

SECOND EDITION

INTRODUCTORY  
ALGEBRA  
A Modern Approach

Keedy-Bittinger

## PREFACE TO THE SECOND EDITION

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The users of the first edition have been so enthusiastic in their response and so helpful in offering constructive criticism, the authors first wish to say "thank you."

### WHAT'S NEW IN THE SECOND EDITION?

**Suggestions from Users.** In various places errors have been corrected, changes in wording have been made, boldface type has been used, and exercises have been added, deleted, or changed. Chapter 4 in the first edition, which was rather long, has been divided into two chapters, and Chapters 5 and 6 of the first edition have been combined. There is also a new, more detailed index, and the headings of chapters, sections, exercise sets, and answers have been redesigned to make them easier to use.

**The Metric System.** To aid transition to the metric system, we have used metric units in approximately 50% of the applied problems. Students do not need to know what the metric units are in order to solve the problems, but if they wish they can take a "short course" on the metric system using the newly-added appendix on "The Metric System" and the familiarization material on the inside front cover.

### SUGGESTIONS FOR USING THIS BOOK

In the preface to the first edition, it was pointed out that there are many ways in which this book can be used, and that flexibility in that regard is one of its important features. The book has been very effective in lecture, math lab, and independent study situations.

Worth special mention is a teaching method developed by some users that works well in classes of all sizes, notably large ones. The instructor does not lecture, but makes assignments which students do on their own, including working exercise sets. The following class period the instructor spends answering questions. Students have an additional day or two to polish their homework before handing it in. In the meantime, they are working on the next assignment. This method has the advantage of providing individualization while at the same time keeping the class together and working as a group. It also minimizes the number of instructor man-hours required.

### SUPPLEMENTS THAT ACCOMPANY THE SECOND EDITION

In order to further enhance flexibility in using this book, the following (optional) supplements are now available:

- *Audio-Tape Cassettes* are available for use in audio-tutorial or math lab situations.
- *An Instructor's Manual* contains commentary and a sample course syllabus.
- *A Test Booklet* contains five alternate forms of each chapter test and the final examination, with answers spaced for easy grading.
- *An Answer Booklet* contains the answers to all of the exercises.
- *A Student's Solutions Booklet* contains worked out solutions to all of the margin exercises.

### ACKNOWLEDGEMENTS

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October, 1974

M.L.K.  
M.L.B.

## PREFACE TO THE FIRST EDITION

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This book covers beginning algebra. It is intended for use by students who have not been exposed to the subject. There is strong emphasis on the development of algebraic skills, but the “why’s” of algebra also receive significant attention.

It is the emphasis on the “why’s” that prompts the authors to call this book “modern”. We feel that emphasis on understanding is essentially what makes a book such as this modern, rather than the use of such things as excessive set notation and sophisticated theory.

In this book, as well as its companion volumes, the authors bring to bear not only their own experience in teaching remedial courses but also the suggestions and recommendations of instructors in scores of two-year institutions across the country.

One of the principal and distinguishing features of the book is its design and format. Each page has an outer margin, which is used for materials of several types. For each lesson the objectives are stated in behavioral terms at the top of the page. These can easily be seen by the student and they tell him clearly what is expected of him in terms of performance. We hope that this will help answer the question all-too-often heard, “What material are we responsible for?” The most important items in the margins, however, are the sample or developmental exercises. These are placed with the text development so that the student can become involved actively in the development of the topic and gain some practice on exercises of the type he will be expected to do as homework for the lesson. The text refers to these exercises in the margins at the appropriate places.

There are many ways in which this book can be used. Flexibility in this regard is indeed one of its important features. The instructor who wishes to use it as he would an ordinary textbook can do so very easily. All he need do is ignore, and have students ignore, the exercises in the margins.

If an instructor wishes to use the lecture method primarily, but would like to bring some student-centered activity into the class, he can very easily do so. He would merely stop lecturing and have the students do the exercises in the margins at the appropriate times. On the other hand, the book is well suited for use in a learning laboratory situation. Because of its design, it can be used by a student with minimal instructor guidance. Yet it retains the flavor of the ordinary textbook, without the often deadly quality of the programmed textbook.

