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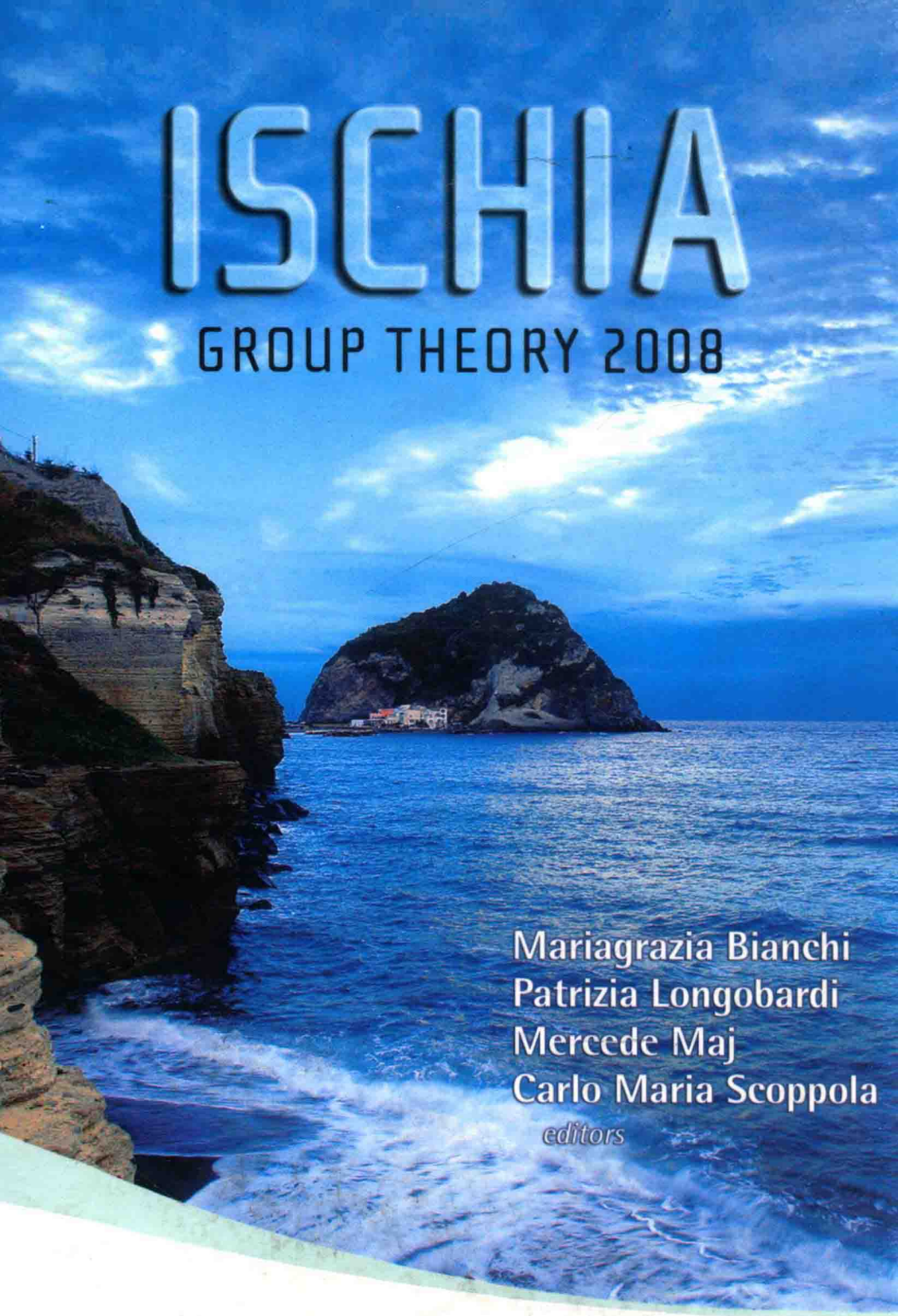
GROUP THEORY 2008



