Werner Vogel
Dirk-Gunnar Welsch

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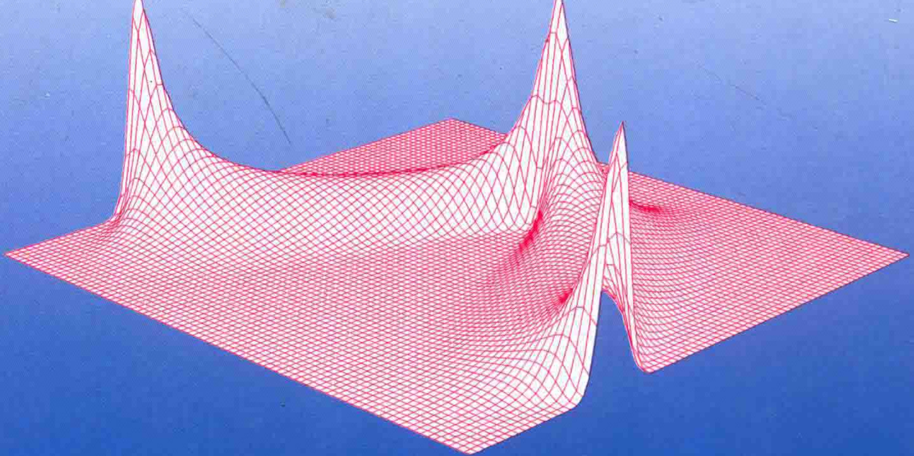
Quantum Optics

Third, Revised and Extended Edition



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Preface

The refinement of experimental techniques has greatly stimulated progress in quantum optics. Understanding of the quantum nature of matter and light has been significantly widened and new insights have been gained. A number of fundamental predictions arising from the concepts of quantum physics have been proved by means of optical methods.

In our book *Quantum Optics*, which arose from lectures that we have given for many years in Jena, Güstrow and Rostock, an attempt is made to develop the theoretical concepts of modern quantum optics, with emphasis on current research trends. It is based on our book, *Lectures on Quantum Optics* (Akademie Verlag/VCH Publishers, Berlin/New York, 1994) and its revised and enlarged second edition, *Quantum Optics – An Introduction* (Wiley-VCH, Berlin, 2001), which we wrote together with S. Wallentowitz. Taking into account representative developments in the field, in the second edition we have included new topics such as quantization of radiation in dispersing and absorbing media, quantum-state measurement and reconstruction, and quantized motion of laser-driven trapped atoms. Following this line, in the present edition we have again included new topics. The new Chapter 10 is devoted to medium-assisted electromagnetic vacuum effects, with special emphasis on spontaneous emission and van der Waals and Casimir forces. In the substantially revised and extended Chapter 8, a unified concept of measurement-based nonclassicality and entanglement criteria for bosonic systems is presented. The new measurement principles needed in this context are explained in Chapter 6. Two sections are added to Chapter 9 in which the problem of unwanted losses in quantum-state extraction from leaky optical cavities is studied. A consideration of decoherence effects in the motion of trapped atoms is added to Chapter 13.

Quantum Optics should be useful for graduate students in physics as well as for research workers who want to become familiar with the ideas of quantum optics. A basic knowledge of quantum mechanics, electrodynamics and classical statistics is assumed.

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W. Vogel and D.-G. Welsch Rostock and Jena, March 2006

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Appendix

1
Introduction

Since the first experimental demonstration of nonclassical light in 1977, quantum optics has been a very rapidly developing and growing field of modern physics. There are a number of books on the subject [e. g., Agarwal (1974); Allen and Eberly (1975); Carmichael (1993, 1998); Cohen-Tannoudji, Dupont-Roc and Grynberg (1989, 1992); Gardiner (1991); Gerry and Knight (2004); Haken (1985); Klauder and Sudarshan (1968); Loudon (1983); Louisell (1973); Mandel and Wolf (1995); Meystre and Sargent (1990); Orszag (2000); Peřina (1985, 1991); Schleich (2001); Scully and Zubairy (1997); Shore (1990); Vogel and Welsch (1994); Vogel, Welsch and Wallentowitz (2001); Walls and Milburn (1994)], and it is covered in many journals.[1] Presently, in one journal alone (Physical Review A) hundreds of articles on a broad spectrum of quantum-optical and related topics appear every year. Moreover, there are close connections to other traditional fields, such as nonlinear optics, laser spectroscopy and optoelectronics, and the boundaries have often been flexible. The recent improvements in experimental techniques allow one to control the quantum states of various systems with increasing precision. These possibilities have also stimulated the development of rapidly increasing new fields of research such as atom optics and quantum information.

