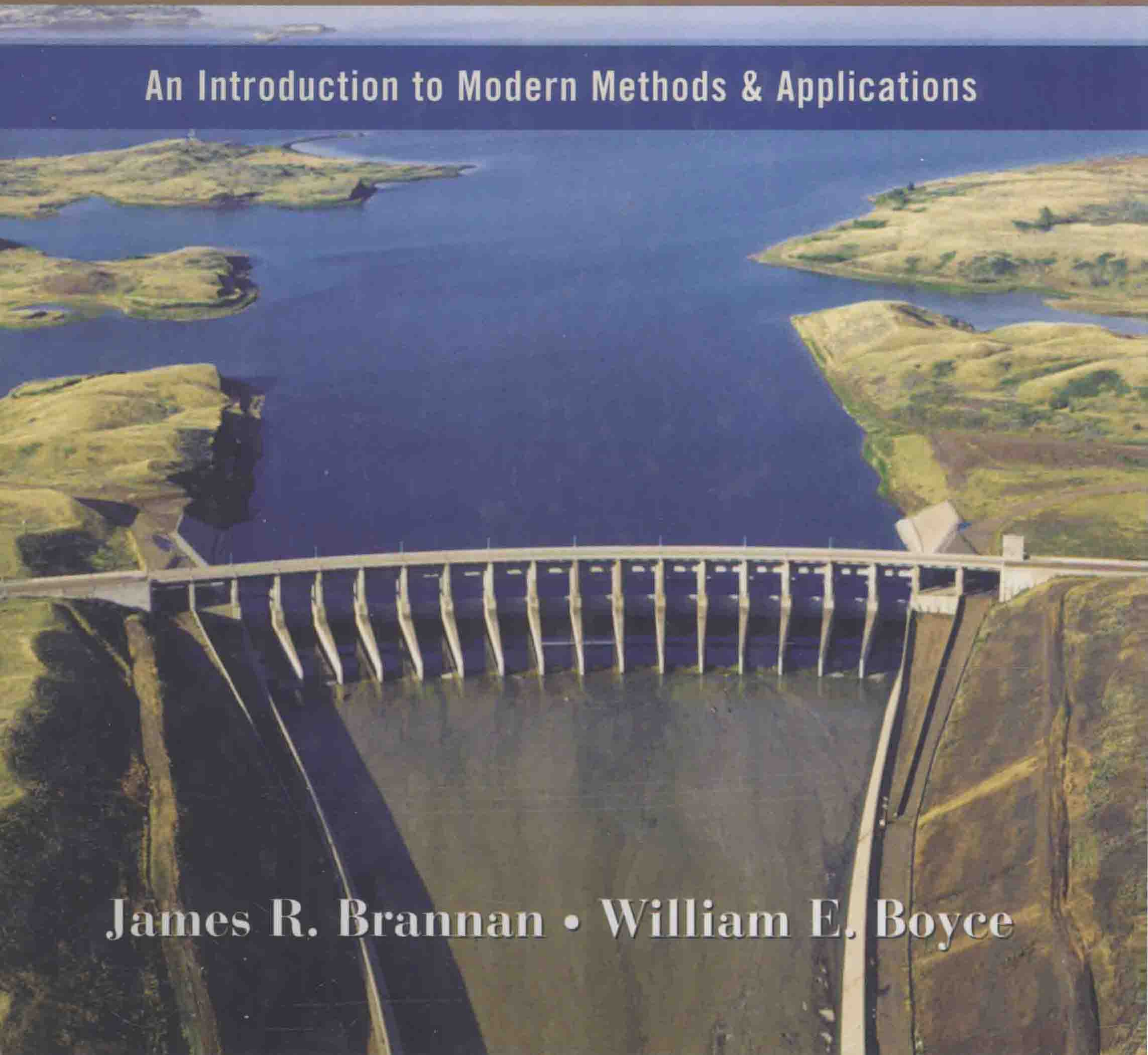
DIFFERENTIAL EQUATIONS

with BOUNDARY VALUE PROBLEMS

An Introduction to Modern Methods & Applications



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DIFFERENTIAL EQUATIONS WITH BOUNDARY VALUE PROBLEMS

An Introduction to Modern Methods and Applications

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An Introduction to Modern Methods and Applications

PREFACE

This is a textbook for a first course in differential equations. The book is intended for science and engineering majors who have completed the calculus sequence, but not necessarily a first course in linear algebra. It emphasizes a systems approach to the subject and integrates the use of modern computing technology in the context of contemporary applications from engineering and science.