Mariagrazia Bianchi
Patrizia Longobardi
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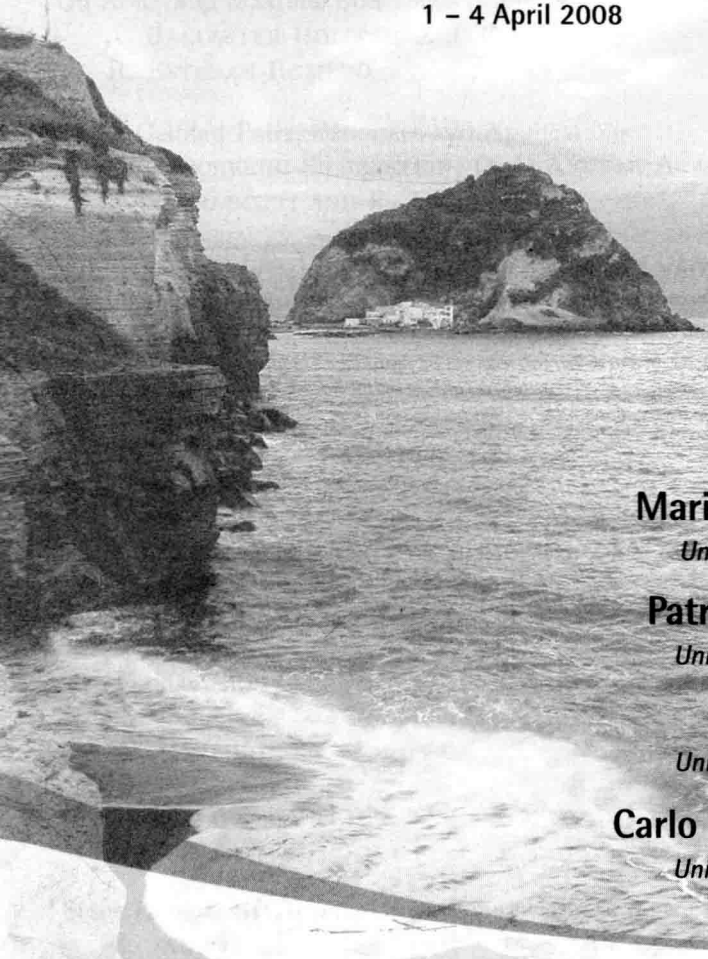
Proceedings of the Conference
Naples, Italy
1 – 4 April 2008



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ISCHIA

GROUP THEORY 2008

Preface

The research articles included in this volume represent the proceedings of the Conference "Ischia Group Theory 2008", the third in a series that previously included "Ischia Group Theory 2004, in honour of Marcel Herzog" and "Ischia Group Theory 2006: in Honor of Akbar Rhemtulla". The Conference took place at the Jolly Hotel, Ischia (Naples, Italy) from April 2nd to April 4th, 2008.

The scientific committee was formed by Mariagrazia BIANCHI (Università di Milano), Patrizia LONGOBARDI (Università di Salerno), Mercede MAJ (Università di Salerno), Carlo Maria SCOPPOLA (Università dell'Aquila), while the local organizing commitee, at Università di Salerno, consisted of Costantino DELIZIA, Annamaria LUCIBELLO, Chiara NICOTERA, Carmela SICA, Antonio TORTORA, Maria TOTA.

The papers in this volume are contributions by speakers and participants of the conference.

The span of the research topics represented in these Proceedings is rather large, and includes classification of classes of p-groups, character theory, finiteness conditions, profinite groups and pro-p-groups, subgroup structure, linear groups, Gelfand pairs, varieties of groups, groups and graphs.

The schedule of the talks and the list of all participants are included in these Proceedings.

A poster session on various research topics augmented the scientific program.

It is our pleasure to thank our colleagues of Università di Salerno, who made our stay very smooth and pleasant; the speakers, who greatly contributed to the scientific interest of the conference; the participants, who created an intense and lively atmosphere; the authors of all papers, for their interesting contributions; the referees of the papers, for their effective and accurate effort in providing the editors with reliable information; the staff of the Jolly Hotel for their patient, effective and accommodating support; the Publisher, for the professional and timely production of these Proceedings.

