

ISCHIA

GROUP THEORY 2008

Mariagrazia Bianchi
Patrizia Longobardi
Mercede Maj
Carlo Maria Scoppola
editors

ISCHIA

GROUP THEORY 2008

Proceedings of the Conference

Naples, Italy

1 – 4 April 2008

Editors

Mariagrazia Bianchi

University of Milano, Italy

Patrizia Longobardi

University of Salerno, Italy

Mercede Maj

University of Salerno, Italy

Carlo Maria Scoppola

University of L'Aquila, Italy

 **World Scientific**

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

ISCHIA GROUP THEORY 2008

Proceedings of the Conference in Group Theory

Copyright © 2009 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

ISBN-13 978-981-4277-79-2 (pbk)

ISBN-10 981-4277-79-7 (pbk)

Printed by FulIsland Offset Printing (S) Pte Ltd, Singapore

ISCHIA

GROUP THEORY 2008



Preface

The research articles included in this volume represent the proceedings of the Conference “Ischia Group Theory 2008”, the third in a series that previously included “Ischia Group Theory 2004, in honour of Marcel Herzog” and “Ischia Group Theory 2006: in Honor of Akbar Rhemtulla”. The Conference took place at the Jolly Hotel, Ischia (Naples, Italy) from April 2nd to April 4th, 2008.

The scientific committee was formed by Mariagrazia BIANCHI (Università di Milano), Patrizia LONGOBARDI (Università di Salerno), Mercede MAJ (Università di Salerno), Carlo Maria SCOPPOLA (Università dell’Aquila), while the local organizing committee, at Università di Salerno, consisted of Costantino DELIZIA, Annamaria LUCIBELLO, Chiara NICOTERA, Carmela SICA, Antonio TORTORA, Maria TOTA.

The papers in this volume are contributions by speakers and participants of the conference.

The span of the research topics represented in these Proceedings is rather large, and includes classification of classes of p -groups, character theory, finiteness conditions, profinite groups and pro- p -groups, subgroup structure, linear groups, Gelfand pairs, varieties of groups, groups and graphs.

The schedule of the talks and the list of all participants are included in these Proceedings.

A poster session on various research topics augmented the scientific program.

It is our pleasure to thank our colleagues of Università di Salerno, who made our stay very smooth and pleasant; the speakers, who greatly contributed to the scientific interest of the conference; the participants, who created an intense and lively atmosphere; the authors of all papers, for their interesting contributions; the referees of the papers, for their effective and accurate effort in providing the editors with reliable information; the staff of the Jolly Hotel for their patient, effective and accommodating support; the Publisher, for the professional and timely production of these Proceedings.

Last but not least, we thank our Sponsors, listed in the next page.

The Editors

Sponsors

INdAM – GNSAGA

Istituto Nazionale di Alta Matematica “F. Severi” – Gruppo Nazionale per le Strutture Algebriche, Geometriche e le loro Applicazioni

Università degli Studi di Milano

Università degli Studi di Salerno

Università degli Studi dell’Aquila

PRIN – Programmi di Ricerca di Interesse Nazionale

Progetto dal titolo: “Gruppi, Algebre di Lie, Crittografia”

