SCHAUM'S OUTLINE SERIES

THEORY AND PROBLEMS OF

ELECTRIC CIRCUITS

JOSEPH A. EDMINISTER

INCLUDING 350 SOLVED PROBLEMS

SCHAUM'S OUTLINE SERIES IN ENGINEERING

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SCHAUM'S OUTLINE OF

THEORY AND PROBLEMS

OF

ELECTRIC CIRCUITS

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SCHAUM'S OUTLINE SERIES
McGRAW-HILL BOOK COMPANY
New York, St. Louis, San Francisco, Toronto, Sydney

ISBN 07-018974-9

18 19 20 21 22 23 24 25 26 27 28 29 30 SH SH 8 7 6 5 4 3 2 1 0

Preface

This book is designed for use as a supplement to all current standard texts or as a textbook for a first course in circuit analysis. Emphasis is placed on the basic laws, theorems and techniques which are common to the various approaches found in other texts.

The subject matter is divided into chapters covering duly-recognized areas of theory and study. Each chapter begins with statements of pertinent definitions, principles and theorems together with illustrative and other descriptive material. This is followed by graded sets of solved and supplementary problems. The solved problems serve to illustrate and amplify the theory, present methods of analysis, provide practical examples, and bring into sharp focus those fine points which enable the student to apply the basic principles correctly and confidently. The large number of supplementary problems serve as a complete review of the material of each chapter.

Topics covered include fundamental circuit responses, analysis of waveforms, the complex number system, phasor notation, series and parallel circuits, power and power factor correction, and resonance phenomena. Considerable use of matrices and determinants is made in the treatment of mesh current and node voltage methods of analysis. Matrix methods are also employed in the development of wye-delta transformations and network theorems such as superposition and reciprocity. Mutually coupled circuits are very carefully explained. Polyphase circuits of all types are covered, with emphasis on the one-line equivalent circuit which has important applications. The trigonometric and exponential Fourier series are treated simultaneously, and the coefficients of one are frequently converted to coefficients of the other to show their relationship. Direct and alternating current transients are treated using classical differential equations so that this topic can precede the phasor notation of Chapter 5, and this is recommended for those whose proficiency in mathematics will permit this arrangement. The Laplace transform method is introduced and applied to many of the same problems treated in Chapter 16 by differential equations. This permits a convenient comparison of the two methods and emphasizes the strong points of the Laplace method.

I wish to avail myself of this opportunity to express my gratitude to the staff of the Schaum Publishing Company, especially to Mr. Nicola Miracapillo, for their valuable suggestions and helpful cooperation. Thanks and more are due my wife, Nina, for her unfailing assistance and encouragement in this endeavor.

JOSEPH A. EDMINISTER

The University of Akron
August 21, 1965

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Chapter 1

Definitions and Circuit Parameters

MECHANICAL UNITS

The rationalized MKS system of units is used in electrical engineering.

In this system the fundamental mechanical units are the meter (m) of length, the kilogram (kg) of mass, and the second (sec) of time. The corresponding derived force unit, the newton (nt), is that unbalanced force which will produce an acceleration of 1 m/sec^2 in a mass of 1 kg.

$$\text{Force (newtons)} = \text{mass (kilograms)} \times \text{acceleration (m/sec}^2)$$

It follows that the mks unit of work and energy is the newton-meter, called the joule, and the unit of power is the joule/sec or watt. (1 newton-meter = 1 joule, 1 joule/sec = 1 watt)

COULOMB'S LAW

The force F between two point charges q and q' varies directly as the magnitude of each charge and inversely as the square of the distance r between them.

$$F = k\frac{qq'}{r^2}$$

where k is a (dimensional) proportionality constant which depends on the units used for charge, distance and force. F is given in newtons if q and q' are in coulombs, r in meters, and

$$k = 9 \times 10^9 \text{ nt-m}^2/\text{coul}^2$$

If we now define $k = \dfrac{1}{4\pi\epsilon_0}$, then $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{qq'}{r^2}$ where $\epsilon_0 = \dfrac{1}{4\pi k} = 8.85 \times 10^{-12}$ coul^2/nt-m^2.

When the surrounding medium is not a vacuum, forces caused by charges induced in the medium reduce the resultant force between free charges immersed in the medium. The net force is now given by $F = \dfrac{1}{4\pi\epsilon}\dfrac{qq'}{r^2}$. For air ϵ is only slightly larger than ϵ_0 and for most purposes is taken equal to ϵ_0. For other materials ϵ is given by

$$\epsilon = K\epsilon_0$$

where K is a dimensionless constant called the *dielectric constant* or *specific inductive capacity* of the material between the charges, $\epsilon = K\epsilon_0$ is called the *permittivity* of the material, and ϵ_0 the *permittivity of free space.* For a vacuum, $K=1$ and $\epsilon = \epsilon_0$.

The unit of charge, the coulomb, may be defined as the quantity of charge which, when placed 1 meter from an equal and similar charge in vacuum, repels it with a force of 9×10^9 newtons. Convenient submultiples of the coulomb are

$$1\ \mu\text{c} = 1 \text{ microcoulomb} = 10^{-6} \text{ coulomb}$$
$$1\ \mu\mu\text{c} = 1 \text{ micromicrocoulomb} = 10^{-12} \text{ coulomb}$$

The charge carried by an electron ($-e$) or by a proton ($+e$) is $e = 1.602 \times 10^{-19}$ coulomb.

POTENTIAL DIFFERENCE v

The potential difference v between two points is measured by the work required to transfer unit charge from one point to the other. The *volt* is the potential difference (p.d.) between two points when 1 joule of work is required to transfer 1 coulomb of charge from one point to the other: 1 volt = 1 joule/coulomb.

If two points of an external circuit have a potential difference v, then a charge q in passing between the two circuit points does an amount of work qv as it moves from the higher to the lower potential point.