Differential equations is an old and venerable subject that will always play a central role in describing phenomena that change over time. Indeed, at least one course in differential equations is part of the curriculum of most engineering and science students. Our goal in writing this text is to provide these students with both an introduction to, and a survey of, modern methods, applications, and theory of this beautiful and powerful mathematical apparatus that is likely to serve them well in their chosen field of study. The subject matter is presented in a manner consistent with the way practitioners use differential equations in their work; technology is used freely, with more emphasis on methods, modeling, graphical representation, qualitative concepts, and geometric intuition than on theoretical issues. For example, some important theoretical results, such as theorems guaranteeing existence and uniqueness of solutions to initial value problems, are not proved. Nevertheless, they are carefully stated, illustrated by examples, and used frequently.

Any student who studies major portions of this book, does a reasonable number of the section exercises, and completes some of the chapter projects will surely develop an appreciation for the power of differential equations to shed light on issues of societal importance. In addition, he or she will acquire skills in modeling, analysis, and computer simulation that will be useful in a wide variety of situations. It is for such students that we have written this book.

Major Features

- Flexible Organization. Chapters are arranged, and sections and projects are structured, to facilitate choosing from a variety of possible course configurations depending on desired course goals, topics, and depth of coverage.
- Numerous and Varied Problems. Throughout the text, section exercises of varying levels of difficulty give students hands-on experience in modeling, analysis, and computer experimentation.
- Emphasis on Systems. Systems of first order equations, a central and unifying theme of the text, are introduced early, in Chapter 3, and are used frequently thereafter.
- Linear Algebra and Matrix Methods. Two-dimensional linear algebra sufficient for the study of two first order equations, taken up in Chapter 3, is presented in Section 3.1. Linear algebra and matrix methods required for the study of linear systems of dimension n (Chapter 6) are treated in Appendix A.
- ODE Architect. The companion ODE Architect provides students with a user-friendly software tool for computing numerical approximations to solutions of systems of differential equations, and for constructing component plots, direction fields, and phase portraits.
- Optional Computing Exercises. In most cases, problems requesting computer generated solutions and graphics are optional.
- Visual Elements. In addition to a large number of illustrations and graphs within the text, physical representations of dynamical systems and animations available in ODE Architect enable students to visualize solutions routinely.

- **Contemporary Project Applications.** Optional projects at the ends of Chapters 2 through 10 integrate subject matter in the context of exciting, contemporary applications in science and engineering. Among these are controlling the attitude of a satellite, ray theory of wave propagation, uniformly distributing points on a sphere, and vibration analysis of tall buildings.
- **Laplace Transforms.** A detailed chapter on Laplace transforms discusses systems, discontinuous and impulsive input functions, transfer functions, feedback control systems, poles, and stability.
- **Control Theory.** Ideas and methods from the important application area of control theory are introduced in some examples, some projects, and in the last section on Laplace Transforms. All of this material is optional.
- **Recurring Themes and Applications.** Important themes, methods, and applications, such as dynamical system formulation, phase portraits, linearization, stability of equilibrium solutions, vibrating systems, and frequency response are revisited and reexamined in a variety of mathematical models under different mathematical settings.
- **Chapter Summaries.** A summary at the end of each chapter provides students and instructors with a birds-eye view of the most important ideas in the chapter.
- **Answers to Problems.** Answers to the problems are provided at the end of the book; many of them are accompanied by a figure.