Dipartimento di Matematica e Informatica dell’Università di Salerno

Dipartimento di Matematica “F. Enriques” dell’Università di Milano

Dipartimento di Matematica Pura ed Applicata dell’Università dell’Aquila

Dipartimento di Fisica “E.R. Caianiello” dell’Università di Salerno

Provincia di Salerno

Conference Program

WEDNESDAY, APRIL 2

09.20 Welcome Greetings

Chairman: Otto Kegel

09.30 M. ISAACS

Characters of groups with self-normalizing Sylow subgroups

10.20 A. BALLESTER-BOLINCHES

On abnormal maximal subgroups of finite groups

11.10 Coffee Break

11.30 J. COSSEY

Quasinormal subgroups in finite p -groups

12.20 Z. JANKO

Some exceptional minimal situations by finite p -groups

13.30 Lunch Break

Chairman: Francesco de Giovanni

15.30 H. SMITH

Residual finiteness in locally nilpotent groups

16.20 P. NEUMANN

Non-separating groups and synchronization

17.10 Coffee Break

Chairman: Dimitrios Varsos

17.30 B. AMBERG

Products of groups and nearrings

18.20 K. ERSOY

Centralizers in locally finite simple groups of Lie type

18.40 A. FACCHINI

Equivalence of diagonal matrices over local rings

21.30 Recital of classical Neapolitan songs

THURSDAY, APRIL 3

Chairman: Giovanni Zacher

09.30 O. KEGEL

An example of a “concrete and universal” existentially closed group

10.20 L. DI MARTINO

Minimal irreducibility and unipotent characters of finite groups of Lie type

11.10 Coffee Break

11.30 A. RHEMTULLA

Ordered groups - finitely determined orders

12.00 D. CHILLAG

A congruence of Blichfeldt and exact conjugacy classes - characters analogs

12.50 M. DE FALCO Groups with few normalizer subgroups

13.10 Picture

13.30 Lunch Break

Chairman: Hermann Heineken

15.30 M. HERZOG

On a commuting graph related to conjugacy classes in groups

16.20 W. HERFORT

Finitely generated pro-p groups that act on pro-p trees

17.10 Coffee Break

Chairman: Mariagrazia Bianchi

17.30 M. GIORGETTI

Goppa codes: automorphism groups and equivalence

17.50 P. SPIGA

Finite groups whose irreducible characters vanish only on p-elements

18.10 C. TAMBURINI

Constructive generation of matrix groups of low rank by elements of prescribed orders

