Study Guide

Jay Davis

TRIGONOMETRY

Margaret L. Lial/Charles D. Miller

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This Study Guide is dedicated to
Paula, Joel, Kama and Meara.

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Study Guide

TRIGONOMETRY

TO THE STUDENT

To use this Study Guide, proceed as follows.

1. Read the text Trigonometry by Lial and Miller, and go to class (if you are enrolled in one). Participate in class discussions.
2. If you feel you already know some or all of the material in a new chapter, try taking the Pretest. Check your answers carefully.
3. Turn to the Objectives you missed on the Pretest. (These are indicated in the left-hand column next to the Pretest.) Cut a strip of heavy cardboard or paper to cover the right-hand columns of answers and hints. Uncover them only after you have done your best to answer the questions yourself. Then check your results.
4. If they aren't in agreement and you don't understand the answer on the right, then carefully rework the appropriate sections of the text. If you still are having trouble, see your instructor or tutor or seek help in the mathlab, if your school has one.
5. When you are able to complete a unit in the Study Guide, go back to the text and work the assigned problem in the appropriate sections. Check your answers in the back of the text or in the Solutions Guide.
6. At the end of each chapter, test yourself by taking the Posttest in the guide or the Chapter Test in the text. Check your answers carefully, and rework the sections where you missed problems. Then take the other test and check your answers again. You should now be ready for the instructor's test.

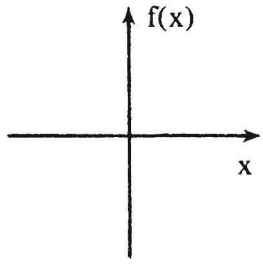
CONTENTS

UNIT 1 THE TRIGONOMETRIC FUNCTIONS

(See Chapter 1, pages 1-26, in Trigonometry.)

See Objective	PRETEST
1.1 #1	1. $\|7\| + \|-7\| =$ _____
1.1 #7	2. In which quadrant does (8,-3) lie?
1.1 #13	3. If the point (x,y) is in Quadrant II, determine the signs for x and y.
1.1 #20	4. If the point (x,y) is in Quadrant III and r is a positive number, determine if the ratio r/x is positive or negative.
1.1 #21	5. Find the distance from (3,8) to (-2,5).
1.1 #24	6. If $f(x) = 3x^2 - x + 5$, find the value of f(2).
1.1 #27	7. If $f(x) = 3x - 2$, find the values of f(x) for x = -4, -2, 0, 1, 2, and 4. Draw the graph of f(x).

x	f(x)
-4	
-2	
0	
1	
2	
4	



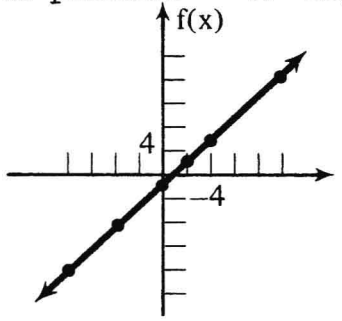
1.1 #29	8. Find the domain and range of $y = x^2 + 4$. Is this a function?
1.1 #31	9. Find the domain of $y = \frac{3}{x + 2}$.
1.2 #1	10. Find the angle of smallest possible positive measure coterminal with 370°.
1.2 #7	11. Find the measure of two angles, one positive and one negative, that are coterminal with 80°.
1.2 #11	12. Let (3,3) be a point on the terminal side of θ which is an angle in standard position. Measure θ with a protractor.
1.2 #15	13. A boat propeller rotates 500 times per minute. Find the

		number of degrees that a point on the tip of the propeller blade will rotate in 1 second.
1.3	#5	14. Evaluate $\cos 90^\circ + 3 \sec 90^\circ$.
1.3	#11	15. Find the values of the six trigonometric functions of θ if θ is an angle in standard position and (-4,3) is on its terminal side.
1.4	#18	16. Is it possible for $\sin \theta = -3$?
1.4	#23	17. If $\emptyset$ is in Quadrant III and $\tan \emptyset = 3/5$, find the values of the other five trigonometric functions of $\emptyset$.