This book contains some other features not usually found in a college textbook. There are tests at the ends of chapters, in addition to a final examination. Besides these, there is a pretest which can be used diagnostically. The exercise sheets which the student removes from the book are designed for quick and easy grading or scoring. The answers in the *Instructor’s Manual* are arranged so that they match the spacing on the exercise sheets. The *Manual* also contains alternative forms of the chapter tests and the final examination. The book contains a great number of exercises (about 3500), and the authors have attempted to use language sparingly, so that the student has a maximal chance to learn the mathematics by reading it.

The material herein is suitable for a semester or quarter course for students who have not been exposed to algebra. This is the second in a series of books written in the same style. The preceding and following books are *Arithmetic: A Modern Approach*, and *Intermediate Algebra: A Modern Approach*.

The authors wish to thank the numerous people who helped make this book what it is. These include the many instructors in two-year colleges with whom we visited and who made many constructive suggestions. Professors Jerry Ball of Chabot College, Ralph Mansfield of Chicago City College, Loop Campus, and others made many helpful suggestions for improving the manuscript, and the staff of the Addison-Wesley Publishing Company has prodded and encouraged us most appropriately. Last, but not least, we thank our wives for their patience and helpful encouragement.

M. L. K.  
M. L. B.

January, 1971

**PRETEST**

NAME \_\_\_\_\_

CLASS \_\_\_\_\_ SCORE \_\_\_\_\_ GRADE \_\_\_\_\_

The purpose of this test is to determine your background in algebra and help you or your instructor decide where to begin your study. When you have completed the test, read the test analysis on the answer page at the end of the book. The answers are given along with page numbers that refer you to the material for that question. Remove the test from the book and begin.

**ANSWERS**

**Chapter 1**

1. What does  $x^2$  represent if  $x$  represents 3?

2. What does  $x^2 + 2$  represent if  $x$  represents 3?

3. What is the reciprocal of  $\frac{x}{2}$ ?

4. Which symbol,  $>$  or  $<$ , should be inserted to make a true statement?  
3                  12

1.

2.

3.

4.

**Chapter 2**

5. Which symbol,  $>$  or  $<$ , should be inserted to make a true statement?  
-8                  -2

6. Simplify.  
 $|-12|$

7. Add.  
 $-7 + 3$

8. Subtract.  
 $12 - (-20)$

9. Add.  
 $-3 + 12 + (-8) + 2$

10. Simplify.  
 $-3 \cdot 2 + 7 \cdot 3 - (-2 \cdot 5)$

5.

6.

7.

8.

9.

10.

ANSWERS

11. Divide and simplify.

$$-\frac{3}{5} \div \frac{5}{7}$$

12. Multiply.

$$(-3)(-2)\left(\frac{1}{2}\right)(-1)$$

11.

12.

13.

13. Rename, using a negative exponent.

$$\frac{1}{4^3}$$

14. Simplify.

$$x^{-4}x^2$$

15. Simplify.

$$\frac{y^4}{y^{-3}}$$

14.

15.

Chapter 3

16. Solve.

$$4x + 5 = 3x + 8$$

17. Solve.

$$4(x - 1) - 3(x + 3) = -11$$

16.

18. The sum of two consecutive odd integers is 64. What are the integers?

17.

18.

Chapter 4

19. Collect like terms.

$$3x^2 + 2x - 5x^2 + 3x - 2 + 4$$

20. Arrange in descending order.

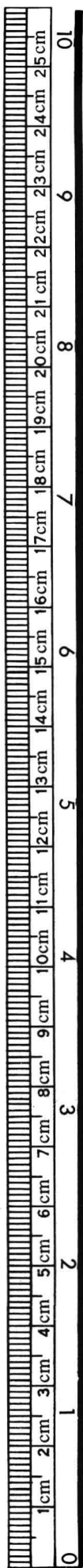
$$3x - 2 + 5x^4 - 4x^2$$

19.

20.



Centimeters (cm); 1 millimeter =  $\frac{1}{10}$  centimeter



# **TABLES OF MEASURES** (see pages 425–437 for more on the metric system)

## **LENGTH**

1 kilometer (km)	=	1000 meters (m)
1 hectometer (hm)	=	100 meters
1 dekameter (dam)	=	10 meters
1 decimeter (dm)	=	0.1 meter
1 centimeter (cm)	=	0.01 meter
1 millimeter (mm)	=	0.001 meter

## **MASS OR WEIGHT**

1 kilogram (kg)	=	1000 grams (g)
1 hectogram (hg)	=	100 grams
1 dekagram (dag)	=	10 grams
1 decigram (dg)	=	0.1 gram
1 centigram (cg)	=	0.01 gram
1 milligram (mg)	=	0.001 gram
1 metric ton (MT or t)	=	1000 kilograms

## **AREA**

1 hectare (ha)	=	100 are (a), or 10,000 sq m (m <sup>2</sup> )
1 are (a)	=	100 sq m (m <sup>2</sup> )
1 centare (ca)	=	0.01 are, or 1 m <sup>2</sup>

The word “are” is pronounced “AIR.”