Last but not least, we thank our Sponsors, listed in the next page.

The Editors

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Conference Program

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09.20 Welcome Greetings

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On abnormal maximal subgroups of finite groups

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11.30 J. COSSEY
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12.20 Z. JANKO
Some exceptional minimal situations by finite p-groups

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15.30 H. SMITH
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16.20 P. NEUMANN
Non-separating groups and synchronization

17.10 Coffee Break

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17.30 B. AMBERG
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18.20 K. ERSOY
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18.40 A. FACCHINI
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12.50 M. DE FALCO Groups with few normalizer subgroups

13.10 Picture

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17.50 P. SPIGA
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10.20 A. CARANTI
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11.30 M. EVANS
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15.30 R. BRANDL
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17.00 K. AZIZIHERIS
Counting the number of nonlinear irreducible characters of a finite group

17.30 M. DIXON
Some simple locally (soluble-by-finite) groups

19.00 Concert of Baroque Music

20.30 Social Dinner

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Contents

On abnormal maximal subgroups of finite groups

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In this survey we show the influence of the abnormal maximal subgroups of finite groups in their structure.

Keywords: Finite groups; supersoluble group; nilpotent group; maximal subgroup; abnormal subgroup.

1. Introduction

This survey is about finite groups. Hence the unspoken rule is that all groups considered are finite.

> "What is the role of the abnormal subgroups in the structure of a group?"

is the motivating question in this survey. In fact, the results we present here are contributions to the long-running investigation of the influence on a group of its proper abnormal structure.

*Corresponding author

Consequently, abnormality, a subgroup embedding property introduced by P. Hall in his Cambridge lectures, is one of the central concepts here. Recall that a subgroup U of a group G is said to be *abnormal* in G if for all $g \in G$, $g \in \langle U, U^g \rangle$.

Theorem 1.1 (Hall, Cambridge lectures). *The following conditions are together both necessary and sufficient for U to be abnormal in G:*

(1) Every subgroup of G containing U is self-normalising.
(2) U is not contained in two distinct conjugate subgroups of G.

Condition 1 is already sufficient if G is soluble. However A. Feldman[1] showed that it is not sufficient in the general case. The unitary group $U_3(3)$ has a non-abnormal subgroup U isomorphic to Σ_4 such that every subgroup containing it is self-normalising.

Recall the obvious but convenient fact that a maximal subgroup is either normal or abnormal in G. Hence the abnormal maximal subgroups of a group are precisely its non-normal maximal subgroups. Moreover, every abnormal subgroup is contained in an abnormal maximal subgroup.

The history of our results probably begins with Dedekind groups: groups with all subgroups normal. They were first investigated by R. Dedekind.[2] The motivation was algebraic number theory. Dedekind wanted to determine the algebraic number fields with the property that every subfield is normal.

Dedekind groups form a proper subclass of the one composed of all groups with all maximal subgroups normal: the class of all nilpotent groups.

Typical examples of groups in which the set of the abnormal maximal subgroups is non-empty but nonetheless restricted are critical groups with respect to classes of groups containing the class of all nilpotent groups. Recall that a group G is said to be $\mathcal{X}$-critical, for a class of groups $\mathcal{X}$, if $G \notin \mathcal{X}$, but all proper subgroups of G are in $\mathcal{X}$. It seems clear that detailed knowledge of the structure of $\mathcal{X}$-critical groups can give some insight into what makes a group to belong to $\mathcal{X}$. For example, when $\mathcal{X}$ is closed under taking subgroups, a group of least order which does not belong to $\mathcal{X}$ is $\mathcal{X}$-critical. In this case, it is enough to check the condition on proper subgroups only for the maximal subgroups.

Many authors have studied $\mathcal{X}$-critical groups for a number of classes of groups. The focus here is on the classes of nilpotent and supersoluble groups.

The critical groups for the class of all nilpotent groups were studied by O. J. Schmidt.[3] These groups are called nowadays Schmidt's groups. He