Answers to Pretest

1. 14 2. Quadrant IV 3. x is negative, y is positive 4. negative
5. $\sqrt{25 + 9} = \sqrt{34}$ 6. 15 7.

x	f(x)
-4	-14
-2	-8
0	-2
1	1
2	4
4	10



8. domain is all real numbers; range is all real numbers ≥ 4, yes 9. all real numbers except $x = -2$ 10. 10° 11. 440°, -280° (other answers are possible) 12. 45° 13. 3000° 14. not defined 15. $\sin \theta = 3/5$; $\cos \theta = -4/5$; $\tan \theta = 3/-4$; $\csc \theta = 5/3$; $\sec \theta = 5/-4$; $\cot \theta = -4/3$ 16. no
17. $\sin \emptyset = -3/\sqrt{34}$; $\cos \emptyset = -5/\sqrt{34}$; $\cot \emptyset = 5/3$; $\sec \emptyset = -\sqrt{34}/5$; $\csc \emptyset = \sqrt{34}/(-3)$

1.1 BASIC TERMS (pages 1-7, in Trigonometry.)

Objectives: Upon completing this section, you should be able to

- work with absolute values (#1-6)
- determine the quadrant of a point (#7-12)
- determine signs of coordinates (#13-20)
- compute the distance between points (#21-23)
- use functional notation (#24-26)
- plot points and functions (#27-28)
- determine domains and ranges (#29-33)

Complete each of the following.

1. $|7| + |-7| = \underline{7} + \underline{7} = \underline{14}$

   The distance from 0 to 7 and the distance from 0 to -7 are both 7, so the absolute value of 7 and the absolute value of -7 are both 7.

2. $|7| + |-5| = 7 + \_\_\_ = \_\_\_\_$ — 5, 12

3. $|-6| + 4 = \_\_\_ + 4 = \_\_\_\_$ — 6, 10

4. $|-5| - |-4| = \_\_\_ - \_\_\_ = \_\_\_$ — 5, 4, 1

5. $|8 - 9| + |-3| = |-1| + \_\_\_$ — 3

   $= \_\_\_ + \_\_\_ = \_\_\_$ — 1, 3, 4

6. $|3 - (-2)| - |-7 - (-5)| = |5| - |\_\_\_\_|$ — -2

   $= \_\_\_ - \_\_\_ = \_\_\_$ — 5, 2, 3

In Problems 7-11, determine the quadrant in which each point lies.

7. (8,-3) is in Quadrant ____. — IV

8. (-2,-1) is in Quadrant _____. — III

9. (7,0) is in Quadrant none.

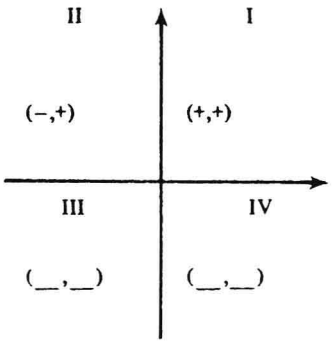
   This point is on an axis, and not in a Quadrant.

10. (7,4) is in Quadrant ____. — I

11. (0,-8) is in Quadrant _____. — none

12. (2,5) is in Quadrant ___. — I

13. Complete the following figure to give the sign of each coordinate for points in the four quadrants.



Using the figure above, find the sign of each coordinate if the point is located in the given quadrant.

14. Quadrant II: x is ___, y is ___. −, +

15. Quadrant I: x is ___, y is ___. +, +

16. Quadrant III: x is ___, y is ___. −, −

17. Quadrant IV: x is ___, y is ___. +, −

Assume r is a positive number and point (x,y) is in the indicated quadrant. Decide if the given ratio is positive (+) or negative (−).