## **VOLUME**

1000 cubic centimeters (cc or cm <sup>3</sup> )	=	1 liter (ℓ)
1 cubic centimeter (cc)	=	1 milliliter (mℓ)
1 mℓ of water weighs	=	1 g
1 stere	=	1 cubic meter

## **METRIC-AMERICAN CONVERSIONS** (Approximate)

### **LENGTH**

1 m	=	39.37 in. = 3.3 ft
1 in.	=	2.54 cm
1 km	=	0.62 mi
1 mi	=	1.6 km
1 cm	=	$\frac{3}{8}$ in.

### **MASS OR WEIGHT**

1 kg	=	2.2 lb
1 MT	=	1.1 tons
1 lb	=	454 g
1 oz	=	28 g
1 g	=	weight of a raisin, or weight of a paperclip

### **AREA**

1 hectare	=	2.47 acres
1 are	=	120 sq yd

### **VOLUME**

1 liter	=	1.057 qt = 2.1 pt
1 cup	=	240 mℓ
1 ounce (liquid)	=	30 mℓ
1 gallon	=	3.78 liters
1 tablespoon	=	15 mℓ
1 teaspoon	=	5 mℓ

## **Some Familiarization Exercises**

1. Measure yourself in centimeters.

Height = \_\_\_\_\_ centimeters

Chest/  
Bust = \_\_\_\_\_ centimeters

Waist = \_\_\_\_\_ centimeters

Hips = \_\_\_\_\_ centimeters

2. Measure your weight in kilograms.

Weight = \_\_\_\_\_ kilograms

3. Measure this book in centimeters.

Length = \_\_\_\_\_ centimeters

Width = \_\_\_\_\_ centimeters

Thickness = \_\_\_\_\_ centimeters

4. Measure the weight of this book in grams.

Weight of book = \_\_\_\_\_ grams

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21. Add.

$$\begin{array}{r}
 4x^3 - 3x^2 + 2x - 5 \\
 5x^2 - 4x + 8 \\
 7x^2 - 12x - 2 \\
 \hline
 \end{array}$$

22. Subtract.

$$(8x^2 - 5x + 2) - (-3x^2 + 4)$$

23. Add.

$$(3y^4 + 2y^2 - 5) + (7y^2 - 3y + 7)$$

24. Multiply.

$$3x(2x^2 + 5)$$

25. Multiply.

$$(x - 2)(x^2 + 2x - 3)$$

26. Multiply.

$$(y + 3)(y - 3)$$

**Chapter 5**

27. Factor.

$$6x - 12$$

28. Factor.

$$9y^2 - 1$$

29. Factor.

$$x^2 + 3x - 10$$

30. Factor.

$$2y^2 - y - 6$$

NAME \_\_\_\_\_

CLASS \_\_\_\_\_

ANSWERS

21. \_\_\_\_\_

22. \_\_\_\_\_

23. \_\_\_\_\_

24. \_\_\_\_\_

25. \_\_\_\_\_

26. \_\_\_\_\_

27. \_\_\_\_\_

28. \_\_\_\_\_

29. \_\_\_\_\_

30. \_\_\_\_\_

## ANSWERS

31.

32.

33.

34.

35.

36.

38.

39.

40.

31. Solve.

$$x^2 + 6 = 5x$$

32. One more than a number times one less than the number gives 24. What is the number?

## Chapter 6

33. In which quadrant is the graph of the point  $(5, -2)$ ?

34. Solve.

$$x^2 - 7x = -6$$

35. What is the  $x$ -intercept of  $3x + 2y = 12$ ?

36. What is the  $y$ -intercept of  $3x + 2y = 12$ ?

37. Graph the equation

$$y = -x + 2.$$

Attach your graph paper to this sheet.