Systems

The book emphasizes differential equations and applications within the systems framework. Two-dimensional linear systems appear early, in Chapter 3, following a treatment of first order equations in Chapters 1 and 2. Second order equations, taken up in Chapter 4, are not only dealt with directly, but are also presented in the context of first order systems. Higher order equations are subsumed within the logical extension from two dimensions to n dimensions in Chapter 6. We feel an early introduction to systems offers advantages over more traditional presentation formats for several reasons:

- The systems paradigm has not only permeated all fields of engineering and the natural sciences, but quantitative areas of the business sciences, such as economics and finance, as well. Most realistic problems in these disciplines consist of two or more components that interact in some manner. For such problems, the systems approach often facilitates modeling and analysis.
- Virtually all initial value problem solvers require that second and higher order scalar equations and systems be written as systems of first order equations. Since many of the problems and projects in this text are enhanced by numerical simulations and graphical presentations, it is important for the student to be able to cast problems as systems of first order equations early in the course.
- The student gets early exposure to the geometry of phase portraits along with the important relationship between eigenvalues and qualitative concepts such as long-term behavior of solutions and stability of equilibrium points.
- The systems approach helps promote a unified view of differential equations, in contrast to a perception shared by many students that the subject is a collection of somewhat distinct topics (first order equations, second order equations, nth order equations, and first order systems), each with its own method of solution.

Matrices and Linear Algebra

Students come to a differential equations course with widely varying knowledge of linear algebra and matrices. Some have had a full semester course, but many have had a much smaller exposure to the subject. In addition, many instructors prefer, or have time, to cover only two-dimensional systems, while others may wish to discuss systems in n dimensions. To accommodate these variations we present a two-tiered approach to systems of first order differential equations.

Chapter 3 deals primarily with systems of two linear homogeneous differential equations with constant coefficients. The algebraic skills that are needed to handle such systems are (1) solving two-dimensional linear algebraic systems, and (2) solving quadratic equations. These are skills that all differential equations students have, and Section 3.1 provides the opportunity to review them. What may be new to students in this section is the terminology and geometry associated with the eigenvalue problem for 2×2 matrices. Our experience is that students readily understand how to solve the eigenvalue problem in a two-dimensional setting, since they already know the underlying mathematics.

We want to emphasize that the treatment of two-dimensional linear systems in Chapter 3 is sufficient to make the rest of the book, except for Chapter 6, accessible to students. In other words, a quite limited knowledge of linear algebra and matrix methods is sufficient to handle most of the material in this book.

However, many important applications require consideration of systems of dimension greater than two. For those instructors who want to go beyond two-dimensional systems, we include a consideration of n-dimensional systems in Chapter 6. To deal with such systems effectively, we require additional machinery from linear algebra and matrix theory. Appendix A provides a summary of the necessary results from these areas required by Chapter 6. Readers with little or no previous exposure to this material will find it necessary to study some, or all, of Appendix A before proceeding on to Chapter 6. An alternative way to utilize Appendix A is to undertake the study of Chapter 6 directly, drawing upon necessary results from Appendix A as needed. The topics covered in Appendix A are treated in as independent a manner as possible, allowing the instructor to pick and choose on the fly. The depth of coverage will, of course, be determined by the background of the students and the course goals of the instructor. This approach may be preferred if the students already have an adequate background in matrix algebra, or if the calculation of eigenvalues and eigenvectors is to be performed primarily by using a computer or a calculator. Using this approach, coverage of Chapter 6 can be greatly streamlined. Appendix A provides all that is needed (and only what is needed) from matrix algebra to handle the systems of differential equations in Chapter 6. It is not an introduction to abstract linear algebra; rather, it focuses on the computational issues that are needed in Chapter 6.

Technology

Due to the availability of modern interactive computing environments such as MATLAB®, *Maple*®, and *Mathematica*®, it is now easy for students to approximate solutions of differential equations numerically, and to visualize them graphically. Indeed, most of the numerical calculations required by the problems in this book can be performed with the companion software, ODE Architect, a numerical differential equations solver package with a convenient user interface. In this text, the computer is used in two different ways:

- It is employed as a tool to help convey the proper subject matter of differential equations, especially through the use of graphics.