18. In Quadrant II, $\frac{x}{y} = \frac{(\ )}{+} = (\quad)$ $\frac{(-)}{+}$, (−)

19. In Quadrant III, $\frac{y}{x} = \frac{(\ )}{(\ )} = (\quad)$ $\frac{(-)}{(-)}$, (+)

20. In Quadrant III, $\frac{y}{r} = \frac{(\ )}{(\ )} = (\quad)$ $\frac{(-)}{(+)}$, (−)

Find the distance, d, between the following pairs of points.

21. (-2,5) and (3,8)

Let $(x_1,y_1) = (-2,5)$ and $(x_2,y_2) = (3,8)$. The distance formula says $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Therefore

$$d = \sqrt{(3 - (-2))^2 + (8 - 5)^2}$$
$$= \sqrt{5^2 + 3^2}$$
$$= \sqrt{25 + 9}$$
$$= \sqrt{\quad}$$ $\sqrt{34}$

22. (7,6) and (-1,4); d = _____ $\sqrt{68} = 2\sqrt{17}$

23. (-4,-5) and (-8,-12); d = _____ $\sqrt{65}$

Let $f(x) = 3x^2 - x + 5$. Find each of the following values.

24. $f(2) = 3(2)^2 - 2 + 5 = 12 - 2 + 5 =$ _____ 15

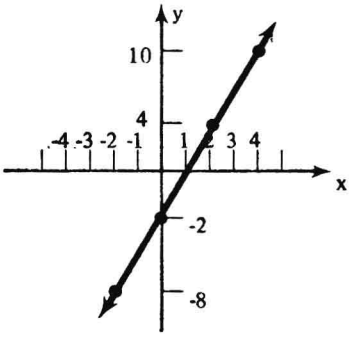
25. $f(3) = 3(\quad)^2 - (\quad) + 5 =$ _____ 3, 3, 29

26. $f(-2) = 3(\quad)^2 - (\quad) + 5 =$ _____ -2, -2, 19

For each of the following relations, replace x, in turn, with -4, -2, 0, 1, 2, 4. Use the results to graph the relations.

27. $f(x) = 3x - 2$

x	f(x)
-4	-14
-2	-8
0	-2
1	( )
2	( )
4	( )



1

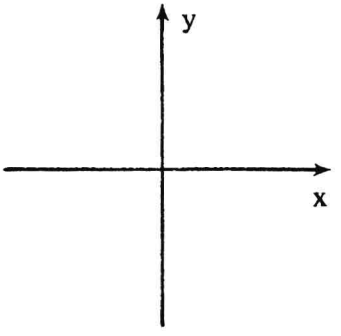
4

10

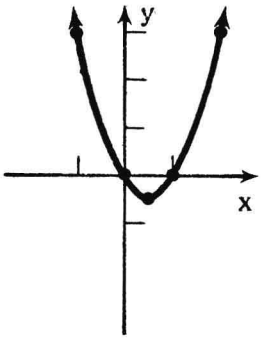
Is this relation a function? _____ (There is just one y-value for each x-value.) yes

28. $y = x^2 - 2x$

x	y
-4	( )
-2	( )
0	( )
1	( )
2	( )
4	( )



y
(24)
(8)
(0)
(-1)
(0)
(8)



Is this a function? _____ yes

Find the domain and range for each relation below. Identify those which are functions.

29. $y = x^2 + 4$

The domain is the set of all real numbers. Since $x^2 \geq 0$, $y \geq 4$ describes the range. This is a function, since there is only one y-value for each x-value.

30. $x = y^2 + 4$

domain = _____ — all real numbers ≥ 4

range = _____ — all real numbers

This (is)(is not) a function. — is not

Find the domain of the following relations.

