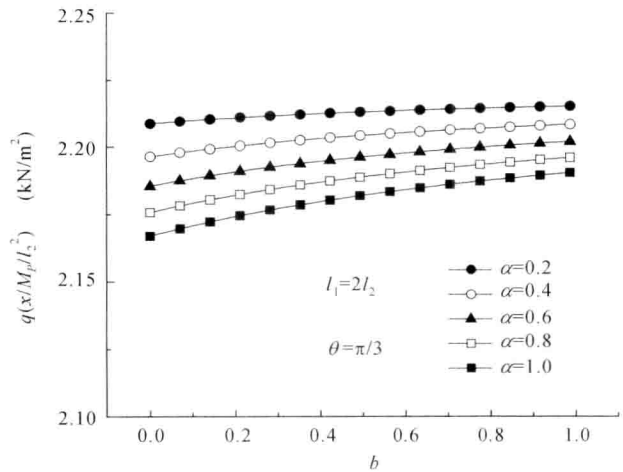


Fig. 0.1. Limit loads of oblique plate ($\theta = \pi/3$, $l_1 = 2l_2$) for different materials

The solution with $b = 0$ is the same as the solution of the Mohr-Coulomb material, and the solution with $b = 0$ and $\alpha = 1.0$ is the same for the Tresca material. The solution with $b = 1.0$ is for the generalized twin-shear criterion and the solution with $b = \alpha = 1.0$ is the solution of the twin-

shear stress criterion. Other serial solutions between the single-shear theory (Tresca-Mohr-Coulomb theory) and the twin-shear theory are new solutions for different materials. Therefore the unified solution can be adopted for more materials and structures. It can be noted that all the solutions for the bearing capacity of structures with $b > 0$ are higher than those with the Tresca or Mohr-Coulomb criterion. The application of the unified solution is economical in the use of materials and energy. The other example is the determination of the limit pressure and thickness of pressure vessels in design. The relationship between limit pressure and wall thickness of a thin-walled vessel with the unified strength theory parameter b is shown in Fig.0.2 and Fig.0.3 respectively.