The aim of this book is to describe the fundamentals of quantum optics, and to introduce the basic theoretical concepts to a depth sufficient to apply them practically and to understand and treat specialized problems which have arisen in recent research. On the basis of a general quantum-field-theoretical approach, important topics are presented in a unified manner. Keeping in mind that any real light field is due to sources, time-dependent commutation rules are considered carefully. Nonclassical light is studied and a detailed analysis of measurement schemes is given, including the effect of passive optical instruments, such as beam splitters, spectral filters and leaky cavities. From this background, the basic concepts are developed that allow one to de-

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1) For example, see Europhysics Letters, European Physical Journal D, Journal of Modern Optics, Journal of Optics B, Journal of Physics A and B, Journal of the Optical Society of America B, Nature, Optics Communications, Optics Letters, Physical Review A, Physical Review Letters, Physics Letters A, Science.

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termine the quantum states of various systems from measured data. Methods of quantum-state preparation are outlined for particular systems, such as propagating light fields, cavity fields and the quantized motion of a trapped atom.

Any attempt to give a complete overview on the present state of the field, together with a complete list of references, would be a hopeless venture. We have therefore decided to refer to selected work that may be useful in the context of particular topics, with special emphasis on textbooks, review articles and research-stimulating original articles. Before giving a guide to the topics covered, we mention two important fields that, apart from some basic ideas, are not considered, although they are closely related to quantum optics. These are the large fields of nonlinear optics [see, e. g., Bloembergen (1965); Boyd (1991); Peřina (1991); Schubert and Wilhelmi (1986); Shen (1984)] and laser physics and laser spectroscopy [see, e. g., Sargent, Scully and Lamb (1977); Haken (1970); Levenson and Kano (1988); Milonni and Eberly (1988); Stenholm (1984)].

1.1
From Einstein's hypothesis to photon anti-bunching

At the beginning of the last century, one of the unresolved problems in physics was the photoelectric effect. When light falls on a metallic surface, photoelectrons may be ejected (Fig. 1.1), whose energy is insensitive to the intensity, but increases with the frequency of the incident light. This result is obviously in contradiction to the concepts of classical physics. From a classical point of view, one would expect the energy of the emitted electrons to increase with the light intensity. Einstein's explanation of the photoelectric effect in 1905,

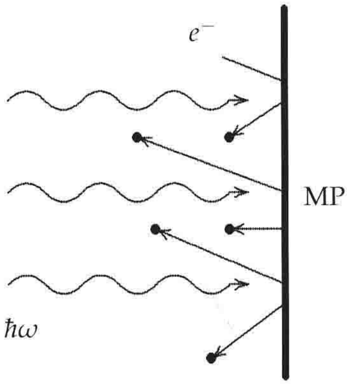

Fig. 1.1 Photoelectric effect: light of frequency ω falls on a metallic plate (MP) and ejects electrons (e^-).

by postulating the existence of light quanta, photons, may be regarded as the birth of quantum optics. He assumed that light is composed of quanta of energy

$$E = \hbar\omega \tag{1.1}$$

and momentum

$$p = \hbar k = \frac{h}{\lambda} \,. \tag{1.2}$$

In this way, quantities that typically describe the wave aspects of light are related to those that describe particle aspects with the "coupling constant" between wave and particle features being given by the Planck constant $\hbar$. Hence the kinetic energy of an emitted electron, E_{kin}, is given by the difference between the energy of the absorbed photon, $\hbar\omega$, and the binding energy of the electron in the metal, E_{b}:

$$E_{\text{kin}} = \hbar\omega - E_{\text{b}} \,, \tag{1.3}$$

which implies that, in agreement with observations, the energy of the photoelectrons increases with the frequency of the incident light. Increasing the intensity of the light corresponds to increasing the number of light quanta falling on the metal surface, which gives rise to an increasing number of photoelectrons.