31. $y = \frac{3}{x + 2}$

The domain is the set of all possible values of x. Since division by zero is not defined, -2 cannot be used as a replacement value for x. Therefore, the domain is the set of all real numbers x, such that $x \neq$ ____. — -2

32. $y = \frac{5}{x}$; domain = _____ — all real numbers x, such that $x \neq 0$

33. $y = \frac{7}{(x - 3)(x + 2)}$; domain = _____ — all real numbers x, such that $x \neq 3$ or -2

1.2 ANGLES (pages 8-14 in Trigonometry)

Objectives: Upon completing this section, you should be able to

- measure coterminal angles (#1-10)
- use a protractor (#11-14)
- convert from revolutions to degrees (#15-16)

Find the angles of the smallest possible positive measure coterminal with each of the following angles.

1. 370° — $370^\circ - (1 \cdot 360^\circ) = 10^\circ$

2. 840° — $840^\circ - (2 \cdot 360^\circ) = 840^\circ - 720^\circ =$ _____ — 120°

3. 900° — $900^\circ - (\_\_\_ \cdot 360^\circ) = 900^\circ -$ _____ = _____ — 12, 720°, 180°

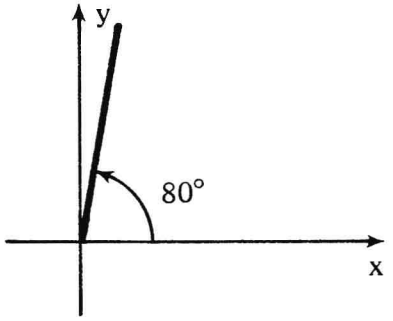
4. -55° — $-55^\circ + (1 \cdot 360^\circ) =$ _____ — 305°

5. -210° — $-210^\circ + (\_\_\_ \cdot 360^\circ) = -210^\circ +$ _____ = _____ — 1, 360°, 150°

6. -843° $\quad -843^\circ + (\_\_\_ \cdot 360^\circ) = -843^\circ + \_\_\_\_\_ = \_\_\_\_\_$ | 3, 1080°, 137°

Place the following angles in standard position. Draw an arrow to show the correct amount of rotation. Find the measure of two other angles, one positive and one negative, that are coterminal with the given angle.

7. 80°



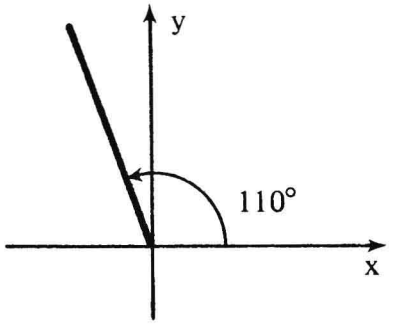
[An infinite number of answers are possible]

$80^\circ + 360^\circ = 440^\circ$ is a positive angle coterminal with 80°

$80^\circ - 360^\circ = -280^\circ$ is a negative angle coterminal with 80°

$80^\circ \pm n \cdot 360^\circ$ are also correct for any positive integer n

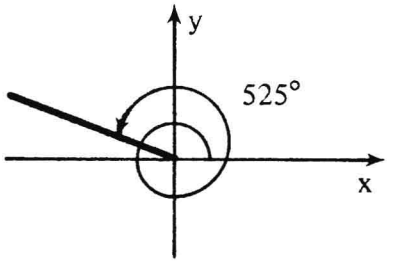
8. 110°



$110^\circ +$ _____ $=$ _____ is a positive angle coterminal with 110° | 360°, 470°

$110^\circ -$ _____ $=$ _____ is a negative angle coterminal with 110° | 360°, -250°

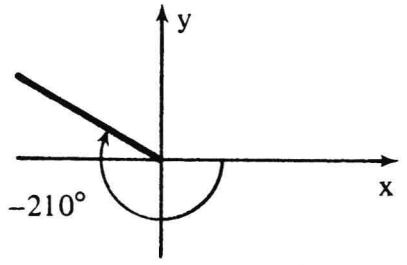
9. 525°



$525^\circ - \_\_\_ \cdot 360^\circ =$ _____ is a positive angle coterminal with 525° | 1, 165°

$525^\circ - \_\_\_ \cdot 360^\circ =$ _____ is a negative angle coterminal with 525° | 2, -195°

