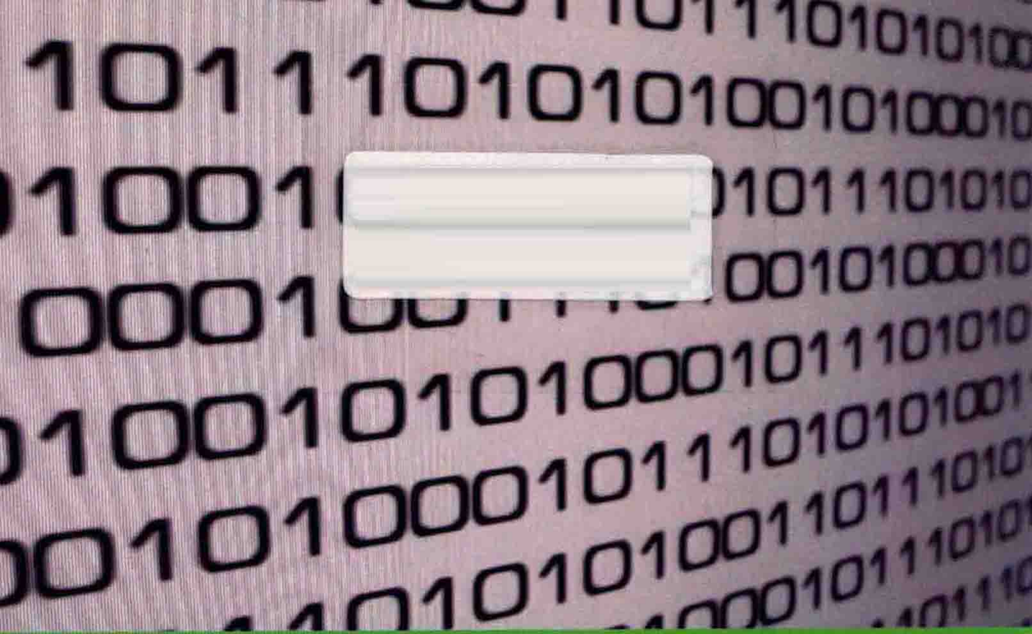

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FUZZY DIMENSION OF MODULES OVER RINGS (Monograph)

Fuzzy Dimension, Uniform Submodule, Linearly Independent Elements

VDM
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FUZZY DIMENSION OF MODULES OVER RINGS

(A Monograph)

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FUZZY DIMENSION OF MODULES OVER RINGS

(A Monograph)

PREFACE

This monograph entitled "Fuzzy Dimension of Modules over Rings", is divided into three chapters.

In Chapter 1, we present some results on Modules with Finite Goldie Dimension (FGD), and some Dimension Conditions in Modules with FGD. In Chapter 2, we present some results on Fuzzy Modules. In Chapter-3, we present work done on "Fuzzy Dimension of a Module with DCC on Submodules".

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Satyanarayana Bhavanari
and
Shaik Mohiddin Shaw

INTRODUCTION

In recent decades interest has arisen in algebraic systems with binary operations addition and multiplication. 'Ring' is one of such systems. A ring is an algebraic system (R, +, .) satisfying the conditions:

i) (R, +) is an Abelian group;

ii) (R, .) is a semi-group; and

iii) $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$ for all $a, b, c \in R$.

Ring theory became an important part of Algebra.
Moreover, if there exists an element $1 \in R$ such that $1a = a = a1$ for all $a \in R$, then we say that R is a ring with identity.

Modern Algebra presently, the basis for developing several new areas mentioned below. The past 30 years have seen an enormous expansion in several new areas of technology. These new areas include Digital Computing, Data Communication, Sequential Machines, Computer Systems and Radar Solar Systems. Work in each of these areas relies heavily on Modern Algebra. This fact has made the study of *Modern Algebra* important to Applied Mathematicians, Engineers and Scientists who use Digital Computers or who work in the other areas of Technology mentioned above.

Let R be an associative ring. An Abelian group (M, +) is said to be a **module** over R if there exists a mapping (called **scalar multiplication**) $f : R \times M \to M$ (the image of (r, m) is denoted by rm) satisfying the following three conditions:

(i) $r(a + b) = ra + rb$;

(ii) $(r + s)a = ra + sa$; and

(iii) $r(sa) = (rs)a$ for all $a, b \in M$ and $r, s \in R$.

Moreover, if R is ring with identity 1 and $1m = m$ for all $m \in M$, then M is called a ***unital R–Module***.