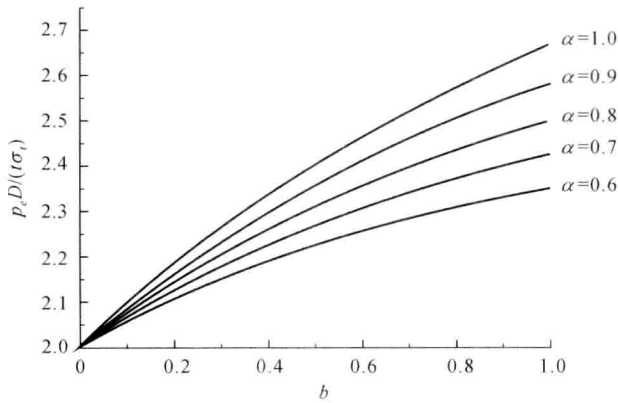


Fig. 0.2. Limit pressure versus unified strength theory parameter b

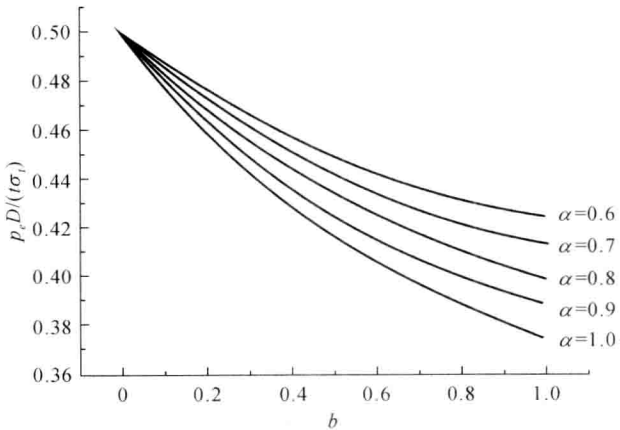


Fig. 0.3. Wall thickness versus unified strength theory parameter b

It can be seen that

- The conventional solution is a single solution ($b = 0$ in Fig.0.2 and Fig.0.3), which can be adopted only for one kind of material. The new solution is a unified solution, a serial solution, which can be adopted for more materials and structures.
- The solution for Tresca material ($b = 0$ and $\alpha = 1.0$) is identical to the solution for the Mohr-Coulomb material ($b = 0$ and $\alpha \neq 1.0$). It appears that the SD effect of materials ($\alpha \neq 1.0$ material) cannot be considered by the Mohr-Coulomb strength theory in this case.
- All the solutions for the bearing capacity of structures with $b > 0$ are higher than those for the Tresca-Mohr-Coulomb criterion. All the solutions for the wall thickness of a pressure vessel with $b > 0$ are lower than the solution using the conventional Tresca criterion or Mohr-Coulomb criterion.
- The applications of the unified strength theory and the unified solutions are more economical in the use of materials and the use of energy, leading to a reduction in environmental pollution.
- The wider application of the enhancement-factor concept on a global scale is, on the one hand, going to bring tremendous energy saving and pollution mitigation. It calls, on the other hand, for a theoretical support on which the concept can be based. Engineering practice in general has a desire to have a new strength theory, which should be more rational and more consistent with the experimental data than what can be achieved by using the Tresca-Mohr-Coulomb single-shear strength theory.

A series of the unified solutions for various structures are described in this book. It is organized as:

Chapters 2~4 give a brief introduction to the fundamental stress state, yield function and limit analysis theorem.

Chapters 5~9 deal with plastic limit analyses for circular plates, annular plates, oblique plates, rhombus plates, rectangular plates, square plates and cylinders by using the unified strength theory.

In Chapters 10, 11, the unified solutions of the dynamic plastic analysis of plates, and the limit velocity of rotating discs and cylinders are emphasized.

Penetration, wellbore analyses and orthogonal circular plates are presented in Chapters 12~14.

Chapters 15~17 are devoted to the shakedown theorem and shakedown analysis of pressure vessels, simply supported and clamped circular plates and rotating discs, using the unified strength theory. Brief summaries and references are given at the end of each chapter.

The unified strength theory and unified solutions provide a fundamental theory for the application of strength design of engineering structures. They can also be used for increasing the admissible loads or decreasing the cross-sections and the weight of structures. This results in a reduction in materials

and energy consumption and a reduction in environmental pollution and the cost of structures.

The applications of the unified strength theory in plastic limit analysis and shakedown analysis for different structures are still developing. This book summarizes the research results obtained up to now. It is expected that the unified strength theory will have more and more applications in the future in addition to the plate and cylindrical structures discussed in this book. The applications of the unified strength theory in computational analysis are still growing. We hope that the Chinese idiom “Throwing out a brick to attract a piece of jade” becomes real and this book can serve as a solid brick.

The results of bearing capacity and shakedown loads obtained by using various yield criteria are very different. The results are influenced strongly by the selection of the yield criterion. We need to use a new efficient criterion. The straight-line segments on the unified strength theory make it convenient for analytical treatment of plasticity problems. The unified strength theory provides us with a very effective approach to studying the effect of yield criterion for various engineering problems. The serial results can be appropriate for most materials, from metallic materials to geomaterials.

Appreciation must be expressed for the support of the China Academy of Launch Vehicle Technology in Beijing, the Aircraft Strength Research Institute of China in Xi'an, the MOE Key Lab for Strength and Vibration at Xi'an Jiaotong University, Nanyang Technological University in Singapore, Springer and Zhejiang University Press.

I am pleased to acknowledge Dr. Guowei Ma, Dr. Jianchun Li, Dr. Fang Wang, Dr. Junhai Zhao, Dr. Xueying Wei, Dr. Yanping Wang, Dr. Shuanqiang Xu and others for their research in plastic analysis and design using the unified yield criterion and the unified strength theory. A series of unified solutions for various structures were presented. The pioneering work on the unified solution of the static and dynamic limit-bearing capacity of structures for non-SD materials using the unified yield criterion was performed by Dr. Ma during the period 1993 to 1999. The work on the unified solution of static and dynamic limit analysis of structures for SD materials was conducted by Drs. Li, Wei, Wang, Zhao, Fan and others. The work of unified solutions of shakedown for cylinders, rotating discs and circular plates was carried out by Prof. Xu Shuanqiang and Dr. Li.