An agent such as a battery or generator has an electromotive force (emf) if it does work on the charge moving through it, the charge receiving electrical energy as it moves from the lower to the higher potential side. Emf is measured by the p.d. between the terminals when the generator is not delivering current.

CURRENT i

A material containing free electrons capable of moving from one atom to the next is a conductor. The application of a potential difference causes these electrons to move.

An electric current exists in a conductor whenever charge q is being transferred from one point to another in that conductor. If charge is transferred at the uniform rate of 1 coulomb/sec, then the constant current existing in the conductor is 1 ampere: 1 ampere = 1 coulomb/sec. In general, the instantaneous current i in a conductor is

$$i\,(\text{amperes}) \;=\; \frac{dq\,(\text{coulombs})}{dt\,(\text{seconds})}$$

The positive current direction is, by convention, opposite to the direction in which the electrons move. See Fig. 1-1.

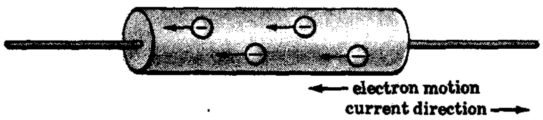


Fig. 1-1

POWER p

Electrical power p is the product of impressed voltage v and resulting current i.

$$p\,(\text{watts}) \;=\; v\,(\text{volts}) \times i\,(\text{amperes})$$

Positive current, by definition, is in the direction of the arrow on the voltage source; it leaves the source by the + terminal as shown in Fig. 1-2. When p has a positive value the source transfers energy to the circuit.

If power p is a periodic function of time t with period T, then the

$$\text{Average power } P \;=\; \frac{1}{T}\int_0^T p\,dt$$

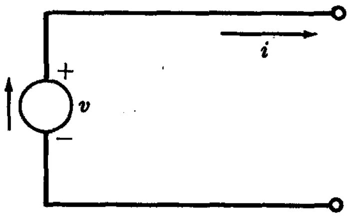


Fig. 1-2

ENERGY w

Since power p is the time rate of energy transfer,

$$p = \frac{dw}{dt} \qquad \text{and} \qquad W = \int_{t_1}^{t_2} p\,dt$$

where W is the energy transferred during the time interval.

RESISTOR, INDUCTOR, CAPACITOR

When electrical energy is supplied to a circuit element, it will respond in one or more of the following three ways. If the energy is consumed, then the circuit element is a pure *resistor*. If the energy is stored in a magnetic field, the element is a pure *inductor*. And if the energy is stored in an electric field, the element is a pure *capacitor*. A practical circuit device exhibits more than one of the above and perhaps all three at the same time, but one may be predominant. A coil may be designed to have a high inductance, but the wire with which it is wound has some resistance; hence the coil has both properties.

RESISTANCE R

The potential difference $v(t)$ across the terminals of a pure resistor is directly proportional to the current $i(t)$ in it. The constant of proportionality R is called the resistance of the resistor and is expressed in volts/ampere or ohms.

$$v(t) = R\,i(t) \qquad \text{and} \qquad i(t) = \frac{v(t)}{R}$$

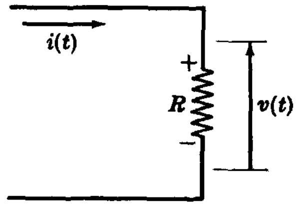


Fig. 1-3

No restriction is placed on $v(t)$ and $i(t)$; they may be constant with respect to time, as in D.C. circuits, or they may be sine or cosine functions, etc.

Lower case letters (v, i, p) indicate general functions of time. Capital letters (V, I, P) denote constant quantities, and peak or maximum values carry a subscript (V_m, I_m, P_m).

INDUCTANCE L

When the current in a circuit is changing, the magnetic flux linking the same circuit changes. This change in flux causes an emf v to be induced in the circuit. The induced emf v is proportional to the time rate of change of current if the permeability is constant. The constant of proportionality is called the *self-inductance* or *inductance* of the circuit.

$$v(t) = L\frac{di}{dt} \qquad \text{and} \qquad i(t) = \frac{1}{L}\int v\,dt$$

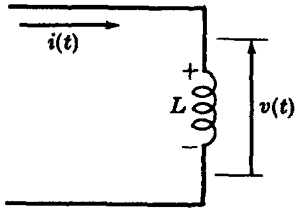


Fig. 1-4

When v is in volts and di/dt in amperes/sec, L is in volt-sec/ampere or *henries*. The self-inductance of a circuit is 1 henry (1 h) if an emf of 1 volt is induced in it when the current changes at the rate of 1 ampere/sec.

CAPACITANCE C

The potential difference v between the terminals of a capacitor is proportional to the charge q on it. The constant of proportionality C is called the *capacitance* of the capacitor.

$$q(t) = C\,v(t), \quad i = \frac{dq}{dt} = C\frac{dv}{dt}, \quad v(t) = \frac{1}{C}\int i\,dt$$

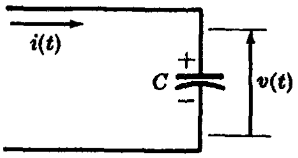


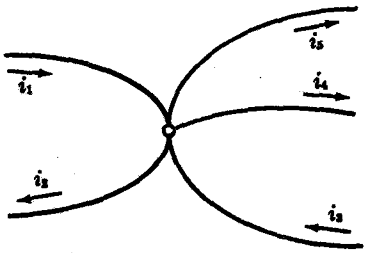
Fig. 1-5

When q is in coulombs and v in volts, C is in coulombs/volt or *farads*. A capacitor has capacitance 1 farad (1 f) if it requires 1 coulomb of charge per volt of potential difference between its conductors. Convenient submultiples of the farad are

$$1\,\mu\text{f} = 1\text{ microfarad} = 10^{-6}\text{ f} \quad \text{and} \quad 1\,\mu\mu\text{f} = 1\text{ micromicrofarad} = 10^{-12}\text{ f}$$

KIRCHHOFF'S LAWS

1. The sum of the currents entering a junction is equal to the sum of the currents leaving the junction. If the currents toward a junction are considered positive and those away from the same junction negative, then this law states that the algebraic sum of all the currents meeting at a common junction is zero.



