ALPHA C. CHIANG

FUNDAMENTAL METHODS OF MATHEMATICAL ECONOMICS

SECOND EDITION

FUNDAMENTAL METHODS OF MATHEMATICAL ECONOMICS

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**FUNDAMENTAL METHODS
OF MATHEMATICAL ECONOMICS**

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PREFACE TO THE SECOND EDITION

Since the publication of the first edition, numerous economists have favored me with their generous comments and thoughtful suggestions. It is to find embodiment for many of these suggestions, as well as for certain ideas that have crystallized in my own mind in the meantime, that I am bringing out a new edition. With quite a few sections now duly amplified, and some others duly simplified, it is hoped that the present version will prove more useful than the previous one.

The most important change in this edition consists of an all-new chapter on nonlinear programming (Chap. 20), where I introduce the famous Kuhn-Tucker theorem (concave programming), as well as the subsequent Arrow-Enthoven generalization (quasiconcave programming). Substantial attention is paid in that discussion to the clarification of the important but tricky concept of the so-called "constraint qualification." The second major change is the explicit use of the implicit-function theorem as the analytical foundation for the entire discussion of the comparative statics of general-function models (Chap. 8). In the latter chapter, I now emphasize the total-differential method rather than the total-derivative method of obtaining the comparative-static derivatives.

Other changes are widely scattered in the various chapters. Among other things, I have adopted as standard terminology the names of *concave* and *convex functions* (Chap. 9 and thereafter), and also included new materials on quasiconcave and quasiconvex functions (Chaps. 12, 20). As an alternative means of testing the sign-definiteness of a quadratic form, the notion of the characteristic equation of a matrix has been introduced (Chap. 11), which is later compared with the characteristic equation of differential equations and difference equations (Chap. 17). There is in this edition also a discussion of L'Hôpital's rule (Chap. 12) and integration by parts (Chap. 13). The Lagrange multiplier is now given a full economic interpretation (Chap. 12), and this is later tied to the dual choice variable in linear programming (Chap. 19). Exact differential equations are now tackled by a simpler method (Chap. 14). In the dynamic models (Chaps. 14 to 17), a careful distinction is now made between the *intertemporal* and the *market-clearing* senses of equilibrium. New exercises have been added in many chapters. Also, in response to a suggestion by several people, I have attempted to "toughen" the exercises somewhat. But I have by no means abandoned my original intention of letting the exercises serve as drills to firm up the student's grasp and bolster his or her confidence, rather than as intellectual challenges that could unwittingly frustrate or intimidate the novice.

Despite the multitude of changes, the basic format and approach of the book has been kept intact. However, I have come to the realization that the book permits much more flexibility in its use than was suggested in the preface to the first edition. Upon completing the study of matrix algebra (Chap. 5), for instance, the reader may proceed directly to linear programming (Chaps. 18 and 19) and game theory (Chap. 21) without difficulty. Similarly, after finishing the topic of constrained optimization (Chap. 12), it is possible to proceed to nonlinear programming (Chap. 20) with or without the background of linear programming. Readers who are primarily interested in optimization problems may also omit the comparative-static analysis of general-function models (Chap. 8) and go from Chap. 7 directly to Chap. 9. In that case, though, it may be necessary also to omit Sec. 11.6 and the comparative-static portion of Sec. 12.4.

In one way or another, the following persons have influenced the final shape of this edition, and I wish to thank them all: Professors Nancy S. Barrett (The American University), Thomas Birnberg (Yale University), E. J. R. Booth (The University of Connecticut), Harald Dickson (University of Gothenburg, Sweden), Roger N. Folsom (Naval Postgraduate School), Jack Hirshleifer (University of California, Los Angeles), James C. Hsiao (Southern Connecticut State College), Ki-Jun Jeong (Seoul National University, Korea), J. Frank Sharp (New York University Graduate School of Business Administration), Dennis R. Starleaf (Iowa State University), and Mr. Chiou-Nan Yeh (University of Massachusetts). Since not all of their recommendations have been accepted, however, I alone must remain responsible for the finished product. In particular, I have regretfully decided to discard the suggestion to include in this edition an introduction to dynamic optimization. This topic cannot possibly be treated adequately without an extended excursion into the mathematical methods of calculus of variations, optimal control theory, and dynamic programming, but to undertake such an excursion would likely cause this book to burst at the seams. A better alternative is, therefore, to relegate the topic to a separate volume where it can be presented free from a stringent space constraint.

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PREFACE TO THE FIRST EDITION

This book is written for economists. Its chief purpose is twofold: (1) to render a systematic exposition of certain basic mathematical methods, and (2) to relate these mathematical techniques to the various types of economic analysis in such a way that the mutual relevance of the two disciplines is clearly brought out. Two types of readers may therefore find it useful: first, those who possess the mathematical background but are looking for a bridge to link it to economics, and secondly, those who have yet to learn the mathematics. Since most readers will probably fall into the latter category, I have endeavored to develop the technical materials with considerable patience—assuming very little foreknowledge and proceeding in a step-by-step manner designed to minimize the likelihood of the reader getting lost along the way. Moreover, I have resorted to a much greater degree of informality in the presentation than would please the mathematical purist in the belief that, especially in a book of this type, *readability* should be an overriding consideration.