10. -210°

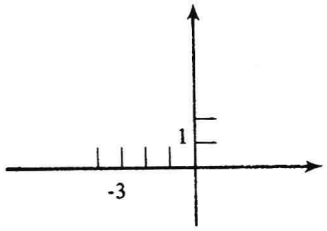


-210° + _____ = _____ is a positive angle coterminal with -210° — 360°, 150°

-210° - _____ = _____ is a negative angle coterminal with -210° — 360°, -570°

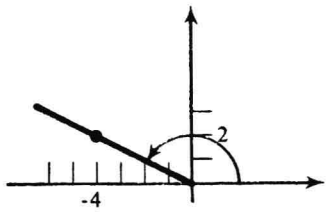
Locate the following points in a coordinate system. Draw a ray through the given point, starting at the origin. Use a protractor to measure the angle made by the positive x-axis and the ray that you drew.

11. (1,-3)



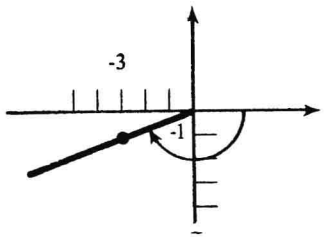
Locate the point in the coordinate system and draw a ray through the point. Measuring with a protractor the angle you have formed gives -71°.

12. (-4,2)



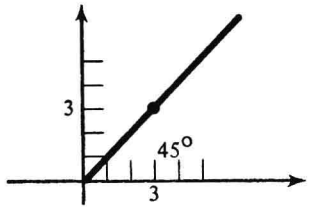
Measuring with a protractor we get _____. — 153°

13. (-3,-1)



Measuring with a protractor we get _____. — -162°

14. (3,3)



Measuring with a protractor we get ___. — 45°

15. A boat propeller rotates 500 times per minute. Find the number of degrees that a point on the tip of the propeller blade will travel in 1 second.

$$500\ \frac{\text{revolutions}}{\text{min}} \cdot \frac{1\ \text{min}}{60\ \text{sec.}} \cdot \frac{360^\circ}{1\ \text{revolution}} = \frac{500 \cdot 6^\circ}{\text{sec.}} =$$

$$\frac{3000^\circ}{\text{sec.}} = 3000^\circ \text{ per second}$$

16. A record rotates at $33\ \frac{1}{3}$ revolutions per minute. Change this RPM to degrees per second.

$$33\ \frac{1}{3}\ \frac{\text{revolutions}}{\text{min}} = \frac{100}{3}\ \frac{\text{revolutions}}{\text{min}}$$

$$\frac{100}{3}\ \frac{\text{revolutions}}{\text{min}} \cdot \frac{1\ \text{min}}{(\quad)\ \text{sec.}} \cdot \frac{(\quad)}{1\ \text{revolution}}$$

$$= \frac{(\quad)}{\text{sec.}}$$

360°, 200°

60

1.3 DEFINITION OF THE TRIGONOMETRIC FUNCTIONS (pages 15-19 in Trigonometry.)

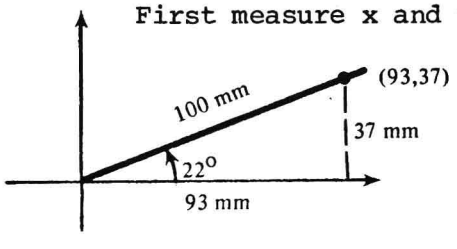
Objectives: Upon completing this section, you should be able to

- determine trigonometric values from protractor-drawn angles (#1-4)
- evaluate trigonometric functions of quadrantal angles (#5-10)
- find functional values (#11-14)

Use a protractor for each of the following angles. Pick a point on the terminal side of each angle to represent 100 mm. Construct a perpendicular line from that point to the x-axis, using a compass or a plastic triangle with a 90° angle. Then find sine, cosine, and tangent for each angle.