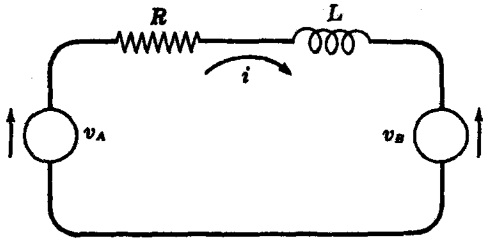
Σ currents entering = Σ currents leaving

$$i_1 + i_3 = i_2 + i_4 + i_5$$

or

$$i_1 + i_3 - i_2 - i_4 - i_5 = 0$$

Fig. 1-6



Σ potential rises = Σ potential drops

$$v_A - v_B = Ri + L(di/dt)$$

or

$$v_A - v_B - Ri - L(di/dt) = 0$$

Fig. 1-7

2. The sum of the rises of potential around any closed circuit equals the sum of the drops of potential in that circuit. In other words, the algebraic sum of the potential differences around a closed circuit is zero. With more than one source when the directions do not agree, the voltage of the source is taken as positive if it is in the direction of the assumed current.

Circuit Response of Single Elements

Element	Voltage across element	Current in element
Resistance R	$v(t) = R\,i(t)$	$i(t) = \dfrac{v(t)}{R}$
Inductance L	$v(t) = L\dfrac{di}{dt}$	$i(t) = \dfrac{1}{L}\int v\,dt$
Capacitance C	$v(t) = \dfrac{1}{C}\int i\,dt$	$i(t) = C\dfrac{dv}{dt}$

Units in the MKS System

Quantity		Unit		Quantity		Unit	
Length	l	meter	m	Charge	Q, q	coulomb	c
Mass	m	kilogram	kg	Potential	V, v	volt	v
Time	t	second	sec	Current	I, i	ampere	amp
Force	F, f	newton	nt	Resistance	R	ohm	Ω
Energy	W, w	joule	j	Inductance	L	henry	h
Power	P, p	watt	w	Capacitance	C	farad	f

Solved Problems

1.1. In the circuit shown in Fig. 1-8 the applied constant voltage is $V = 45$ volts. Find the current, the voltage drop across each resistor, and the power in each resistor.

The sum of the voltage rises equals the sum of the voltage drops around any closed loop; thus

$$V = I(2) + I(6) + I(7), \qquad 45 = 15I, \qquad I = 3 \text{ amp}$$

The voltage drop across the 2 ohm resistor is $V_2 = IR_2 = 3(2) = 6$ volts. Similarly, $V_6 = 3(6) = 18$ volts, and $V_7 = 21$ volts.

The power in the 2 ohm resistor is $P_2 = V_2 I = 6(3) = 18$ watts or $P_2 = I^2 R_2 = 3^2(2) = 18$ watts. Similarly, $P_6 = V_6 I = 54$ watts, and $P_7 = V_7 I = 63$ watts.

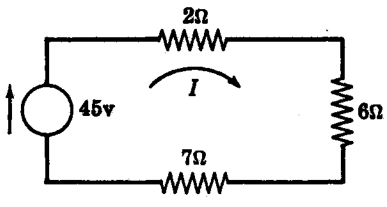


Fig. 1-8

1.2. A current I_T divides between two parallel branches having resistances R_1 and R_2 respectively as shown in Fig. 1-9. Develop formulas for the currents I_1 and I_2 in the parallel branches.

The voltage drop in each branch is the same, i.e. $V = I_1 R_1 = I_2 R_2$. Then

$$I_T = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= I_1 R_1 \left(\frac{R_2 + R_1}{R_1 R_2}\right) = I_1 \left(\frac{R_2 + R_1}{R_2}\right)$$

from which $I_1 = I_T\left(\dfrac{R_2}{R_1 + R_2}\right)$. Similarly, $I_2 = I_T\left(\dfrac{R_1}{R_1 + R_2}\right)$.

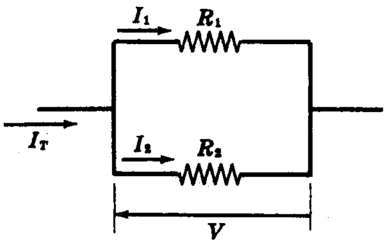


Fig. 1-9

1.3. Three resistors R_1, R_2, R_3 are in parallel as shown in Fig. 1-10. Derive a formula for the equivalent resistance R_e of the network.

Assume a voltage A to B of $v(t)$, and let the currents in R_1, R_2, R_3 be $i_1(t), i_2(t), i_3(t)$ respectively. The current in R_e must be the total current $i_T(t)$. Then $v(t) = R_1 i_1(t) = R_2 i_2(t) = R_3 i_3(t) = R_e i_T(t)$ and

$$i_T(t) = i_1(t) + i_2(t) + i_3(t) \quad \text{or} \quad \frac{v(t)}{R_e} = \frac{v(t)}{R_1} + \frac{v(t)}{R_2} + \frac{v(t)}{R_3}$$

or

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

For a two-branch parallel circuit, $\dfrac{1}{R_e} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ or $R_e = \dfrac{R_1 R_2}{R_1 + R_2}$.

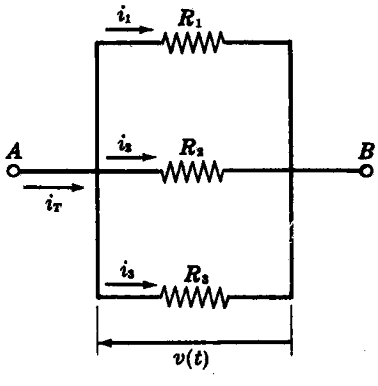


Fig. 1-10

1.4. The two constant voltage sources V_A and V_B act in the same circuit as shown in Fig. 1-11. What power does each deliver?