I am also indebted to thank many other researchers for their research on the unified solution of structures in the fields of soil mechanics, rock mechanics, concrete mechanics and computational mechanics. I would like to express sincere thanks to Academician Shen ZJ, Academician Yang XM, Academician Chen SY, Academician Shen ZY, Academician Chen HQ, Prof. Fan SC at Nanyang Technological University, Singapore, Prof. Jiang MJ, Prof. Zhao JH, Prof. Fan W, Prof. Fung XD, Prof. Zhou XP, Prof. Yang XL, Prof. Liao HJ, Prof. Zhu XR, Prof. Chen CF, Dr. Zhang LY, Dr. Zhang SQ, et al. In addition, acknowledgement is made to Prof. Bingfeng Yu, Vice President

of the Science and Technology Council, IIR and Director of Xi'an Jiaotong University Library and Dr. Maozheng Yu, Professor of the School of Energy and Power Engineering, Xi'an Jiaotong University for their kind discussions and suggestions. I would like to express my sincere thanks to the editors of Springer and Zhejiang University Press for their excellent editorial work on our manuscript.

Maohong Yu
Singapore

Contents

1	Introduction	1
1.1	Background	1
1.2	Unification of Yield and Strength Criteria	3
1.3	Plastic Limit Analysis	4
1.4	Plastic Limit Analysis of Rotating Solids	6
1.5	Shakedown Analysis of Structures	7
1.6	Plastic Limit Analysis Based on the Unified Strength Theory	8
1.7	Summary	9
	References	9
2	Fundamental Concepts of Stress and Strain	16
2.1	Stress Components and Invariants	16
2.2	Deviatoric Stress Tensor and the Tensor Invariants	18
2.3	Principal Shear Stresses	19
2.4	Octahedral Shear Stress	21
2.5	Strain Components	22
2.6	Equations of Equilibrium	24
2.7	Generalized Hooke's Law	24
2.8	Compatibility Equations	25
2.9	Governing Equations for Plane Stress Problems	26
2.10	Governing Equations in Polar Coordinates	27
2.11	Bending of Circular Plate	29
2.12	Summary	31
	References	31
3	Yield Condition	32
3.1	Introduction	32
3.2	Conventional Yield Criteria	33
3.2.1	Maximum Normal Stress Criterion	33
3.2.2	Maximum Shear Stress-based Criteria — Single-shear Theory	33

3.2.3	Octahedral Shear Stress-based Criteria—Three-shear Theory	36
3.2.4	Twin-shear Stress-based Criterion—Twin-shear Theory	40
3.3	Unified Yield Criterion for Metallic Materials (Non-SD Materials)	44
3.4	Unified Strength Theory for SD Materials	45
3.4.1	Mechanical Model of the Unified Strength Theory	46
3.4.2	Mathematical Modelling of the Unified Strength Theory	47
3.4.3	Mathematical Expression of the Unified Strength Theory	48
3.4.4	Yield Surfaces and Yield Loci of the Unified Strength Theory	48
3.5	Significance of the Unified Strength Theory	49
3.6	Unified Strength Theory in the Plane Stress State	52
3.6.1	$\sigma_1 \geq \sigma_2 > 0, \sigma_3 = 0$	53
3.6.2	$\sigma_1 \geq 0, \sigma_2 = 0, \sigma_3 < 0$	53
3.7	Summary	55
3.8	Problems	56
	References	61
4	Theorems of Limit Analysis	64
4.1	Introduction	64
4.2	Perfectly Plastic Solid	66
4.3	Power of Dissipation	66
4.4	Lower-bound Theorem	67
4.5	Upper-bound Theorem	68
4.6	Fundamental Limit Theorems	68
4.7	Important Remarks	69
4.7.1	Exact Value of the Limit Load (Complete Solution) ...	69
4.7.2	Elastic-plastic and Rigid-plastic Bodies	69
4.7.3	Load-bearing Capacity	70
4.7.4	Uniqueness	70
	References	71
5	Plastic Limit Analysis for Simply Supported Circular Plates	74
5.1	Introduction	74
5.2	Basic Equations of Circular Plate	75
5.3	Unified Solutions of Simply Supported Circular Plate for Non-SD Materials	76
5.3.1	Uniformly Distributed Load	78
5.3.2	Arbitrary Axisymmetrical Load	80
5.4	Unified Solutions of Simply Supported Circular Plate for SD Materials	95
5.4.1	Partial-uniform Load	97