In order to equip the reader with sufficient mathematics to wade through the current economic literature with confidence rather than trepidation, a wide range of mathematical topics is covered in the ensuing pages. Even though the treatment of each topic is of necessity limited to an elementary level, the reader who faithfully ploughs his way through the volume should acquire at least a reading knowledge, or even a working knowledge, of the concepts of sets, set operations, relations and functions, matrix algebra, differential and integral calculus, simple differential equations and difference equations, and the rudimentary notions of convex sets.

To integrate these mathematical subjects with economic analysis, numerous illustrations of mathematically formulated economic models are given in the text. Better yet, the entire book has actually been organized along *economic* rather than mathematical lines. After a brief introduction (Part 1), discussing the nature and structure of mathematical models, the remainder of the book is divided into five parts, each dealing with a distinct type of economic study:

Part 2: Static (or Equilibrium) Analysis

Part 3: Comparative-Static Analysis

Part 4: Optimization Problems
(a special case of equilibrium analysis)

Part 5: Dynamic Analysis

Part 6: Mathematical Programming and Game Theory
(a different framework of optimization)

The mathematical tools appropriate for each are then introduced in due order within the economic framework. By fitting the mathematics into the economic context, rather than the reverse, I believe the reader can gain a better perspective of the interrelation between the two disciplines, as well as a better motivation in the reading of the technical materials.

The arrangement of the economic topics, as outlined above, follows a natural order—from statics to comparative statics to dynamics. As it turns out, this arrangement leads also to a meaningful and convenient order of presentation for the relevant mathematical materials. The discussion of equilibrium analysis in Part 2 provides the setting for the introduction of elementary matrix algebra, because equilibrium analysis often involves the solving of a simultaneous linear-equation system. This early introduction of matrix algebra, a feature not usually found in the other books in the field, proves extremely desirable, since it permits the explicit use of vectors, matrices, and determinants throughout the remaining parts of the volume. In Part 3, the study of comparative statics leads to the notion of rate of change and of derivatives, including partial and total derivatives; these are then applied, in Part 4, to problems of optimization. In Parts 3 and 4, however, use is made of the matrix algebra learned earlier. When we proceed to dynamics in Part 5, the mathematics moves from the realm of differential calculus into that of integral calculus and differential equations, followed by a parallel discussion of difference equations. Here again, the reader will find matrix algebra of service. And finally, in mathematical programming and game theory, Part 6, matrix algebra again figures prominently, although elementary concepts of convex sets are also discussed there. In short, there is a systematic buildup of the "tool kit" in the text. For this reason, the reader is advised to read the first four parts (the first twelve chapters) in the exact order given. Part 5 and Part 6, on the other hand, may be read in reverse sequence. Mathematically, it is more desirable to let differential equations (Part 5) follow differential calculus (Part 4) directly, but in terms of economics, mathematical programming and game theory (Part 6) should follow optimization (Part 4) before tackling dynamics. The reader can take his choice.

The major guiding principle in the writing of this book is to make it readable and teachable. To this end, graphs are employed, wherever needed, to elucidate the mathematical discussion. Also, a liberal quantity of cross references are supplied so that the reader may review, compare, and integrate the various subjects presented at different points in the volume. Toward the same goal, intuitive and economic explanations are frequently given to clarify the *why* of a particular mathematical operation. Exercises are presented at the end of almost every section; for maximum benefit, the reader should work out as many of these exercises as possible, if indeed not all.

Although the emphasis within the book falls primarily on methodology, the text does contain detailed discussions of numerous economic models, including several models of the market, models of the firm and of the consumer, national-income models, input-output models, as well as models of economic growth. Consequently, aside from its obvious relevance to the standard courses of mathematics for economists and mathematical economics, it should also be helpful as a supplementary text in such courses as price theory, national-income analysis, business cycles, economic development, and economic growth.

The materials presented herein have been used by my students during the years past. Their questions and comments, especially those of Mrs. Roberta Grower Carey, often provided valuable guidance in the revisions which culminated in the present version. In addition, Professor Marc Nerlove of Northwestern University was kind enough to read the entire manuscript and to provide me with detailed suggestions that have led to substantial improvement. Professor John C. H. Fei of Yale University also read portions of the manuscript and made useful comments. To all of them, I am deeply grateful. My thanks go also to The University of Connecticut for lightening my teaching load while the book was under way. Last, but not least, I must express my profound appreciation to my wife, who not only had to relinquish countless hours of my time that rightfully belonged to her, but also cheerfully contributed her talent as a critic and as a typist to this undertaking.

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ONE

INTRODUCTION

1

THE NATURE OF MATHEMATICAL ECONOMICS

Mathematical economics is not a distinct branch of economics in the sense that public finance or international trade is. Rather, it is an *approach* to economic analysis, in which the economist makes use of mathematical symbols in the statement of his problem and also draws upon known mathematical theorems to aid in his reasoning. As far as the specific subject matter of analysis goes, it can be micro- or macroeconomic theory, or public finance, or the economics of underdeveloped countries, or what not.

Using the term *mathematical economics* in the broadest possible sense, one may very well say that every elementary textbook of economics today exemplifies mathematical economics insofar as geometrical methods are frequently utilized to derive theoretical results. Such a usage of the term is obviously too general. Conventionally, mathematical economics is reserved to describe cases employing mathematical techniques beyond simple geometry, such as matrix algebra, differential and integral calculus, differential equations, difference equations, and set theory. It is the purpose of this book to introduce the reader to the most fundamental aspects of these mathematical methods—those encountered daily in the current economic literature.

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