The sum of the potential rises is equal to the sum of the potential drops around a closed circuit; hence

$$20 - 50 = I(1) + I(2), \qquad I = -10 \text{ amp}$$

Power delivered by $V_A = V_A I = 20(-10) = -200$ watts.

Power delivered by $V_B = V_B I = 50(10) = 500$ watts.

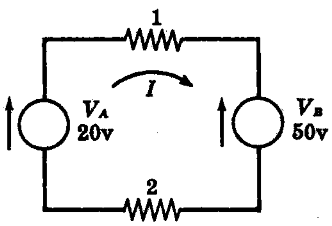


Fig. 1-11

1.5. In the circuit shown in Fig. 1-12(*a*), the voltage function is $v(t) = 150 \sin \omega t$. Find the current $i(t)$, the instantaneous power $p(t)$, and the average power P.

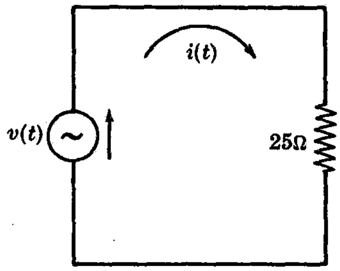


Fig. 1-12(*a*)

$$i(t) \;=\; \frac{1}{R}v(t) \;=\; \frac{150}{25}\sin \omega t \;=\; 6 \sin \omega t \text{ amperes}$$

$$p(t) \;=\; v(t)\,i(t) \;=\; (150 \sin \omega t)(6 \sin \omega t) \;=\; 900 \sin^2 \omega t \text{ watts}$$

$$P \;=\; \frac{1}{\pi}\int_0^{\pi} 900 \sin^2 \omega t \, d(\omega t) \;=\; \frac{900}{\pi}\int_0^{\pi} \tfrac{1}{2}(1 - \cos 2\omega t)\, d(\omega t)$$

$$\;=\; \frac{900}{2\pi}\left[\omega t - \tfrac{1}{2}\sin 2\omega t\right]_0^{\pi} \;=\; 450 \text{ watts}$$

The current $i(t)$ is seen to be related to the voltage $v(t)$ by the constant R. The instantaneous power plot could have been obtained by a point by point product of the v and i plots shown in Fig. 1-12(*b*) below. Note that v and i are both positive or both negative at any instant; the product must therefore always be positive. This agrees with the statement that whenever current flows through a resistor, electrical energy is delivered by the source.

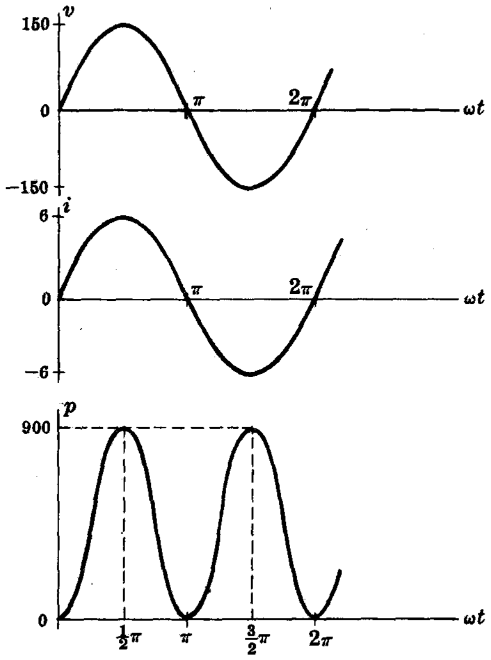


Fig. 1-12(*b*)

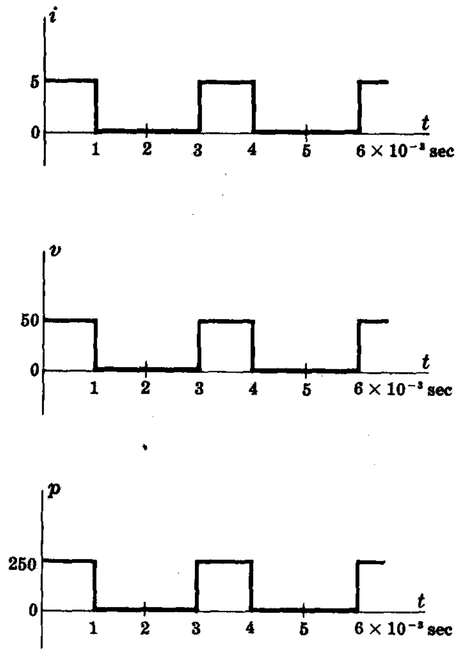


Fig. 1-13

1.6. The current function shown in Fig. 1-13 above is a repeating square wave. With this current existing in a pure resistor of 10 ohms, plot voltage $v(t)$ and power $p(t)$.

Since $v(t) = R\,i(t)$, the voltage varies directly as the current. The maximum value is $Ri_{max} = 5(10) = 50$ volts.

Since $p = vi$, the power plot is a point by point product. The maximum value is $i_{max}\,v_{max} = 50(5) = 250$ watts.

1.7. The current function shown in Fig. 1-14 below is a repeating sawtooth and exists in a pure resistor of 5 ohms. Find $v(t)$, $p(t)$, and average power P.

Since $v(t) = R\,i(t)$, $v_{max} = Ri_{max} = 5(10) = 50$ volts.