5.4.2	Linearly Distributed Load	102
5.5	Summary	107
5.6	Problems	108
	References	110
6	Plastic Limit Analysis of Clamped Circular Plates	112
6.1	Introduction	112
6.2	Unified Solutions of Clamped Circular Plate for Non-SD Materials	112
6.2.1	Uniformly Distributed Load	112
6.2.2	Arbitrary Loading Radius	117
6.2.3	Arbitrary Loading Distribution	122
6.3	Unified Solutions of Clamped Circular Plate for SD Materials	127
6.4	Summary	133
6.5	Problems	134
	References	134
7	Plastic Limit Analysis of Annular Plate	136
7.1	Introduction	136
7.2	Basic Equations for Annular Plate Based on UYC	137
7.2.1	Case (1)	140
7.2.2	Case (2)	141
7.2.3	Special Case	142
7.3	Unified Solutions of Annular Plate for Non-SD Materials	142
7.4	Unified Solutions of Limit Load of Annular Plate for SD Materials	145
7.4.1	Unified Strength Theory	145
7.4.2	Basic Equations for Annular Plate Based on the UST .	146
7.4.3	Limit Analysis	147
7.4.4	Results and Discussions	149
7.5	Summary	150
7.6	Problems	152
	References	152
8	Plastic Limit Analyses of Oblique, Rhombic, and Rectangular Plates	154
8.1	Introduction	154
8.2	Equations for Oblique Plates	156
8.2.1	The Equilibrium Equation in Ordinary Coordinate System	156
8.2.2	Field of Internal Motion	158
8.2.3	Moment Equation Based on the UST	158
8.3	Unified Solution of Limit Analysis of Simply Supported Oblique Plates	159
8.4	Limit Load of Rhombic Plates	162

8.5	Limit Load of Rectangular Plates	163
8.6	Unified Limit Load of Square Plates	168
8.7	Tabulation of the Limit Load for Oblique, Rhombic and Square Plates	169
8.8	Summary	170
8.9	Problems	172
	References	173
9	Plastic Limit Analysis of Pressure Vessels	175
9.1	Introduction	175
9.2	Unified Solution of Limit Pressure of Thin-walled Pressure Vessel	176
9.3	Limit Pressure of Thick-walled Hollow Sphere	180
9.3.1	Elastic Limit Pressure of Thick-walled Sphere Shell ...	181
9.3.2	Plastic Limit Pressure of Thick-walled Sphere Shell ...	182
9.4	Unified Solution of Elastic Limit Pressure of Thick-walled Cylinder	184
9.5	Unified Solution of Plastic Limit Pressure of Thick-walled Cylinder	190
9.5.1	Stress Distribution	190
9.5.2	Plastic Zone in the Elasto-plastic Range	192
9.5.3	Plastic Zone Radius in the Elasto-plastic Range	193
9.5.4	Plastic Limit Pressure	193
9.6	Summary	198
9.7	Problems	200
	References	202
10	Dynamic Plastic Response of Circular Plate	205
10.1	Introduction	205
10.2	Dynamic Equations and Boundary Conditions of Circular Plate	206
10.2.1	First Phase of Motion ($0 \leq T \leq \tau$)	209
10.2.2	Second Phase of Motion ($\tau \leq t \leq T$)	212
10.3	Static and Kinetic Admissibility	214
10.4	Unified Solution of Dynamic Plastic Response of Circular Plate	216
10.5	Special Cases of the Unified Solutions	221
10.6	Summary	229
	References	229
11	Limit Angular Speed of Rotating Disc and Cylinder	231
11.1	Introduction	231
11.2	Elastic Limit of Discs	232
11.3	Elasto-plastic Analysis of Discs	233
11.4	Elasto-plastic Stress Field of Rotating Disc	236