FRIDAY, April 4Chairman: Inmaculada Lizasoain

09.30 M. LEWIS

Generalizing Camina groups and their character tables

10.20 A. CARANTI

Some questions concerning the algebra of Rijndael/AES

11.10 Coffee Break

11.30 M. EVANS

Free abelianized extensions of groups

12.20 E. KHUKHRO

Large characteristic subgroups and their applications

13.30 Lunch Break

Chairman: Alfio Ragusa

15.30 R. BRANDL

Permutable conjugates

16.00 N. GAVIOLI

Pro p -groups with few normal subgroups

16.40 Coffee Break

17.00 K. AZIZIHERIS

Counting the number of nonlinear irreducible characters of a finite group

17.30 M. DIXON

Some simple locally (soluble-by-finite) groups

19.00 Concert of Baroque Music

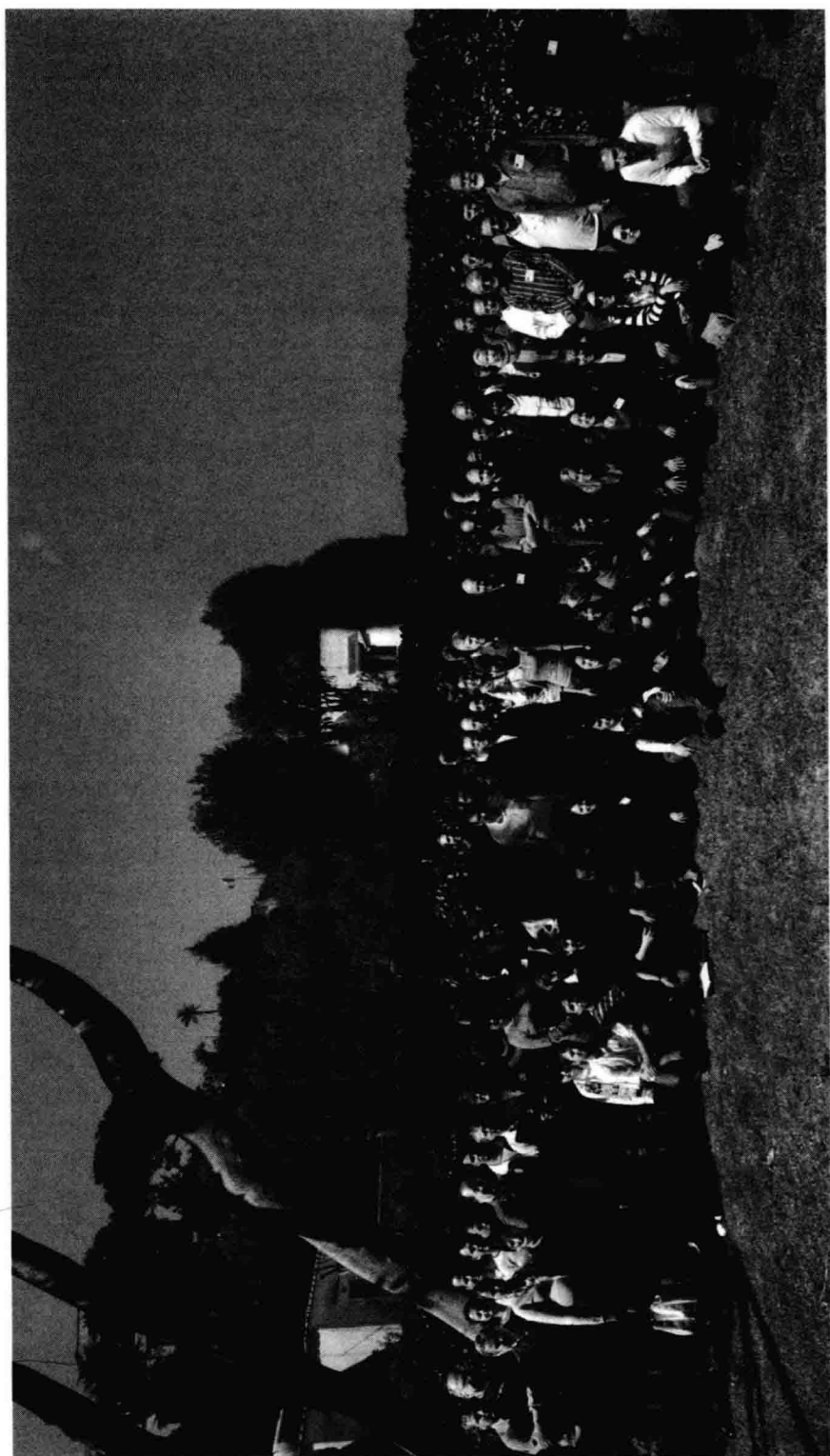
20.30 Social Dinner

Registered Participants

Bernhard Amberg, *Johannes Gutenberg-Universität Mainz, Germany*
 Marina Avitabile, *Università di Milano - Bicocca, Italy*
 Kamal Aziziheris, *Tabriz University, Iran*
 Adolfo Ballester-Bolinches, *Universidad de Valencia, Spain*
 Mariagrazia Bianchi, *Università di Milano, Italy*
 Celestina Bonzini, *Università di Milano, Italy*
 Victor Bovdi, *University of Debrecen, Hungary*
 Rolf Brandl, *Universität Würzburg, Germany*
 Clara Calvo, *Universidad de Valencia, Spain*
 Andrea Caranti, *Università di Trento, Italy*
 Luisa Carini, *Università di Messina, Italy*
 Tullio G. Ceccherini-Silberstein, *Università del Sannio, Italy*
 Maria Rosaria Celentani, *Università di Napoli Federico II, Italy*
 David Chillag, *Technion-Israel Institute of Technology Haifa, Israel*
 Valentina Colombo, *Università di Padova, Italy*
 John Cossey, *Australian National University, Australia*
 Eleonora Crestani, *Università di Padova, Italy*
 Francesca Dalla Volta, *Università di Milano - Bicocca, Italy*
 Andrea Damiani, *Conservatorio di Santa Cecilia - Roma, Italy*
 Alma D'Aniello, *Università di Napoli Federico II, Italy*
 Paola D'Aquino, *Seconda Università di Napoli, Italy*
 Ulderico Dardano, *Università di Napoli Federico II, Italy*
 Maria De Falco, *Università di Napoli Federico II, Italy*
 Francesco de Giovanni, *Università di Napoli Federico II, Italy*
 Costantino Delizia, *Università di Salerno, Italy*
 Giovanni Di Maria, *Università di Napoli Federico II, Italy*
 Lino Di Martino, *Università di Milano - Bicocca, Italy*
 Amel Dilmi, *University of Setif, Algeria*
 Martyn Dixon, *University of Alabama, U.S.A.*
 Kivanç Ersoy, *Middle East Technical University, Turkey*