When $0 < t < 2 \times 10^{-3}$ sec, $i = \dfrac{10}{2 \times 10^{-3}}t = 5 \times 10^3 t$. Then

$$v = Ri = 25 \times 10^3 t, \quad p = vi = 125 \times 10^6 t^2, \quad P = \frac{1}{2 \times 10^{-3}}\int_0^{2\times 10^{-3}} 125 \times 10^6 t^2 \, dt = 167 \text{ watts}$$

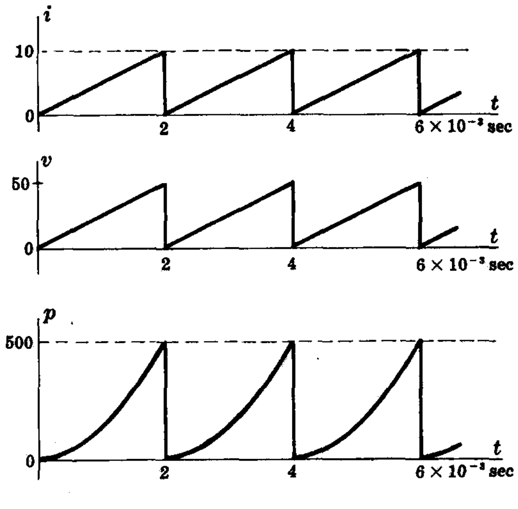


Fig. 1-14

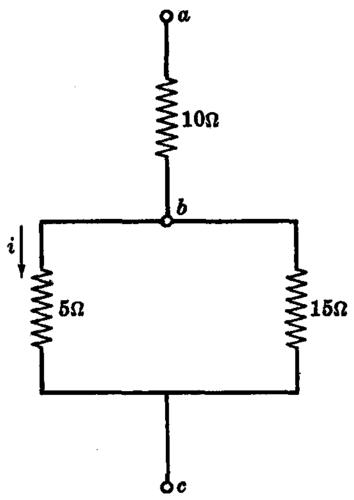


Fig. 1-15

1.8. In the circuit shown in Fig. 1-15 above, the current in the 5 ohm resistor is $i(t) = 6 \sin \omega t$ amperes. (*a*) Determine the current in the 15 and 10 ohm resistors and the voltages *a* to *b* and *b* to *c*. (*b*) Find the instantaneous and average power consumed in each resistor.

(*a*) The same voltage v_{bc} is across the 5 and 15 ohm resistors; then

$$v_{bc} = i_5 R_5 = (6 \sin \omega t)(5) = 30 \sin \omega t \quad \text{and} \quad i_{15} = v_{bc}/R_{15} = 2 \sin \omega t$$

Now $\quad i_{10} = i_{15} + i_5 = 8 \sin \omega t \quad$ and $\quad v_{ab} = i_{10} R_{10} = 80 \sin \omega t$

(*b*) Instantaneous power $p = vi$. Thus $p_5 = (30 \sin \omega t)(6 \sin \omega t) = 180 \sin^2 \omega t$. Similarly, $p_{15} = 60 \sin^2 \omega t$ and $p_{10} = 640 \sin^2 \omega t$.

Average power in 5 ohm resistor is

$$P_5 = \frac{1}{\pi}\int_0^\pi 180 \sin^2 \omega t \, d(\omega t) = \frac{1}{\pi}\int_0^\pi 180[\tfrac{1}{2}(1 - \cos 2\omega t)] \, d(\omega t) = 90 \text{ watts}$$

Similarly, $P_{15} = 30$ watts and $P_{10} = 320$ watts.

1.9. A pure resistor of 2 ohms has an applied voltage $v(t)$ given by

$$v(t) = 50\left[1 - \frac{(\omega t)^2}{2!} + \frac{(\omega t)^4}{4!} - \frac{(\omega t)^6}{6!} + \cdots\right] \text{volts}$$

Determine the current and power for this single circuit element.

Expanding $\cos x$ as a power series in x, $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$.

Hence $v(t) = 50 \cos \omega t$, $i(t) = 25 \cos \omega t$, $p(t) = 1250 \cos^2 \omega t$, and $P = 625$ watts.

1.10. A pure inductance $L = .02$ henrys has an applied voltage $v(t) = 150 \sin 1000t$. Determine the current $i(t)$, the instantaneous power $p(t)$, and the average power P.

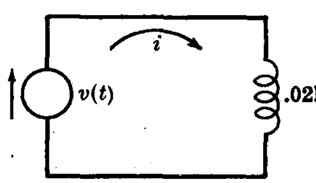


Fig. 1-16(*a*)

$$i(t) = \frac{1}{L}\int v(t)\,dt = \frac{1}{.02}\int 150 \sin 1000t \, dt$$

$$= \frac{150}{.02}\left(\frac{-\cos 1000t}{1000}\right) = -7.5 \cos 1000t \text{ amp}$$

$p = vi = -150(7.5)(\tfrac{1}{2} \sin 2000t) = -562.5 \sin 2000t$ watts. [$\sin x \cos x = \tfrac{1}{2} \sin 2x$.] The average power P is obviously zero, as shown in Fig. 1-16(*b*) below.

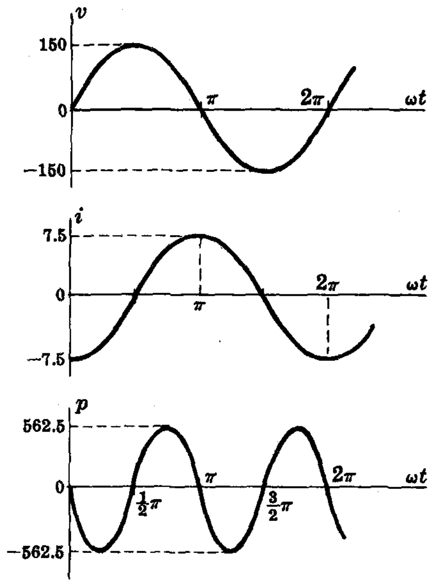


Fig. 1-16(b)