38. Solve.

$$x + y = 13$$

$$x - y = 1$$

39. Solve.

$$2x + 3y = -1$$

$$3x - 2y = 18$$

40. The sum of two numbers is 23. The first number minus twice the second is 8. Find the numbers.

Note: Pretest covers only six chapters. This is because students who do well on this much should proceed to *Intermediate Algebra*, the next book in this series.

# 1 THE NUMBERS OF ORDINARY ARITHMETIC AND THEIR PROPERTIES

## 1.1 NUMBERS AND ALGEBRA

Numbers are abstract ideas. The idea that comes to mind when we answer the question “how many?” is a number, for example. The symbols we write to represent, or *name* numbers are called *numerals*. Thus symbols such as 4, 8, XVI, and  $\frac{3}{4}$  are numerals. In algebra as in arithmetic, we use symbols which represent numbers to calculate and to solve problems. The basic properties of numbers are needed so that we will know what kinds of procedures are correct. As you learn better how to handle algebraic symbols, you will be able to solve problems that you could not have solved before.

### NUMBERS OF ORDINARY ARITHMETIC

There are several kinds of numbers. The numbers we use for counting are called *natural numbers*. They are

1, 2, 3, 4, 5, 6, 7, 8, and so on.

The *whole numbers* include the natural numbers and zero. The whole numbers are

0, 1, 2, 3, 4, 5, 6, 7, 8, and so on.

The *numbers of ordinary arithmetic* include the whole numbers and the fractions, such as  $\frac{2}{3}$  and  $\frac{9}{5}$ . These numbers are sometimes called simply *the numbers of arithmetic*. These numbers are used in algebra as well as arithmetic.

All of the numbers of arithmetic can be named by fractional symbols. Such symbols are called *fractional numerals*, or *fractional notation*.

#### Examples

a) The number *three-fourths* can be named by the fractional numerals

$\frac{3}{4}$ ,  $\frac{6}{8}$ ,  $\frac{300}{400}$  and many more.

b) The number *one-third* can be named by the fractional numerals

$\frac{1}{3}$ ,  $\frac{2}{6}$ ,  $\frac{10}{30}$ , and so on.

Any fractional numeral in which the denominator is three times the numerator names the number one-third.

The whole numbers can also be named with fractional notation.

#### Examples

a)  $0 = \frac{0}{1} = \frac{0}{4}$  etc.    b)  $2 = \frac{2}{1} = \frac{6}{3} = \frac{200}{100}$  etc.    c)  $1 = \frac{2}{2} = \frac{5}{5}$  etc.

Do exercises 1 through 4 at the right.

#### OBJECTIVES

You should be able to:

- Write several numerals for any number of arithmetic (whole number or fraction) by multiplying by 1.
- Find the simplest fractional numeral for a number of arithmetic.
- Add, subtract, and multiply numbers of arithmetic using fractional numerals.

1. Write three fractional numerals for  $\frac{2}{3}$ .

2. Write three fractional numerals for  $\frac{1}{2}$ .

3. Write three fractional numerals for 1.

4. Write three fractional numerals for 4.

5. Multiply by 1 to find three different names for  $\frac{4}{5}$ .

6. Multiply by 1 to find three different names for  $\frac{8}{7}$ .

Simplify.

7.  $\frac{18}{27}$

8.  $\frac{38}{18}$

Every number of arithmetic has many fractional numerals, or fractional *names*. To find different names we can use the notion of multiplying by 1.

*Examples*

$$\text{a) } \frac{2}{3} = \frac{2}{3} \times 1 = \frac{2}{3} \times \frac{5}{5} = \frac{10}{15} \quad (\text{Multiplying a number by 1 does not change the number})$$

$$\text{b) } \frac{7}{5} = \frac{7}{5} \times 1 = \frac{7}{5} \times \frac{11}{11} = \frac{77}{55}$$

**Do exercises 5 and 6 at the left.**

The simplest fractional numeral for a number has the smallest possible numerator and denominator. To simplify we can reverse the above process.