 Ramon Esteban-Romero, *Universidad Politécnica de Valencia, Spain*
 Martin Evans, *University of Alabama, U.S.A.*
 Alberto Facchini, *Università di Padova, Italy*
 Asadollah Faramarzi Salles, *University of Isfahan, Iran*
 Carla Fiori, *Università di Modena e Reggio Emilia, Italy*

- Maria Rosaria Formisano, *Università di Napoli Federico II, Italy*
 Norberto Gavioli, *Università dell'Aquila, Italy*
 Fares Gherbi, *University of Setif, Algeria*
 Marta Giorgetti, *Università dell'Insubria, Italy*
 Hermann Heineken, *Universität Würzburg, Germany*
 Wolfgang Herfort, *University of Technology, Vienna, Austria*
 Marcel Herzog, *Tel Aviv University, Israel*
 Diana Imperatore, *Università di Napoli Federico II, Italy*
 Martin Isaacs, *University of Wisconsin - Madison, U.S.A.*
 Zvonimir Janko, *Ruprecht-Karls-Universität Heidelberg, Germany*
 Manfred Karbe, *EMS Publishing House, Switzerland*
 Otto Kegel, *Universität Freiburg, Germany*
 Evgenii Khukhro, *University of Wales College, U.K.*
 Antonella Leone, *Università di Napoli Federico II, Italy*
 Mark Lewis, *Kent State University, U.S.A.*
 Inmaculada Lizasoain, *Universidad Pública de Navarra, Spain*
 Patrizia Longobardi, *Università di Salerno, Italy*
 Annamaria Lucibello, *Università di Salerno, Italy*
 Mercede Maj, *Università di Salerno, Italy*
 Primož Moravec, *University of Ljubljana, Slovenia*
 Cristina Moreno, *Universidad Pública de Navarra, Spain*
 Alexander Moreto, *Universidad de Valencia, Spain*
 Jose M. Muñoz-Escolano, *Universidad de Zaragoza, Spain*
 Carmela Musella, *Università di Napoli Federico II, Italy*
 Peter Neumann, *The Queen's College - Oxford, U.K.*
 Chiara Nicotera, *Università di Salerno, Italy*
 Peyman Niroomand Shirvan, *Ferdowsi University of Mashhad, Iran*
 Niamh O'Sullivan, *Dublin City University, Ireland*
 Daniele Otera, *Université de Neuchâtel, Switzerland*
 Emauele Pacifici, *Università di Milano, Italy*
 Massimiliano Patassini, *Università di Padova, Italy*
 Tatiana Pedraza, *Universidad Politécnica de Valencia, Spain*
 Marco Pellegrini, *Università di Milano - Bicocca, Italy*
 Primož Potocnik, *University of Ljubljana, Slovenia*
 Alfio Ragusa, *Università di Catania, Italy*
 Rashid Rezaei, *Ferdowsi University of Mashhad, Iran*
 Akbar Rhemtulla, *University of Alberta, Canada*
 Silvana Rinauro, *Università della Basilicata, Italy*
 Jesús Rodríguez-López, *Universidad Politécnica de Valencia, Spain*