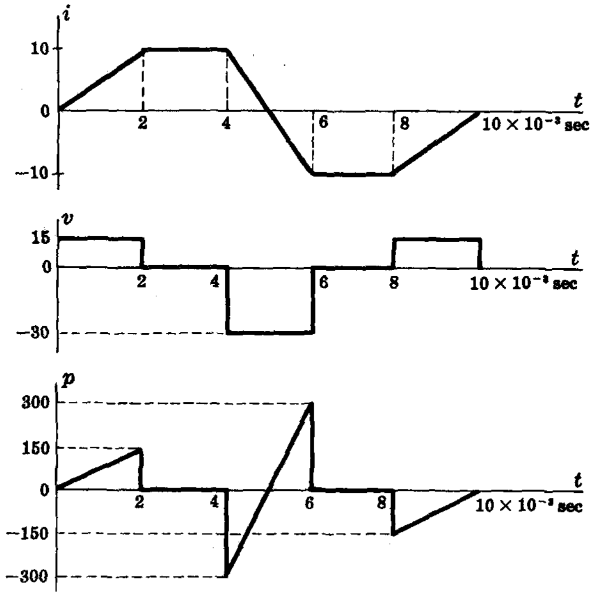


Fig. 1-17

1.11. A single pure inductance of 3 millihenrys passes a current of the waveform shown in Fig. 1-17 above. Determine and sketch the voltage $v(t)$ and the instantaneous power $p(t)$. What is the average power P?

The instantaneous current $i(t)$ is given by (see Fig. 1-17 above):

(1) $0 < t < 2$ ms $\quad i = 5 \times 10^3 t$

(2) $2 < t < 4$ ms $\quad i = 10$

(3) $4 < t < 6$ ms $\quad i = 10 - 10 \times 10^3(t - 4 \times 10^{-3}) = 50 - 10 \times 10^3 t$

(4) $6 < t < 8$ ms $\quad i = -10$

(5) $8 < t < 10$ ms $\quad i = -10 + 5 \times 10^3(t - 8 \times 10^{-3}) = -50 + 5 \times 10^3 t$

The corresponding voltages are:

(1) $v_L = L\dfrac{di}{dt} = 3 \times 10^{-3}\dfrac{d}{dt}(5 \times 10^3 t) = 15$ volts

(2) $v_L = L\dfrac{di}{dt} = 3 \times 10^{-3}\dfrac{d}{dt}(10) = 0$

(3) $v_L = L\dfrac{di}{dt} = 3 \times 10^{-3}\dfrac{d}{dt}(50 - 10 \times 10^3 t) = -30$ volts, etc.

The corresponding instantaneous power values are:

(1) $p = vi = 15(5 \times 10^3 t) = 75 \times 10^3 t$ watts

(2) $p = vi = 0(10) = 0$ watts

(3) $p = vi = -30(50 - 10 \times 10^3 t) = -1500 + 300 \times 10^3 t$ watts, etc.

The average power P is evidently zero.

1.12. A voltage $v(t)$ is applied across two inductances L_1 and L_2 in series. Determine the equivalent inductance L_e which can replace them and yield the same current.

Applied voltage = voltage drop across L_1 + drop across L_2

$$v(t) = L_e\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt}$$

from which $L_e = L_1 + L_2$.

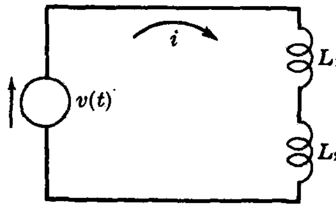


Fig. 1-18

1.13. Find the equivalent inductance L_e of two inductances L_1 and L_2 in parallel as shown in Fig. 1-19 below.

Assume a voltage $v(t)$ exists across the parallel combination and let the currents in L_1 and L_2 be i_1 and i_2 respectively. Since the total current i_T is the sum of the branch currents,

$$i_T = i_1 + i_2 \quad \text{or} \quad \frac{1}{L_e}\int v\,dt = \frac{1}{L_1}\int v\,dt + \frac{1}{L_2}\int v\,dt$$

Then
$$\frac{1}{L_e} = \frac{1}{L_1} + \frac{1}{L_2} \quad \text{or} \quad L_e = \frac{L_1 L_2}{L_1 + L_2}$$

The reciprocal of the equivalent inductance of any number of inductors connected in parallel is the sum of the reciprocals of the individual inductances.

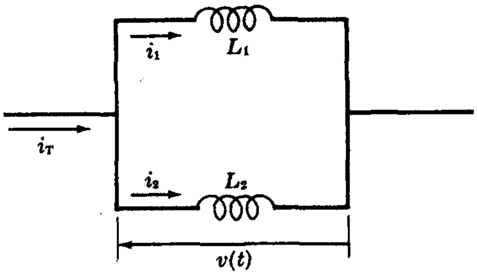


Fig. 1-19

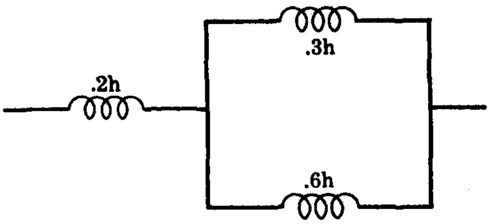


Fig. 1-20

1.14. Three pure inductances are connected as shown in Fig. 1-20 above. What equivalent inductance L_e may replace this circuit?

Equivalent inductance of parallel combination is $L_p = \dfrac{L_1 L_2}{L_1 + L_2} = \dfrac{.3(.6)}{.3 + .6} = .2$ h.

The required equivalent inductance $L_e = .2 + L_p = .4$ h.