*Examples*

$$\text{a) } \frac{10}{15} = \frac{2 \times 5}{3 \times 5} = \frac{2}{3} \times \frac{5}{5} = \frac{2}{3} \quad (\text{We factor numerator and denominator and then "remove" a factor of 1, in this case } \frac{5}{5})$$

$$\text{b) } \frac{36}{24} = \frac{6 \times 6}{4 \times 6} = \frac{3 \times 2 \times 6}{2 \times 2 \times 6} = \frac{3}{2} \times \frac{2 \times 6}{2 \times 6} = \frac{3}{2}$$

You may be in the habit of canceling in cases like this. For example, you might have done this example as follows.

$$\begin{array}{r} 2 \\ \cancel{12} \\ 24 \\ \cancel{36} \\ \cancel{18} \\ 3 \end{array} \quad \text{or} \quad \frac{24}{36} = \frac{2 \times \cancel{12}}{3 \times \cancel{12}} = \frac{2}{3}$$

Canceling causes many errors, so you should *avoid doing it*. Rather, use the method of removing factors of one, at least for now.

**Do exercises 7 and 8 at the left.**

It is sometimes helpful to include a factor of 1 in the numerator or denominator.

*Examples*

$$a) \frac{18}{72} = \frac{2 \times 9}{8 \times 9} = \frac{1 \times 2 \times 9}{4 \times 2 \times 9} = \frac{1}{4} \times \frac{2 \times 9}{2 \times 9} = \frac{1}{4} \quad b) \frac{9}{72} = \frac{1 \times 9}{8 \times 9} = \frac{1}{8}$$

Do exercises 9 and 10 at the right.

*Example 1.* Multiply and simplify.

$$\frac{5}{6} \cdot \frac{9}{25} = \frac{5 \cdot 9}{6 \cdot 25} = \frac{1 \cdot 3 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 5 \cdot 5} = \frac{3 \cdot 5 \cdot 1 \cdot 3}{3 \cdot 5 \cdot 2 \cdot 5} = \frac{3 \cdot 5}{3 \cdot 5} = \frac{1 \cdot 3}{2 \cdot 5} = \frac{3}{10}$$

In the preceding example, we have used a dot  $\cdot$  for a multiplication sign. It means exactly the same thing as  $\times$ .

Do exercises 11 and 12 at the right.

We can use multiplying by 1 to find common denominators for addition and subtraction.

*Example 2.* Add and simplify.

$$\frac{2}{8} + \frac{3}{5} = \frac{2 \cdot 5}{8 \cdot 5} + \frac{3 \cdot 8}{5 \cdot 8} = \frac{10}{40} + \frac{24}{40} = \frac{34}{40} = \frac{17 \cdot 2}{20 \cdot 2} = \frac{17}{20}$$

*Example 3.* Add and simplify.

$$\frac{3}{8} + \frac{5}{12} = \frac{3 \cdot 3}{8 \cdot 3} + \frac{5 \cdot 2}{12 \cdot 2} = \frac{9}{24} + \frac{10}{24} = \frac{19}{24}$$

Do exercises 13 and 14 at the right.

*Example 4.* Subtract and simplify.

$$\frac{9}{8} - \frac{4}{5} = \frac{9 \cdot 5}{8 \cdot 5} - \frac{4 \cdot 8}{5 \cdot 8} = \frac{45}{40} - \frac{32}{40} = \frac{13}{40}$$

Do exercises 15 and 16 at the right.

Do exercise set 1.1, p. 35.

Simplify.

9.  $\frac{27}{54}$

10.  $\frac{48}{12}$

Multiply and simplify.

11.  $\frac{6}{5} \cdot \frac{9}{25}$

12.  $\frac{12}{11} \cdot \frac{14}{3} \cdot \frac{22}{35}$

Add and simplify.

13.  $\frac{4}{5} + \frac{2}{3}$

14.  $\frac{5}{12} + \frac{7}{18}$

Subtract and simplify.

15.  $\frac{4}{5} - \frac{2}{3}$

16.  $\frac{5}{12} - \frac{2}{9}$

## OBJECTIVES

You should be able to:

- Write expanded numerals for numbers of arithmetic with or without exponents.
- Given exponential notation such as  $x^3$ , write  $xxx$  and conversely.
- Convert from fractional numerals to decimal numerals and conversely.

17. Write three kinds of expanded numerals for 52,374.

18. Write three kinds of expanded numerals for 234,809.

19. Write an expanded numeral for 752,398 using exponents.

20. Write an expanded numeral for 20,347 using exponents.

## 1.2 DECIMAL NOTATION

The numbers of arithmetic can all be named by fractional numerals. Another familiar and useful notation for these numbers is called *decimal notation*. It is called decimal because it is based on the number ten.\* Our ordinary numerals for whole numbers are decimal numerals ("decimals" for short) based on ten. To show the meaning of decimal numerals we can write *expanded numerals*.