- Emanuela Romano, *Università di Salerno, Italy*
 Tarek Rouabhi, *University of Setif, Algeria*
 Alessio Russo, *Seconda Università di Napoli, Italy*
 Francesco Russo, *Università di Napoli Federico II, Italy*
 Josu Sangroniz, *Universidad del País Vasco, Spain*
 Lucia Sanus, *Universidad de Valencia, Spain*
 Carlo Maria Scoppola, *Università dell'Aquila, Italy*
 Luigi Serena, *Università di Firenze, Italy*
 Carmela Sica, *Università di Salerno, Italy*
 Salvatore Siciliano, *Università del Salento, Italy*
 Howard Smith, *Bucknell University, U.S.A.*
 Xaro Soler-Escrivà, *Universitat d'Alacant, Spain*
 Pablo Spiga, *Università di Padova, Italy*
 Maria Clara Tamburini, *Università Cattolica di Brescia, Italy*
 Antonio Tortora, *Università di Salerno, Italy*
 Natascia Tortora, *Università di Napoli Federico II, Italy*
 Maria Tota, *Università di Salerno, Italy*
 Nadir Trabelsi, *University of Setif, Algeria*
 Erkan Murat Türkan, *Middle East Technical University, Turkey*
 Dimitrios Varsos, *University of Athens, Greece*
 Gabriel Verret, *University of Ljubljana, Slovenia*
 Giovanni Vincenzi, *Università di Salerno, Italy*
 Thomas Weigel, *Università di Milano - Bicocca, Italy*
 Bettina Wilkens, *Martin-Luther-Universität Halle, Germany*
 Giovanni Zacher, *Università di Padova, Italy*



Contents

Preface	vii
Sponsors	ix
Conference Program	xi
Registered Participants	xv
On Abnormal Maximal Subgroups of Finite Groups A. BALLESTER-BOLINCHES, J. COSSEY AND R. ESTEBAN-ROMERO	1
Finite Gelfand Pairs: Examples and Applications T. CECCHERINI-SILBERSTEIN, D. D'ANGELI, A. DONNO, F. SCARABOTTI AND F. TOLLI	7
On Blichfeldt's Like Congruences and Other Close Characters — Conjugacy Classes Analogs D. CHILLAG	42
Permutation Groups Defined by Unordered Relations F. DALLA VOLTA AND J. SIEMONS	56
Transitivity of Properties of 2-generator Subgroups C. DELIZIA, P. MORAVEC AND C. NICOTERA	68
Some Simple Locally (Soluble-by-Finite) Groups M.R. DIXON, M.J. EVANS AND H. SMITH	79
The Fundamental Group of a Connected Undirected Profinite Graph W. HERFORT	90
On a Graph Associated with a Group D. IMPERATORE	100
Some Exceptional Minimal Situations by Finite p -groups Z. JANKO	116

Regular Limits of Infinite Symmetric Groups O. H. KEGEL	120
Large Normal and Characteristic Subgroups Satisfying Outer Commutator Identities and their Applications E.I. KHUKHRO	131
Soluble Linear Groups with Some Restrictions on Subgroups of Infinite Central Dimension L.A. KURDACHENKO, J.M. MUÑOZ-ESCOLANO AND J. OTAL	156
Character Tables of Groups Where All Nonlinear Irreducible Characters Vanish Off the Center M.L. LEWIS	174
Some Results and Questions Related to the Generating Graph of a Finite Group A. LUCCHINI AND A. MARÓTI	183
On the Minimal Irreducibility of the Unipotent Characters of the Finite Unitary Groups M.A. PELLEGRINI	209
A Note on Relative Isoclinism Classes of Compact Groups R. REZAEI AND F. RUSSO	233
Restrictions of Brauer Characters and π -partial Characters J. SANGRONIZ	236
Real Characters and Degrees: An Overview L. SANUS	243
Varieties of Groups and the Restricted Burnside Problem P. SHUMYATSKY AND J.C. SILVA	248
The (2,3)-generation of Matrix Groups Over the Integers M.C. TAMBURINI	258
p -Projective Groups and Pro- p Trees T. WEIGEL	265

