

Edited by

**Andrew Harvey,
Siem Jan Koopman
and Neil Shephard**

State Space and Unobserved Component Models

Theory and Applications

CAMBRIDGE

STATE SPACE AND UNOBSERVED COMPONENT MODELS

Theory and Applications

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Preface

State space methods and unobserved component models are important in a wide range of subjects, including economics, finance, environmental science, medicine and engineering. The conference ‘State space and unobserved component models’, part of the Academy Colloquium programme of the Royal Netherlands Academy of Arts and Sciences, held in Amsterdam from 29 August to 3 September, 2002 brought together researchers from many different areas, but all pursuing a common statistical theme. The papers selected for this volume will give people unfamiliar with state space models a flavour of the work being carried out as well as providing experts with valuable state-of-the-art summaries of different topics.

The conference on state space methods afforded an ideal opportunity to honour Jim Durbin. Jim has been an active researcher in statistics for over fifty years. His first paper, published in 1950, set out the theory of what was to become known as the Durbin–Watson test. He subsequently published in many other areas of statistics, including sampling theory and regression, but over the last fifteen years or so his attention has again been focussed on time series, and in particular on state space models. A steady stream of work has appeared, beginning with the study of the British seat belt law with Andrew Harvey and culminating in the book with Siem Jan Koopman. It is entirely fitting that the first article in the volume should be by Jim. His clear and lucid style has been an inspiration to generations of students at the London School of Economics – including all three editors of this book – and his paper provides an ideal introduction to unobserved components models. We write some words about Jim’s career at the end of this Preface.

The introductory section of the book has two other papers. The first, by Peter Whittle, explores various aspects of the structure of state space models, particularly as they pertain to decision making, from a control engineering perspective. The second, by Simon Maskell, is an introduction

to the use of particle filtering for readers who are familiar with Kalman filtering. Particle filters have proved of enormous value in dealing with non-linear state space models. The potential for using particle filters in areas such as economics and finance is well illustrated by the contribution made by Jonathan Stroud, Nicholas Polson and Peter Müller in the applications section of the book.

The second section deals with testing. Katsuto Tanaka's article is on the use of wavelet-based methods in the estimation of long memory models with additive noise. Like particle filters, wavelets have had significant impact on the engineering literature and it is interesting to see whether they are able to help in the statistical treatment of an unobserved component model. Andrew Harvey's article brings together various tests concerning cycles. Many of the test statistics are linked by the Cramér-von Mises distribution, a distribution that also plays an important role in the paper by Ted Anderson and Michael Stephens. They examine some goodness-of-fit statistics, a field to which Jim Durbin has made major contributions.

The third section contains two papers on Bayesian inference and one on the bootstrap. Sylvia Frühwirth-Schnatter provides the first systematic treatment of the effect of parameterising state space models on the effectiveness of Markov chain Monte Carlo algorithms. The econometricians Gary Koop and Dale Poirier carefully study a Bayesian treatment of a spline model, focusing on the effect of priors on hyperparameters and initial values of states. David Stoffer and Kent Wall provide a survey of the recent work on the use of bootstrap to provide accurate classical inference on state space models.

The last section contains applications. The papers by Ole Barndorff-Nielsen, Bent Nielsen, Neil Shephard and Carla Ysusi and by Stroud *et al.* both deal with stochastic volatility, a key area in financial econometrics. Eric Zivot, Jeffrey Wang and Siem Jan Koopman provide details of a port of **SsfPack** to **S-PLUS** as part of the **S+FinMetrics** module. **SsfPack** is a library of flexible functions for routine Kalman filtering, smoothing and simulation smoothing for general Gaussian, linear state space models. They are sufficiently flexible that they can also be used as components in the analysis of various non-Gaussian and nonlinear models when combined with importance sampling or Markov chain Monte Carlo techniques. The authors illustrate some of the possibilities with applications of state space methods in finance and macroeconomics. William Bell's paper is on regression component models, with particular emphasis on applications within the US Bureau of the Census. Finally, Richard Durbin, Jim's son, shows how hidden Markov chains can be used to find genes in the human genome.

About the conference

The editors of this volume, with the support of the Royal Netherlands Academy of Arts and Sciences, organised the conference ‘State space and unobserved component models’ at the end of summer 2002 (29 August–3 September). The venue was ‘Het Trippenhuis’ in the centre of Amsterdam.