*Example*

An expanded numeral for 3542 is

$$3000 + 500 + 40 + 2$$

Note that the value of a place is ten times that to its right. We can emphasize this further with other expanded numerals.

*Example*

$$\begin{aligned} 3542 &= 3 \times 1000 + 5 \times 100 + 4 \times 10 + 2 \\ &= 3 \times 10 \times 10 \times 10 + 5 \times 10 \times 10 + 4 \times 10 + 2 \end{aligned}$$

Do exercises 17 and 18 at the left.

## EXPONENTIAL NOTATION

We can use shorthand notation for  $10 \times 10 \times 10$  and similar expressions. It is called *exponential notation*. For

$$10 \times 10 \times 10 \text{ we write } 10^3$$

This is read "ten cubed" or "ten to the third power." The number 3 is called an *exponent* and we call 10 the *base*. Similarly, for  $10 \times 10$  we write  $10^2$ , read "ten squared" or "ten to the second power."

*Example*

An expanded numeral with exponents for 82,653 is

$$8 \times 10^4 + 2 \times 10^3 + 6 \times 10^2 + 5 \times 10 + 3$$

Do exercises 19 and 20 at the left.

An exponent tells how many times the base is used as a factor. The base can be a number other than ten.

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\* The Latin word for ten is *decem*.



*Examples*

- a)  $3^5$  means  $3 \times 3 \times 3 \times 3 \times 3$       b)  $7^4$  means  $7 \cdot 7 \cdot 7 \cdot 7$   
 c) If we use  $n$  to stand for a number,  $n^4$  means  $n \cdot n \cdot n \cdot n$

Do exercises 21 and 22 at the right.

**VARIABLES**

A letter can often represent various numbers. In this case, we call it a *variable*. If we write two or more variables together, such as

$nnn$     or     $xyz$

the agreement is that it means multiplication.

*Examples*

- a)  $nnn$  means  $n \cdot n \cdot n$ . It also means  $n^3$   
 b) If  $n$  stands for 10, then  $nnn$  means  $10 \times 10 \times 10$ , or 1000  
 c)  $xyz$  means  $x \cdot y \cdot z$

Do exercises 23 through 26 at the right.

The numbers 1 and 0 are also given meaning as exponents. To see how this is done we can look at an expanded numeral.

$$3795 = 3 \times 10^3 + 7 \times 10^2 + 9 \times 10 + 5$$

If we are to keep the decreasing pattern of exponents we would write

$$3 \times 10^3 + 7 \times 10^2 + 9 \times 10^1 + 5 \times 10^0$$

Accordingly,  $10^1$  should mean 10 and  $10^0$  should mean 1. These are the meanings we assign to 1 and 0 as exponents.

For any number  $n$ ,  $n^1$  means  $n$ .

For any number  $n$ , other than zero,\*  $n^0$  means 1.

*Examples*

- a)  $5^1 = 5$ ,    b)  $8^1 = 8$ ,    c)  $3^0 = 1$ ,    d)  $7^0 = 1$ ,    e)  $5^0 = 1$

Do exercises 27 and 28 at the right.

**DECIMAL NUMERALS FOR FRACTIONS**

Decimal numerals for fractions contain *decimal points*. The place values to the right of a decimal point are tenths, hundredths, and so on. We can show this with expanded numerals.

\* We shall see later why 0 is excluded, i.e.  $0^0$  is meaningless.

21. What is the meaning of  $5^4$ ?

22. What is the meaning of  $x^5$ ?

23. Write exponential notation for  $nnnnn$ .

24. What is the meaning of  $y^3$ ? Do not use  $\times$  or  $\cdot$ .

25. What number does  $xxx$  represent if  $x$  stands for 2?

26. What does  $n^4$  represent if  $n$  stands for 10?

27. Write an expanded numeral for 3562 using exponents. Include exponents of 1 and 0.

28. What is  $5^1$ ? What is  $7^0$ ?

Write an expanded numeral for each of the following numbers.

29. 23.678

30. .27

31. 4.0067

Write fractional numerals. You need not simplify.

32. 1.62

33. 35.431

Write decimal numerals.