1.15. A pure inductor carries a current $i(t) = I_m \sin \omega t$. Assuming the stored energy in the magnetic field is zero at $t = 0$, derive and sketch the energy function $w(t)$.

$$v(t) = L\frac{d}{dt}(I_m \sin \omega t) = \omega L I_m \cos \omega t$$

$$p(t) = vi = \omega L I_m^2 \sin \omega t \cos \omega t = \tfrac{1}{2}\omega L I_m^2 \sin 2\omega t$$

$$w(t) = \int_0^t \tfrac{1}{2}\omega L I_m^2 \sin 2\omega t\,dt = \tfrac{1}{4} L I_m^2 [-\cos 2\omega t + 1] = \tfrac{1}{2} L I_m^2 \sin^2 \omega t$$

At $\omega t = \pi/2, 3\pi/2, 5\pi/2$, etc, the stored energy is maximum and equals $\frac{1}{2}LI_m^2$. At $\omega t = 0, \pi, 2\pi, 3\pi$, etc, the stored energy is zero. See Fig. 1-21 below.

When $p(t)$ is positive the flow of energy is toward the load and the stored energy increases. When $p(t)$ is negative the energy is returning from the magnetic field of the inductor to the source. In a pure inductor no energy is consumed. The average power is zero and there is no net transfer of energy.

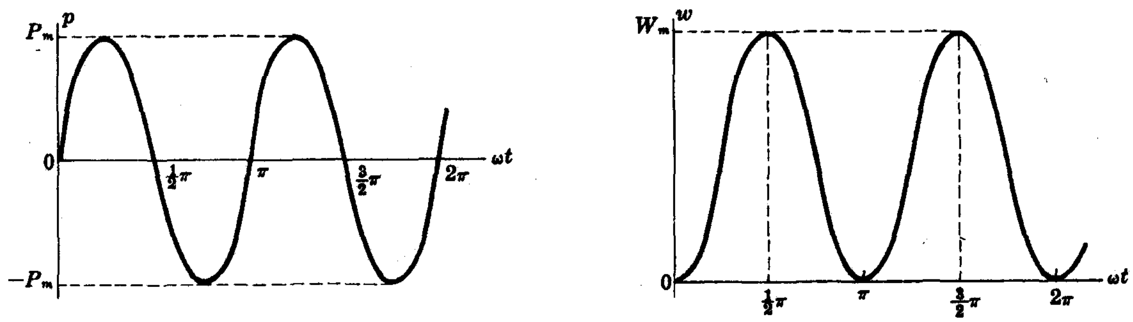


Fig. 1-21

1.16. Consider a pure capacitor with an applied voltage $v(t) = V_m \sin \omega t$. Find the current $i(t)$, the power $p(t)$, the charge $q(t)$, and the stored energy $w(t)$ in the electric field assuming $w(t) = 0$ at $t = 0$.

$$i(t) = C\,dv/dt = \omega C V_m \cos \omega t \text{ amperes}$$
$$p(t) = vi = \tfrac{1}{2}\omega C V_m^2 \sin 2\omega t \text{ watts}$$
$$q(t) = Cv = CV_m \sin \omega t \text{ coulombs}$$
$$w(t) = \int_0^t p\,dt = \tfrac{1}{4}CV_m^2(1 - \cos 2\omega t) = \tfrac{1}{2}CV_m^2 \sin^2 \omega t$$

At $\omega t = \pi/2, 3\pi/2, 5\pi/2$, etc, the stored energy is maximum and equals $\frac{1}{2}CV_m^2$. At $\omega t = 0, \pi, 2\pi, 3\pi$, etc, the stored energy is zero. See Fig. 1-22 below.

When $p(t)$ is positive the flow of energy is from the source to the electric field of the capacitor and the stored energy $w(t)$ is increasing. When $p(t)$ is negative, this stored energy is being returned to the source. The average power P is zero and there is no net transfer of energy.

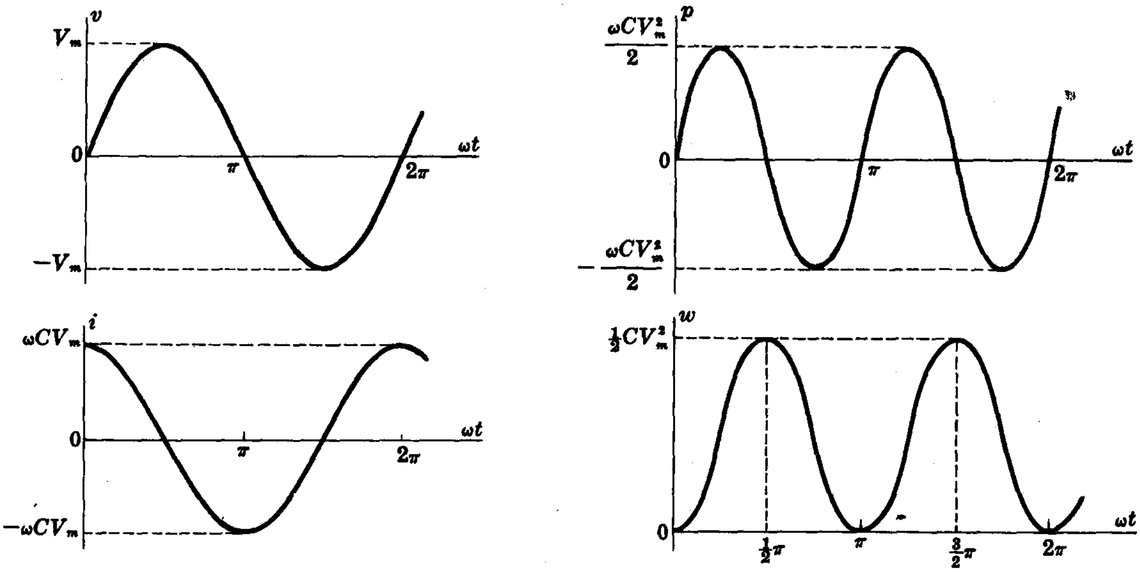


Fig. 1-22

1.17. Determine the equivalent capacitance C_e of the parallel combination of two capacitors C_1 and C_2 shown in Fig. 1-23 below.