The conference focused on the following topics:

- modern computing tools for nonlinear and non-Gaussian models;
- unobserved components: estimation and testing;
- signal extraction and dynamic factor models;
- methods for official statistics;
- applications in areas like biostatistics, economics, engineering and finance.

The first part of the conference (the Academy Colloquium, 29–31 August) was designed as a workshop for about fifty invited researchers in the field. The participants and the programme of the Colloquium are listed below. The second part was the Masterclass that was attended by an international audience of more than 100 researchers and Ph.D. students. The programme of the masterclass is also presented.

We were delighted that many participated in this conference by presenting a paper on a topic related to the overall theme of the conference or by discussing papers or by attending the conference. A selection of the papers

Conference participants

T.W. Anderson,	Stanford University
Richard T. Baillie,	Michigan State University
William R. Bell,	Statistical Research Division, US Bureau of Census
Charles S. Bos,	Free University Amsterdam
Fabio Buseti,	Bank of Italy, Rome
Richard A. Davis,	Colorado State University
Herman van Dijk,	Erasmus University Rotterdam
Arnaud Doucet,	University of Cambridge
Catherine Doz,	University of Cergy-Pontoise, France
J. Durbin,	London School of Economics
Richard M. Durbin,	Sanger Centre, Cambridge
Cristiano A.C. Fernandes,	Pontificada Universidad Catolica, Rio de Janeiro
David F. Findley,	Statistical Research Division, US Bureau of Census
Sylvia Frühwirth-Schnatter,	Johannes Kepler University, Linz
Andrew C. Harvey,	University of Cambridge
Richard H. Jones,	University of Colorado
Genshiro Kitagawa,	Institute of Statistics and Mathematics, Tokyo
Gary Koop,	University of Leicester

Siem Jan Koopman,	Free University, Amsterdam
Hans Rudolf Künsch,	ETH Zurich
Kai Ming Lee,	Tinbergen Institute Amsterdam
Rob E. Luginbuhl,	Free University Amsterdam
Jan R. Magnus,	CentER for Economic Research, Tilburg University
Simon Maskell,	QinetiQ, Malvern
Brendan McCabe,	University of Liverpool
Filippo Moauro,	National Institute of Statistics, Italy
Roderick Molenaar,	ABP Investments and Research
Charles Nelson,	University of Washington, Seattle
Jukka Nyblom,	Stockholm School of Economics
Marius Ooms,	Free University Amsterdam
Keith Ord,	University of Georgetown, Washington DC
Daniel Peña,	Carlos III University, Madrid
Danny Pfeiffermann,	Hebrew University, Tel Aviv
Mike K. Pitt,	Warwick University, Coventry
Tommaso Proietti,	Udine University
Thomas Rothenberg,	University of California, Berkeley
Neil Shephard,	Nuffield College, University of Oxford
Robert H. Shumway,	University of California
Pieter Jelle van der Sluis,	ABP Investments and Research
David Stoffer,	University of Pittsburgh
Jonathan R. Stroud,	University of Chicago
Katsuto Tanaka,	Hitosubashi University
A.M. Robert Taylor,	University of Birmingham
Richard Tiller,	US Bureau of Labour Statistics
Pedro Valls,	IBMEC Business School, Sao Paulo
Aart de Vos,	Free University Amsterdam
Peter Whittle,	University of Cambridge
Peter C. Young,	University of Lancaster
Kenneth F. Wallis,	Warwick University, Coventry
Eric Zivot,	University of Washington, Seattle

Papers presented at the conference

T.W. Anderson,	A goodness-of-fit test for AR(1) models and power against state space alternatives
Richard T. Baillie,	A high frequency perspective on the forward premium anomaly
William R. Bell,	On some applications of RegComponent time series models
Richard A. Davis,	Observation driven models for Poisson counts
Arnaud Doucet,	Optimisation of particle methods using stochastic approximation