1. 22°

First measure x and y.



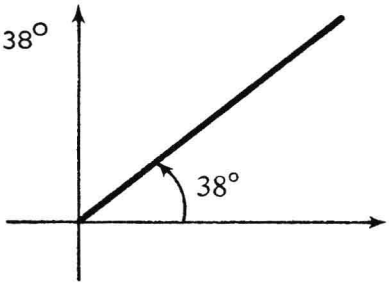
$x \approx 93\text{mm}$, $y \approx 37\text{mm}$

$$\sin 22^\circ = \frac{y}{r} = \frac{37}{100} = .37$$

$\cos 22^\circ = \frac{x}{r} = \frac{93}{100} = .93$

$\tan 22^\circ = \frac{y}{x} = \frac{37}{93} = .40$

2. 38°



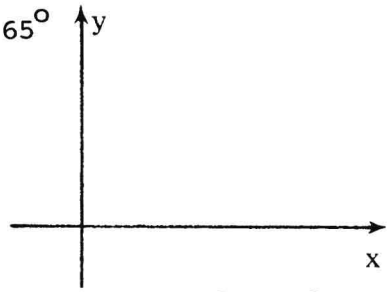
x = _____, y = _____ — 79mm, 62mm

$\sin 38^\circ = \frac{(\quad)}{(\quad)} = $ _____ — $\frac{62}{100}$, .62

$\cos 38^\circ = \frac{(\quad)}{(\quad)} = $ _____ — $\frac{79}{100}$, .79

$\tan 38^\circ = \frac{(\quad)}{(\quad)} = $ _____ — $\frac{62}{79}$, .78

3. 65°



x = _____, y = _____ — 42mm, 91mm

$\sin 65^\circ = \frac{(\quad)}{(\quad)} = $ _____ — $\frac{91}{100}$, .91

$\cos 65^\circ = \frac{(\quad)}{(\quad)} = $ _____ — $\frac{42}{100}$, .42

$\tan 65^\circ = \frac{(\quad)}{(\quad)} = $ _____ — $\frac{91}{42}$, 2.14

4. Complete the following table, using a point (x,y) on the terminal side of each angle such that r = 1.

				sin θ	cos θ	tan θ	cot θ	sec θ	csc θ
θ	x	y	r	$(\frac{y}{r})$	$(\frac{x}{r})$	$(\frac{y}{x})$	$(\frac{x}{y})$	$(\frac{r}{x})$	$(\frac{r}{y})$
0°	1	0	1	$\frac{0}{1} = 0$	$\frac{1}{1} = 1$	$\frac{0}{1} = 0$	$\frac{0}{1}$ none	$\frac{1}{1} = 1$	$\frac{0}{1}$ none
180°	-1		1						
90°	0	1	1						
270°			1						

(none = undefined)

0,0,-1,0,none,-1,none

1,0,none,0,none,1

0,-1,-1,0,none,0,none,-1

Evaluate each of the following, using the definitions and/or the preceding table.

5. $\cos 90^\circ + 3 \sec 90^\circ = 0 + 3 \cdot \text{none} = \text{none}$

6. $\tan 0^\circ + \sec 180^\circ = 0 + $ ____ = ____ — -1, -1

7. $\sin^2 360^\circ + \cos^2 360^\circ = (\quad)^2 + (\quad)^2 = $ ___ — 0, 1, 1

8. $1 + \tan^2 0^\circ + \csc^2 90^\circ = 1 + $ ___ + ___ = ___ — 0, 1, 2

9. $|\sin 90^\circ| + |\cos 180^\circ| = $ ___ + ___ = ___ — 1, 1, 2

10. $\sin 90^\circ + \cos 180^\circ + \cot 270^\circ + \sin 180^\circ =$ ___ + ___ + ___ + ___ = ___ — 1, -1, 0, 0, 0

Find the values of the six trigonometric functions for the angles in standard position having the following points on their terminal sides.