- In most realistic problems it is not possible to obtain closed-form analytic solutions of systems of equations. Even in a first course in differential equations, computer technology permits the treatment of serious, contemporary applications arising from problems in engineering and science. We include many such applications, some in the main text but primarily in the chapter projects.

Problems that require the use of a computer are marked with ODEA, indicating that ODE Architect may conveniently be used, or CAS, indicating that a computer algebra system should be used. While we feel that students will benefit from using the computer on those problems where numerical approximations or computer generated graphics are requested, in most problems it is clear that use of a computer, or even a graphing calculator, is optional. Furthermore, there are a large number of problems that do not require the use of a computer. Thus, the book can easily be used in a course without using any technology.

Projects

At the end of each chapter (except Chapter 1) we have included projects that deal with contemporary problems normally not included among traditional topics in differential equations.

- Many of the projects involve applications, drawn from a variety of disciplines, that illustrate either a physical principle or a prediction, design, control, or management strategy. To engineers and scientists in training, such problems effectively display the utility and importance of differential equations as a tool that can be used to effect change upon the world in which we live. Through these projects we are able to emphasize the active role that differential equations play in modern science and engineering in addition to their important, but more passive, role in simply describing phenomena that change over time.
- The projects integrate and/or extend the theory and methods presented in the core material in the context of specific problems difficult enough to require some critical and imaginative thinking. By demonstrating the usefulness of the mathematics, a project can be a valuable tool for enhancing students' comprehensive understanding of the subject matter.
- Many of the projects require modeling, analysis, numerical calculations, and interpretation of results. They are structured in the sense that the early exercises guide the reader through the modeling, analysis, and computations while later exercises are frequently more open-ended and demand more from the reader in the way of original and critical thinking. It is not necessary for a student to do all of the exercises to benefit from a project. Indeed, even the idea of the application can be an eye-opening experience for students.
- In our experience, a great deal of knowledge transfer occurs as a result of the student–student and teacher–student dialogue over the course of the project.
- For some, mathematical modeling may mean finding an accurate mathematical description of a given set of raw data. For example, following a discussion of a variety of different population models and their properties, the student may be asked to find one that best fits a set of historical population data. This is an interesting and worthwhile endeavor requiring some resourcefulness and knowledge of elementary methods on the part of the student. Lack of structure is, in fact, a desirable aspect and typifies problems that arise in biology, for example. For others, modeling may mean writing down an appropriate set of differential equations based on known physical laws. In such cases, relatively sophisticated methods may be required to analyze the problem. Few, if any, students are able to devise or derive such methods in any reasonable amount of time. Structured projects

may introduce such methods and guide the student through the analysis. These types of projects provide important mathematical tools for the toolbox of the engineer or scientist in training. Most of the projects in this textbook are of this type.

- Projects vary in length and level of difficulty. Less demanding ones may be assigned on an individual basis. More challenging projects may be assigned to small groups or to individual students in an honors class. A few of the projects may require some programming guidance from the instructor. Assuming that all of the problems for each project are worked out, the table below indicates the approximate level of difficulty for the analysis and computational components: Beginning (B), Intermediate (I), or Advanced (A). As discussed above, ODEA and CAS indicate the suggested computational tool, although all computations can be performed using a computer algebra system. Level of difficulty can be adjusted by suitably restricting assigned problems. With respect to the analysis component, a B level of difficulty roughly corresponds to that of an end of the section problem, with I and A representing increasingly more challenging problems. With respect to the computational component, a B level of difficulty means that only direction fields, phase portraits, or component plots for standard differential equations or systems are requested. An I level of difficulty signifies that there may be discontinuities present, or that some data analysis, such as curve fitting, is required. Finally, an A level of difficulty generally means that some elementary programming, possibly involving functions and loop structures, is required.