34.  $\frac{7}{8}$

35.  $\frac{4}{5}$

36.  $\frac{9}{11}$

37.  $\frac{23}{9}$

a)  $34.243 = 3 \times 10 + 4 + 2 \times \frac{1}{10} + 4 \times \frac{1}{100} + 3 \times \frac{1}{1000}$

b)  $.8 = 8 \times \frac{1}{10}$

Do exercises 29 through 31 at the left.

To convert from a decimal numeral to a fractional numeral, we can proceed as in the following examples.

*Examples*

a)  $34.2 = \frac{342}{10}$  (34.2 is 34 and two-tenths, or 342 tenths)

b)  $16.563 = \frac{16563}{1000}$  (16.563 is 16,563 thousandths)

Do exercises 32 and 33 at the left.

To convert from a fractional numeral to a decimal numeral we can divide.

*Example 1*

$\frac{3}{8}$  means  $3 \div 8$ , so we divide

$$\begin{array}{r} .375 \\ 8 \overline{) 3.000} \\ \underline{2 \ 400} \phantom{00} \\ 600 \phantom{00} \\ \underline{560} \phantom{00} \\ 40 \phantom{00} \\ \underline{40} \phantom{00} \\ 0 \end{array}$$

Sometimes we get a repeating decimal when we divide.

*Example 2*

$\frac{4}{11}$  means  $4 \div 11$ , so we divide

$$\begin{array}{r} .3636 \dots \\ 11 \overline{) 4.00} \\ \underline{3 \ 3} \phantom{00} \\ 70 \phantom{00} \\ \underline{66} \phantom{00} \\ 40 \phantom{00} \\ \underline{33} \phantom{00} \\ 70 \phantom{00} \\ \underline{66} \phantom{00} \\ 4 \end{array}$$

Thus  $\frac{4}{11} = .363636 \dots$ . Such decimals are often abbreviated by putting a bar over the repeating part, as follows.

$\frac{4}{11} = .36\overline{36}$

Do exercises 34 through 37 at the left.

Do exercise set 1.2, p. 37.

### 1.3 PROPERTIES OF THE NUMBERS OF ARITHMETIC

The numbers of arithmetic can be named in various ways. These numbers have certain properties which do not depend upon the kind of notation used for them. Some of the basic properties of these numbers are so simple and obvious that they may seem unimportant. This is deceiving, however. They are very important, especially in algebra.

#### ORDER IN ADDITION

One of the basic properties of numbers is that they can be added in any order. For example,  $3 + 2$  and  $2 + 3$  are the same. A similar thing is true for any two numbers. This simple fact goes by the hifalutin name of *commutative law (or property) of addition*.\*

For any numbers  $a$  and  $b$ ,  $a + b = b + a$ .

This is the *commutative law of addition*.

Do exercises 38 and 39 at the right.

#### PARENTHESES, SYMBOLS OF GROUPING

Let us consider  $5 \times 2 + 4$ . What does it mean? If we multiply 5 by 2 and add 4 we get 14. If we add 2 and 4 and multiply by 5 we get 30. To tell which operation to do first, we use parentheses. In other words, parentheses show groupings.

*Example.*  $(3 \times 5) + 6$  means  $15 + 6$ , or 21

$3 \times (5 + 6)$  means  $3 \times 11$ , or 33

Do exercises 40 through 44 at the right.

#### GROUPING IN ADDITION

If we write  $3 + 5 + 4$ , what does it mean? Does it mean  $(3 + 5) + 4$  or  $3 + (5 + 4)$ ? Either way, we get 12, so it doesn't matter. In fact, if we are doing addition only, we can group numbers in any manner. This means we really don't need parentheses if we are doing only addition. This is another basic property.

\* In learning to spell *commutative*, note that there is no n in it.

#### OBJECTIVES

You should be able to:

- Do calculations as shown by parentheses.
- Tell which laws are illustrated by certain sentences.
- State the distributive law.
- Use the distributive law to factor expressions like  $3x + 3y$ .
- Evaluate an expression like  $3x + 3y$  when numbers are specified for the letters.

Do these calculations.

38.  $17 + 10$

39.  $10 + 17$

Do these calculations.

40.  $(5 \times 4) + 2$

41.  $5 \times (4 + 2)$

42.  $(4 \times 6) + 2$

43.  $5 \times (2 \times 3)$

44.  $(6 \times 2) + (3 \times 5)$