Richard M. Durbin,	Finding genes in the human genome with hidden Markov models
Sylvia Frühwirth-Schnatter,	Efficient Bayesian parameter estimation for state space models based on reparametrisations
Andrew C. Harvey,	Testing for cycles
Richard H. Jones,	Binary longitudinal data
Genshiro Kitagawa,	State space modelling for signal extraction problems in seismology
Gary Koop,	Empirical Bayesian inference in nonparametric model
Siem Jan Koopman,	Modelling economic convergence using unobserved converging components
Hans Rudolf Künsch,	Variations on the particle filter
Charles Nelson,	The structural break in the equity premium
Keith Ord,	The single source of error specification for state space models: an appraisal
Daniel Peña,	Dimension reduction in multivariate time series
Danny Pfeiffermann,	State space modelling with correlated measurement errors with application to small area estimation under benchmark constraints
Neil Shephard,	High frequency financial econometrics: extracting information from realised variances
Robert H. Shumway,	Dynamic mixed models for merging multiple time series fragments
David Stoffer,	Resampling in state space models
Jonathan Stroud,	Practical filtering with parameter learning
Katsuto Tanaka,	Wavelet methods for inference problems associated with long-memory signal plus noise models
A.M.Robert Taylor,	Variance shifts, structural breaks and stationarity tests
Peter Whittle,	State structure, decision making and related issues
Peter C. Young,	Data-based mechanistic modelling and state dependent parameter models

Masterclass

Richard A. Davis,	The innovations algorithm and parameter driven models
Jim Durbin,	Introduction to state space models
Simon Maskell,	Monte Carlo filtering
Jukka Nyblom,	Testing for unobserved components models
Keith Ord,	A new look at models for exponential smoothing
Richard Tiller,	Application of state space modeling for official labor force statistics in the USA
Eric Zivot,	State space modelling in macroeconomics and finance using S+FinMetrics

is included in this volume. We would particularly like to thank those scholars who helped us to referee these papers.

About Professor James Durbin

James Durbin was born 1923 in England. He was educated at St John's College, Cambridge. From 1950 he worked at the London School of Economics and Political Science (LSE) until his retirement in 1988; he remains an Emeritus Professor of Statistics there. He visited research institutions in many corners of the world during his period at the LSE and after his retirement. His research areas include serial correlation (13 publications), time series (>30), sample survey methodology (9), goodness-of-fit tests and sample distribution functions (13) and probability and statistical theory (20). He was deputy editor of *Biometrika* (1962–4) and associate editor of *Biometrika* (1960–2, 1964–7), *Annals of Statistics* (1973–5) and *Journal of the Royal Statistical Society, Series B* (1978–81).

He was the President of the Royal Statistical Society for the period 1986–7 and the President of the International Statistical Institute for the period of 1983–5. Further, he is a Fellow of the Econometric Society (since 1967), the Institute of Mathematical Statistics (since 1958) and the American Statistical Society (since 1960). In 2001 he became a Fellow of The British Academy.

More details about the career of Professor Durbin can be found *Econometric Theory* (1988, volume 4, issued) in which the interview with Professor J. Durbin by Professor Peter C. B. Phillips was published. His publications between 1950 and 1988 are listed at the end of the interview. Below we report his publications from 1988 onwards.

Publications of Professor James Durbin since 1988

- (i) Statistics and statistical science (presidential address). *Journal of the Royal Statistical Society, Series A*, 150, 177–191, 1988.
- (ii) Is a philosophical consensus for statistics attainable? *Journal of Econometrics*, 37, 51–61, 1988.
- (iii) Maximum likelihood estimation of the parameters of a system of simultaneous regression equations. *Econometric Theory*, 4, 159–170, 1988.
- (iv) A reconciliation of two different expressions for the first passage density of Brownian motion to a curved boundary. *Journal of Applied Probability*, 25, 829–832, 1988.