11. $(-4,3)$ $\quad r = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$

$\sin\theta = \frac{y}{r} = \frac{3}{5}$ $\qquad \cot\theta = \frac{x}{y} = \frac{-4}{3}$

$\cos\theta = \frac{x}{r} = \frac{-4}{5}$ $\qquad \sec\theta = \frac{r}{x} = \frac{5}{-4}$

$\tan\theta = \frac{y}{x} = \frac{3}{-4}$ $\qquad \csc\theta = \frac{r}{y} = \frac{5}{3}$

12. $(8, -15)$ $\quad r = \sqrt{8^2 + (-15)^2} = \sqrt{289} = 17$

$\sin\theta = \frac{y}{(\quad)} = \frac{-15}{(\quad)}$ $\qquad \cot\theta = \frac{(\quad)}{(\quad)} = \frac{(\quad)}{(\quad)}$

$\cos\theta = \frac{x}{r} = \frac{(\quad)}{17}$ $\qquad \sec\theta = \frac{r}{x} = \frac{(\quad)}{(\quad)}$

$\tan\theta = \frac{(\quad)}{(\quad)} = \frac{(\quad)}{(\quad)}$ $\qquad \csc\theta = \frac{(\quad)}{(\quad)} = \frac{(\quad)}{(\quad)}$

$r,\ 17,\ \frac{x}{y},\ \frac{8}{-15}$

$8,\ \frac{17}{8}$

$\frac{y}{x},\ \frac{-15}{8},\ \frac{r}{y},\ \frac{17}{-15}$

13. $(2\sqrt{3},\ 2)$ $\quad r = \sqrt{(2\sqrt{3})^2 + 2^2} = \sqrt{16} = 4$

$\sin\theta = \frac{(\quad)}{(\quad)} = \frac{(\quad)}{(\quad)}$ $\qquad \cot\theta = \frac{(\quad)}{(\quad)} = \_\_\_\_$

$\cos\theta = \frac{(\quad)}{(\quad)} = \frac{(\quad)}{(\quad)}$ $\qquad \sec\theta = \frac{(\quad)}{(\quad)} = \frac{(\quad)}{(\quad)}$

$\tan\theta = \frac{(\quad)}{(\quad)} = \frac{(\quad)}{(\quad)}$ $\qquad \csc\theta = \frac{(\quad)}{(\quad)} = \_\_\_\_$

$\frac{2}{4},\ \frac{1}{2},\ \frac{2\sqrt{3}}{2},\ \sqrt{3}$

$\frac{2\sqrt{3}}{4},\ \frac{\sqrt{3}}{2},\ \frac{4}{2\sqrt{3}},\ \frac{2}{\sqrt{3}}$

$\frac{2}{2\sqrt{3}},\ \frac{1}{\sqrt{3}},\ \frac{4}{2},\ 2$

14. $(-7,\ \sqrt{15})$ $\quad r = \sqrt{(-7)^2 + (\sqrt{15})^2} = \_\_\_$

$\sin\theta = \_\_\_\_\_$ $\qquad \cot\theta = \_\_\_\_\_$

$\cos\theta = \_\_\_\_\_$ $\qquad \sec\theta = \_\_\_\_\_$

$\tan\theta = \_\_\_\_\_$ $\qquad \csc\theta = \_\_\_\_\_$

8

$\frac{\sqrt{15}}{8},\ \frac{-7}{\sqrt{15}}$

$\frac{-7}{8},\ \frac{8}{-7}$

$\frac{\sqrt{15}}{-7},\ \frac{8}{\sqrt{15}}$

1.4 USING THE DEFINITIONS OF THE TRIGONOMETRIC FUNCTIONS

(pages 20-26 in Trigonometry.)

Objectives: Upon completing this section, you should be able to

- use reciprocal identities (#1-9)
- identify quadrants for given angles (#10-12)
- give all six trigonometric functions for given angles (#13-17), (#23-26)
- use the ranges of trigonometric functions (#18-22)