Assume a voltage $v(t)$ exists across the parallel combination and let the currents in C_1 and C_2 be i_1 and i_2 respectively. Then, if the total current is i_T,

$$i_T = i_1 + i_2 \qquad \text{or} \qquad C_e \frac{d}{dt}v(t) = C_1 \frac{d}{dt}v(t) + C_2 \frac{d}{dt}v(t) \qquad \text{or} \qquad C_e = C_1 + C_2$$

The resultant (equivalent) capacitance of any number of capacitors connected in parallel is the sum of their individual capacitances.

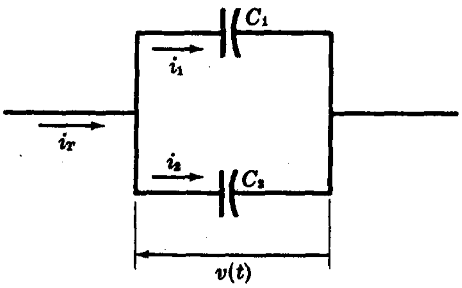


Fig. 1-23

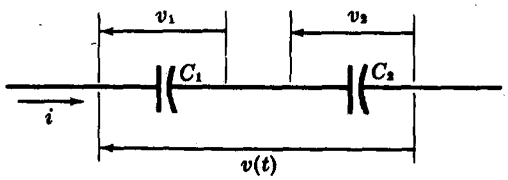


Fig. 1-24

1.18. Determine the equivalent capacitance C_e of the series combination of two capacitors C_1 and C_2 shown in Fig. 1-24 above.

Assume a voltage exists across the series circuit. Then

$$\text{Applied voltage} \;=\; \text{voltage drop across } C_1 + \text{voltage drop across } C_2$$

$$\frac{1}{C_e}\int i(t)\,dt \;=\; \frac{1}{C_1}\int i(t)\,dt + \frac{1}{C_2}\int i(t)\,dt$$

Then
$$\frac{1}{C_e} = \frac{1}{C_1}+\frac{1}{C_2} \qquad \text{or} \qquad C_e = \frac{C_1C_2}{C_1+C_2}$$

The reciprocal of the resultant (equivalent) capacitance of any number of capacitors connected in series is the sum of the reciprocals of the individual capacitances.

1.19. Find the equivalent capacitance C_e of the combination of capacitors shown in Fig. 1-25.

Equivalent capacitance of series branch is

$$C_s = \frac{C_1C_2}{C_1+C_2} = \frac{3(6)}{3+6} = 2\ \mu\text{f}$$

The required equivalent capacitance is

$$C_e = 4 + C_s = 6\ \mu\text{f} = 6\times10^{-6}\ \text{farads}$$

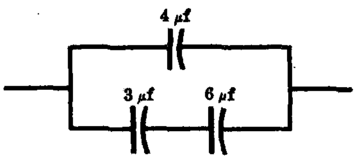


Fig. 1-25

1.20. The given series circuit passes a current $i(t)$ of waveform shown in Fig. 1-26. Find the voltage across each element and sketch each voltage to same time scale. Also sketch $q(t)$, the charge on the capacitor.

Across Resistor: $v_R = Ri$

The plot of v_R is a duplicate of the current function plot, with a peak value of $2(10) = 20$ volts.

Across Inductor: $v_L = L\,di/dt$

(1) $0 < t < 1$ ms $\quad i = 10\times10^3 t$

$$v_L = (2\times10^{-3})(10\times10^3) = 20$$

(2) $1 < t < 2$ ms $\quad i = 10$

$$v_L = (2\times10^{-3})(0) = 0$$

etc.

Across Capacitor: $v_C = \dfrac{1}{C}\displaystyle\int i\,dt$

(1) $0<t<1$ ms
$$v_C = \frac{1}{500\times10^{-6}}\int_0^t (10\times10^3 t)\,dt = 10\times10^6 t^2$$

(2) $1<t<2$ ms
$$v_C = 10 + \frac{1}{500\times10^{-6}}\int_{10^{-3}}^t (10)\,dt = 10 + 20\times10^3(t-10^{-3})$$

etc.

The plot of q is easily made using the relationship $q = Cv_C$. Note that when i is positive, both q and v_C increase, i.e. both the charge on the capacitor and the voltage across the capacitor increase; when i is negative, both decrease.

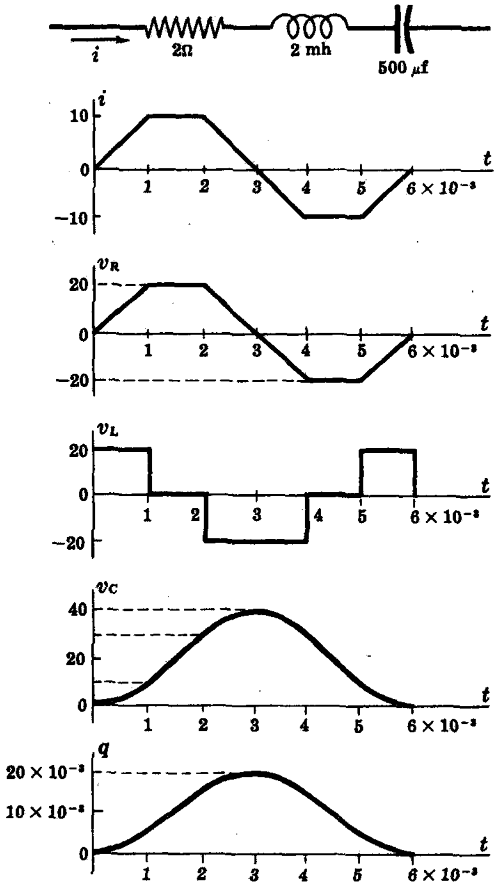


Fig. 1-26