- (v) First passage densities of Gaussian and point processes to general boundaries with special reference to Kolmogorov–Smirnov tests when parameters are estimated. *Proceedings of First World Conference of the Bernoulli Society, Tashkent, USSR*, 1988.
- (vi) Extensions of Kalman modelling to non-Gaussian observations. *Quadermi di Statistica e Matematica applicata*, 12, 3–12, 1990.
- (vii) The first passage density of the Brownian motion process to a curved boundary. *Journal of Applied Probability*, 29, 291–304, 1992.
- (viii) On a test of serial correlation for regression models with lagged dependent variables. *The Art of Statistical Science*, edited K. V. Mardia, pp. 27–32, New York: Wiley, 1992.
- (ix) Optimal estimating equations for state vectors in non-Gaussian and nonlinear state space time series models. *Selected Proceedings of Athens, Georgia, Symposium on Estimating Functions*. Edited I. V. Basawa, V. P. Godambe and R. L. Taylor, 1997.
- (x) (with S. J. Koopman) Monte Carlo maximum likelihood estimation for non-Gaussian state space models. *Biometrika*, 84, 669–684, 1997.
- (xi) (with B. Quenneville) Benchmarking by state space models. *International Statistical Review*, 65, 23–48, 1997.
- (xii) (with J. R. Magnus) Estimation of regression coefficients of interest when other regression coefficients are of no interest. *Econometrica*, 67, 639–643, 1999.
- (xiii) The state space approach to time series analysis and its potential for official statistics (The Foreman Lecture). *Australian and New Zealand Journal of Statistics*, 42, 1–23, 2000.
- (xiv) (with S. J. Koopman) Time series analysis of non-Gaussian observations based on state space models from both classical and Bayesian perspectives (with discussion). *Journal of the Royal Statistical Society, Series B*, 62, 3–56, 2000.
- (xv) (with S. J. Koopman) Fast filtering and smoothing for multivariate state space models. *Journal of Time Series Analysis*, 21, 281–296, 2000.
- (xvi) (with S. J. Koopman) *Time Series Analysis by State Space Methods*. Oxford: Oxford University Press, 2001.
- (xvii) (with S. J. Koopman) A simple and efficient simulation smoother for state space time series analysis. *Biometrika*, 89, 603–616, 2002.
- (xviii) (with S. J. Koopman) Filtering and smoothing of state vector for diffuse state space models. *Journal of Time Series Analysis*, 24, 85–98, 2003.

Acknowledgements

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Part I

State space models

1

Introduction to state space time series analysis

James Durbin

Department of Statistics, London School of Economics and Political Science

Abstract

The paper presents a broad general review of the state space approach to time series analysis. It begins with an introduction to the linear Gaussian state space model. Applications to problems in practical time series analysis are considered. The state space approach is briefly compared with the Box-Jenkins approach. The Kalman filter and smoother and the simulation smoother are described. Missing observations, forecasting and initialisation are considered. A representation of a multivariate series as a univariate series is displayed. The construction and maximisation of the likelihood function are discussed. An application to real data is presented. The treatment is extended to non-Gaussian and nonlinear state space models. A simulation technique based on importance sampling is described for analysing these models. The use of antithetic variables in the simulation is considered. Bayesian analysis of the models is developed based on an extension of the importance sampling technique. Classical and Bayesian methods are applied to a real time series.

1.1 Introduction to state space models

1.1.1 Basic ideas

The organisers have asked me to provide a broad, general introduction to state space time series analysis. In the pursuit of this objective I will try to make the exposition understandable for those who have relatively little prior knowledge of the subject, while at the same time including some results of recent research. My starting point is the claim that state space models provide an effective basis for practical time series analysis in a wide range of fields including statistics, econometrics and engineering.

I will base my exposition on the recent book by Durbin and Koopman (2001), referred to from now on as the DK book, which provides a comprehensive treatment of the subject. Readers may wish to refer to the website <http://www.ssfpack.com/dkbook/> for further information about the book. Other books that provide treatments of state space models and techniques include Harvey (1989), West and Harrison (1997), Kitagawa and Gersch (1996) and Kim and Nelson (1999). More general books on time series analysis with substantial treatments of state space methods are, for example, Brockwell and Davis (1987), Hamilton (1994) and Shumway and Stoffer (2000).

I will begin with a particular example that I will use to introduce the basic ideas that underlie state space time series analysis. This refers to logged monthly numbers of car drivers who were killed or seriously injured in road accidents in Great Britain, 1969–84. These data come from a study by Andrew Harvey and me, undertaken on behalf of the British Department of Transport, regarding the effect on road casualties of the seat belt law that was introduced in February 1983; for details see Durbin and Harvey (1985) and Harvey and Durbin (1986).

Inspection of Figure 1.1 reveals that the series is made up of a trend which initially is increasing, then decreases and subsequently flattens out, plus a seasonal effect which is high in the winter and low in the summer, together with a sudden drop in early 1983 seemingly due to the introduction of the seat belt law. Other features that could be present, though they are not apparent from visual inspection, include cycles and regression effects due to the influence of such factors as the price of petrol, weather variations and traffic density. Thus we arrive at the following model:

$$y_t = \mu_t + \gamma_t + c_t + r_t + i_t + \varepsilon_t, \quad (1.1)$$

where