11.5	Solution Procedure and Results	238
11.6	Unified Solution of Plastic Limit Analysis of Rotating Cylinder	239
11.7	Limit Analysis of a Solid Disc with Variable Thickness	242
11.8	Limit Analysis of an Annular Disc with Variable Thickness ..	246
11.8.1	Case (1) ($1 > \beta \geq \beta_0$)	247
11.8.2	Case (2) ($0 < \beta \leq \beta_0$)	248
11.9	Special Case of $b = 0$	251
11.10	Results and Discussion	252
11.11	Summary	258
11.12	Problems	258
	References	259
12	Projectile Penetration into Semi-infinite Target	261
12.1	Introduction	261
12.2	Spatial Axisymmetric Form of Unified Strength Theory	262
12.3	Fundamental Equations for Concrete Targets	263
12.3.1	Conservation Equations	263
12.3.2	Relation between Pressure and Bulk Strain	264
12.3.3	Failure Criterion Expressed by σ_r and σ_θ	264
12.3.4	Interface Conditions	265
12.4	Cylindrical Cavity Expansion Analysis	265
12.4.1	Elastic Zone ($c_1 t \leq r \leq c_d t$, $\beta_1/\beta \leq \xi \leq 1/\alpha$)	266
12.4.2	Interface of Elastic-cracked Zones ($r = c_1 t$, $\xi = \beta_1/\beta$) ..	270
12.4.3	Radial Cracked Zone ($ct \leq r \leq c_1 t$, $1 \leq \xi \leq \beta_1/\beta$)	271
12.4.4	Interface of the Plastic and Cracked Zones ($r = ct$, $\xi = 1$)	273
12.4.5	Plastic Zone ($v_r t \leq r \leq ct$, $\delta \leq \xi \leq 1$)	275
12.5	Cavity Expansion Pressure and Velocity	276
12.5.1	Incompressible Material	277
12.5.2	Compressible Material	279
12.6	Penetration Resistance Analysis	284
12.7	Analysis and Verification of Penetration Depth	288
12.8	Summary	290
	References	291
13	Plastic Analysis of Orthogonal Circular Plate	293
13.1	Introduction	293
13.2	Orthotropic Yield Criteria	293
13.3	General Solutions	296
13.4	Simply Supported Orthotropic Circular Plate	301
13.4.1	Case I: Point A' Falls on Segment KL	301
13.4.2	Case II: Point A' Falls on Segment LA	302
13.4.3	Case III: Point A' Falls on Segment AB	302
13.4.4	Case IV: Point A' Falls on Segment BC	302

13.4.5	Moment, Velocity Fields and Plastic Limit Load	303
13.5	Fixed Supported Circular Plate	307
13.5.1	Case I: Point A' Falls on Segment KL	307
13.5.2	Case II: Point A' Falls on Segment LA	307
13.5.3	Case III: Point A' Locates on Segment AB	307
13.5.4	Case IV: Point A' Falls on Segment BC'	308
13.5.5	Moment Fields, Velocity Fields, and Plastic Limit Load	308
13.6	Summary	312
	References	313
14	Unified Limit Analysis of a Wellbore	314
14.1	Introduction	314
14.2	Unified Strength Theory	315
14.3	Equations and Boundary Conditions for the Wellbore	316
14.3.1	Strength Analysis for Wellbore	316
14.3.2	Pore Pressure Analysis	318
14.4	Elastic and Plastic Analysis	318
14.4.1	Elastic Phase	318
14.4.2	Plastic Limit Pressure	319
14.4.3	Elastic-plastic Boundary	320
14.4.4	Example	321
14.4.5	Limit Depth for Stability of a Shaft	322
14.5	Summary	324
14.6	Problems	324
	References	325
15	Unified Solution of Shakedown Limit for Thick-walled Cylinder	327
15.1	Introduction	327
15.2	Shakedown Theorem	329
15.2.1	Static Shakedown Theorem (Melan's Theorem)	329
15.2.2	Kinematic Shakedown Theorem (Koiter Theorem)	329
15.3	Shakedown Analysis for Thick-walled Cylinders	330
15.4	Unified Solution of Shakedown Pressure of Thick-walled Cylinders	334
15.5	Connection between Shakedown Theorem and Limit Load Theorem	336
15.6	Shakedown Pressure of a Thick-walled Spherical Shell	339
15.7	Summary	340
15.8	Problems	340
	References	341