On abnormal maximal subgroups of finite groups

A. BALLESTER-BOLINCHES*

*Departament d'Àlgebra, Universitat de València
Dr. Moliner, 50; 46100, Burjassot, València, Spain
E-mail: Adolfo.Ballester@uv.es*

JOHN COSSEY

*Mathematics Department, Mathematical Sciences Institute, Australian National
University
Canberra, ACT 0200, Australia
E-mail: John.Cossey@anu.edu.au*

R. ESTEBAN-ROMERO

*Institut Universitari de Matemàtica Pura i Aplicada, Universitat Politècnica de
València
Camí de Vera, s/n; 46022 València, Spain
E-mail: resteban@mat.upv.es*

In this survey we show the influence of the abnormal maximal subgroups of finite groups in their structure.

Keywords: Finite groups; supersoluble group; nilpotent group; maximal subgroup; abnormal subgroup.

1. Introduction

This survey is about finite groups. Hence the unspoken rule is that all groups considered are finite.

“What is the role of the abnormal subgroups in the structure of a group?”

is the motivating question in this survey. In fact, the results we present here are contributions to the long-running investigation of the influence on a group of its proper abnormal structure.

*Corresponding author

Consequently, abnormality, a subgroup embedding property introduced by P. Hall in his Cambridge lectures, is one of the central concepts here. Recall that a subgroup U of a group G is said to be *abnormal* in G if for all $g \in G$, $g \in \langle U, U^g \rangle$.

Theorem 1.1 (Hall, Cambridge lectures). *The following conditions are together both necessary and sufficient for U to be abnormal in G :*

- (1) *Every subgroup of G containing U is self-normalising.*
- (2) *U is not contained in two distinct conjugate subgroups of G .*

Condition 1 is already sufficient if G is soluble. However A. Feldman¹ showed that it is not sufficient in the general case. The unitary group $U_3(3)$ has a non-abnormal subgroup U isomorphic to Σ_4 such that every subgroup containing it is self-normalising.

Recall the obvious but convenient fact that a maximal subgroup is either normal or abnormal in G . Hence the abnormal maximal subgroups of a group are precisely its non-normal maximal subgroups. Moreover, every abnormal subgroup is contained in an abnormal maximal subgroup.

The history of our results probably begins with Dedekind groups: groups with all subgroups normal. They were first investigated by R. Dedekind.² The motivation was algebraic number theory. Dedekind wanted to determine the algebraic number fields with the property that every subfield is normal.

Dedekind groups form a proper subclass of the one composed of all groups with all maximal subgroups normal: the class of all nilpotent groups.

Typical examples of groups in which the set of the abnormal maximal subgroups is non-empty but nonetheless restricted are critical groups with respect to classes of groups containing the class of all nilpotent groups. Recall that a group G is said to be \mathcal{X} -critical, for a class of groups \mathcal{X} , if $G \notin \mathcal{X}$, but all proper subgroups of G are in \mathcal{X} . It seems clear that detailed knowledge of the structure of \mathcal{X} -critical groups can give some insight into what makes a group to belong to \mathcal{X} . For example, when \mathcal{X} is closed under taking subgroups, a group of least order which does not belong to \mathcal{X} is \mathcal{X} -critical. In this case, it is enough to check the condition on proper subgroups only for the maximal subgroups.

Many authors have studied \mathcal{X} -critical groups for a number of classes of groups. The focus here is on the classes of nilpotent and supersoluble groups.

The critical groups for the class of all nilpotent groups were studied by O. J. Schmidt.³ These groups are called nowadays Schmidt's groups. He