Every vector space is a module. Every Abelian group is a module over the ring of integers $\mathbb{Z}$. Every ring R is a module over itself. So the study of module theory include the study of vector space theory, group theory and ring theory. Thus the module theory became an important part of Algebra.

Let R be a fixed (not necessarily commutative) ring with identity. Throughout this monograph, we are concerned with left R-modules M.

It is well known that the dimension of a vector space is defined as the number of elements in its basis. One can define a basis of a vector space as a maximal set of linearly independent vectors or a minimal set of vectors which span the space. The former, when generalized to modules over rings, becomes the concept of Goldie Dimension.

Goldie [1] introduced the concept of Finite Goldie Dimension (FGD, in short) in modules. A module M is said to have FGD if M contains no infinite direct sum of non-zero submodules. Goldie proved a structure theorem for modules which states that "a module with FGD contains uniform submodules U_1, U_2, …, U_n whose sum is direct and essential in M". The number n obtained here is independent of the choice of U_1, U_2, …, U_n and it is called as Goldie Dimension of M.

Later this dimension theory was studied/developed by the authors like: Anh & Marki [1,2]; Reddy & Satyanarayana [1]; Satyanarayana [1,3,4,7]; and Satyanarayana, Syam Prasad & Nagaraju [1].

Chapter-1 entitled "Preliminary Definitions and Results on Modules" deals some fundamental concepts and results on Modules with Finite Goldie Dimension from the literature. This chapter contains some definitions including module over a ring, direct sum of submodules, cyclic R-module, finitely generated

R-module, essential submodule, Finite Goldie Dimension (FGD) in modules, uniform submodule, and we state some related results form the literature.

For a general submodule K of M the condition: "dim(M/K) = dim M - dim K" is not true. A submodule K of M is said to be a **complement submodule** if there exists a submodule B of M such that K is maximal with respect to the property that $K \cap B = (0)$. Goldie [1] proved that for a complement submodule K, the condition: "dim(M/K) = dim M - dim K" is true. The converse of this result was proved by Reddy-Satyanarayana [1]. Satyanarayana, Syam Prasad & Nagaraju [1] obtained that if a module M has FGD and K_1, K_2 are two submodules of M such that $K = K_1 \cap K_2$ is a complement, then

$$\dim K_1 + \dim K_2 = \dim(K_1 + K_2) + \dim (K_1 \cap K_2).$$

The second chapter deals with the concept 'fuzzy algebra'. Success of fuzzy logic in a wide range application inspired much interest in fuzzy logic among Mathematicians. Lotfi. A. Zadeh (a professor in Electrical Engineering and Computer Science at University of California, Berkeley)(July 1964) introduced a theory whose objects called 'fuzzy sets' (are sets with boundaries that are not precise). In a narrow sense fuzzy logic refers to a logical system that generalizes classical two-valued logic for reasoning under uncertainty. Prof. Zadeh believed that all real world problems could be solved with more efficient and analytic methods by using the concept fuzzy sets. The fuzzy boom (1987 to present) in Japan was a result of the close collaboration and technology transfer between Universities and Industries. In 1988 the Japanese Government launched a careful feasibility study about establishing national research projects on fuzzy logic involving both Universities and Industries. As a result the Japan is able to manufacture fuzzy vacuum cleaner, fuzzy rice cookers, fuzzy refrigerators, fuzzy washing machines, and others.

After the introduction of Fuzzy set by Zadeh [1], the researchers in mathematics were trying to introduce and study this concept of fuzzyness in different mathematical systems under study. Fu-Zheng Pan [1, 2]; and Golan [1] studied the concept 'fuzzy submodule'.

Chapter-2 entitled "Fuzzyness in Modules" contains some fundamental concepts and results on Fuzzyness in Modules over an associative ring R with identity. In this chapter, we present the definitions like fuzzy submodule, level submodule and necessary results from the existing literature. In this chapter, we collect necessary definitions and results related to the concept of fuzzyness in modules. The chapter is divided into four sections.

In section-1, we collected the definitions like fuzzy set, level set and some examples related to these concepts. In section-2, we collected information related to the fundamental operations on fuzzy sets and provided few illustrations. In section-3, we present some fundamental definitions and results related to fuzzy subgroups. In section-4, we consider modules over an associative ring R with identity. We collected two existing definitions of fuzzy submodules and some related fundamental results.

Chapter-3 entitled "On Fuzzy Dimension of a Module with DCC on Submodules" is divided into five small sections. In this Chapter the concepts: minimal element, fuzzy linearly independent element, fuzzy basis, fuzzy dimension in modules were introduced, and proved some important theorems related to these concepts.