16	Unified Solution of Shakedown Limit for Circular Plate . . .	344
16.1	Introduction	344
16.2	Unified Solution of Shakedown Limit for Simply Supported Circular Plate	345
16.2.1	Elastic State	345
16.2.2	Elastic-plastic State	346
16.2.3	Completely Plastic State	348
16.2.4	Shakedown Analysis	348
16.2.5	Discussion	349
16.3	Unified Solution of Shakedown Limit for Clamped Circular Plate	350
16.3.1	Elastic State	350
16.3.2	Elastic-plastic State	350
16.3.3	Completely Plastic State	351
16.3.4	Shakedown Analysis	351
16.3.5	Discussion	352
16.4	Comparison between Shakedown Solution and Limit Results .	353
16.5	Summary	354
16.6	Problems	354
	References	355
17	Shakedown Analysis of Rotating Cylinder and Disc	357
17.1	Introduction	357
17.2	Elasto-plastic and Shakedown Analyses of Rotating Cylinder and Disc	358
17.2.1	Elastic Analyses of Hollow Rotating Circular Bars	358
17.2.2	Elasto-plastic Analyses of Hollow Rotating Circular Bars	360
17.2.3	Shakedown Analyses of Hollow Rotating Circular Bars	362
17.2.4	Elasto-plasticity and Shakedown of Solid Rotating Circular Bars	364
17.3	Summary of Elasto-plasticity and Shakedown Analyses of Rotating Circular Bars	366
17.4	Elasto-plastic and Shakedown Analyses of Rotating Disc . . .	367
17.4.1	Elastic Analyses of Hollow Rotating Discs	368
17.4.2	Plastic Limit Analyses of Hollow Rotating Discs	371
17.4.3	Shakedown of Hollow Rotating Discs	373
17.5	Elasto-plastic and Shakedown Analyses of Solid Rotating Discs	376
17.6	Summary of Elastic and Plastic Analyses of Rotating Discs . .	377
17.7	Summary	378
	References	379
	Index	381

Introduction

1.1 Background

Plasticity is one of the underlying principles in the design of structures, especially metal and reinforced concrete structures. Numerous textbooks and monographs on structural plasticity and plastic design have been published since the 1950s (Baker et al., 1956; Baker, 1956; Neal, 1956; Heyman, 1958; 1971; Hodge, 1959; 1963; Horne, 1964; 1978; Baker and Heyman, 1969; Save and Massonnet, 1972; Chen, 1975; 1982; Morris and Randall, 1979; Horne and Morris, 1981; Zyczkowski, 1981; König and Maier, 1981; König, 1987; Mrazik et al., 1987; Save et al., 1997; Nielsen, 1999). The European Recommendations for the design of steelwork and reinforced concrete structures apply widely the plastic behavior of materials (Horne and Morris, 1981).

The advantages of the plastic method for structural analysis were discussed by Massonnet, Beedle, Heyman and Chen as follows: “The method of plastic design represents reality better than the conventional elastic method; it must lead to better proportioned and more economical structures” (Heyman, 1960). “For plastic design to represent reality means that the collapse load computed from plastic theory can be closely observed in practice” (Heyman, 1960).

“Engineers and research workers have been stimulated to study the plastic strength of steel structures and its application to design for three principal reasons: (a) it has a more logical design basis; (b) it is more economical in the use of steel, and (c) it represents a substantial saving of time in the design” (Beedle, 1960).

“The calculation of load-carrying capacity by use of the limit theorems is much easier than the calculation of stress. Answers obtained are not only physically more meaningful but also simpler. The simplicity of limit analysis opens the way to limit design, to direct design as contrasted with the trial-and-error procedure normally followed in conventional design.” “The estimation of the collapse load is of great value, not only as a simple check for