The photoelectric effect plays an important role in the photoelectric detection of light, the theory of which (Chapter 6) was developed at the end of the 1950s for classical radiation and extended to quantized radiation in the 1960s. Its experimental application has led to a deeper understanding of the statistics of light.

The invention of the laser at the beginning of the 1960s allowed qualitatively new developments in optical research and the growth of new fields such as nonlinear optics and laser spectroscopy. Intensive studies of lasers have stimulated the introduction of a series of basic theoretical concepts in quantum optics: coherent states (Chapter 3), the theory of phase-space functions (Chapter 4) and the quantum theory of damping (Chapter 5).

Modern quantum optics would be unthinkable without the availability of measurement techniques, such as the Hanbury Brown–Twiss experiment, which was first performed in 1956. By using a beam splitter and two photodetectors, the coincidences of photoelectric events were recorded and compared with the product of independently measured events (for the experimental setup see Fig. 8.1, p. 271). In the case of thermal light an excess of coincidences was observed. That is, the measured intensity correlation $G^{(2)}(\tau)$ as a function of the time delay τ, decays from its initial value at $\tau = 0$ towards a stationary value, cf. Fig. 1.2. This effect, which is called photon bunching, can

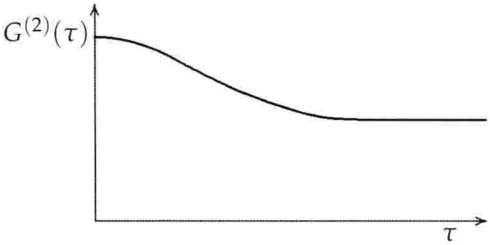

Fig. 1.2 Delay-time dependence of the intensity correlation as typically observed in a Hanbury Brown–Twiss experiment performed with light from a thermal source.

be understood by assuming that the light quanta arrive in bunches, so that the joint probability of events exceeds the product of the two probabilities measured independently of each other. Although this explanation is reasonable, it affords no proof of the existence of photons, since an intensity correlation behavior of the type observed can also be understood classically. It should be emphasized that, in the opposite case, where the measured intensity correlation has a positive initial slope (photon anti-bunching) there is no classical explanation (Chapter 8).

Notwithstanding the success of Einstein's hypothesis, the existence of photons was still a matter of discussion in the 1970s,[2] and the demonstration of photon anti-bunching in 1977 may be regarded as the first direct proof of their existence. The experimental apparatus was of the Hanbury Brown–Twiss type and the detected light was the resonance fluorescence (Chapter 11) from an atomic beam with such a low mean number of atoms that at most one atom contributed to the emitted light. Let us suppose that at a certain instant a single two-level atom that is (resonantly) driven by a laser pump is in the upper quantum state and ready to emit a photon. If the atom emits a photon, it undergoes a transition from the upper to the lower quantum state, which implies that it cannot emit a second photon simultaneously with the first one. The atom can emit a second photon only when it is again excited by the pump field. In other words, the measured intensity correlation vanishes for zero delay, $G^{(2)}(\tau \to 0) = 0$, and in the detection scheme considered there are no equal-time coincidences of photoelectric events. Note that any classical wave or wavepacket is divided by a 50%:50% beam splitter into two parts of equal intensity, which never leads to a vanishing intensity correlation at zero time delay. Photon anti-bunching is essentially a nonclassical property of light and its detection stimulated the formation of quantum optics as a specific field of research.

[2] See, e. g., the paper by Karp (1976), "Test for the non-existence of photons", and the response by Mandel (1977), "Photoelectric counting measurements as a test for the existence of photons".