		Level of Difficulty	
Section	Project Title	Analysis	Computation
2.P.1	Harvesting a Renewable Resource	B	
2.P.2	Designing a Drip Dispenser for a Hydrology Experiment	I	B-CAS
2.P.3	A Mathematical Model of a Groundwater Contaminant Source	I	I-CAS
2.P.4	Monte Carlo Option Pricing: Pricing Financial Options by Flipping a Coin	B	A-CAS
3.P.1	Eigenvalue-Placement Design of a Satellite Attitude Control System	I	B-ODEA
3.P.2	Estimating Rate Constants for an Open Two-Compartment Model	I	A-CAS
3.P.3	The Ray Theory of Wave Propagation	I	I-CAS
3.P.4	A Blood-Brain Pharmacokinetic Model	B	I-CAS
4.P.1	A Vibration Insulation Problem	I	I-CAS
4.P.2	Linearization of a Nonlinear Mechanical System	I	B-ODEA
4.P.3	A Spring-Mass Event Problem	I	B-ODEA
4.P.4	Uniformly Distributing Points on a Sphere	I	A-CAS
4.P.5	Euler-Lagrange Equations	A	
5.P.1	An Electric Circuit Problem	B	B-CAS
5.P.2	Effects of Pole Locations on Step Responses of Second Order Systems	I	B-CAS
5.P.3	The Watt Governor, Feedback Control, and Stability	A	I-CAS
6.P.1	A Compartment Model of Heat Flow in a Rod	B	I-CAS
6.P.2	Earthquakes and Tall Buildings	A	A-CAS
6.P.3	Controlling a Spring-Mass System to Equilibrium	A	A-CAS
7.P.1	Modeling of Epidemics	I	B-ODEA
7.P.2	Harvesting in a Competitive Environment	A	B-ODEA
7.P.3	The Rössler System	I	I-ODEA
8.P.1	Diffraction Through a Circular Aperture	I	
8.P.2	Hermite Polynomials and The Quantum Mechanical Harmonic Oscillator	A	B-CAS
8.P.3	Perturbation Methods	I	I-ODEA
9.P.1	Estimating the Diffusion Coefficient in the Heat Equation	I	I-CAS
9.P.2	The Transmission Line Problem	A	I-CAS
9.P.3	Solving Poisson's Equation by Finite Differences	B	A-CAS
10.P.1	Dynamic Behavior of a Hanging Cable	I	B-CAS
10.P.2	Advection-Dispersion: A Model for Solute Transport in Porous Media	A	A-CAS
10.P.3	Fisher's Equation for Population Growth and Dispersion	I	A-CAS

Computer Simulations

In most realistic problems it is not possible to obtain closed-form analytic solutions of systems of equations. Usually initial value problem (IVP) solvers contained in commercial software packages such as MATLAB, *Maple*, *Mathematica*, or in the companion ODE Architect Tool, are used to obtain numerical approximations to solutions. Our understanding of the problem then depends to a large extent on visualization and interpretation of plots of the graphs of these approximations, usually as one or more parameters in the problem are varied. Computer experiments of this type are called *computer simulations*. In combination with analytical theory, analytical methods, and qualitative techniques, computer simulations provide us with an additional powerful tool for studying the behavior of systems. Many modern engineering and scientific problems, for example, weather prediction, aircraft design, economic forecasting, epidemic modeling, and industrial processes are studied with the aid of computer simulations. Computer simulations of systems of differential equations may be viewed as part of a larger subject area frequently referred to as *computational science*. On a small scale, many of the section exercises and chapter projects expose the student to some of the following pragmatic issues that confront the computational scientist:

- There may be a considerable number of trial and error simulations required enroute to obtaining a successful set of results.
- Errors of one kind or another can occur in a number of different phases of the problem solving process: inaccuracies in the modeling, programming errors, failure of existence or uniqueness in the mathematical problem itself, and ill-behavior of the system relative to the numerical method employed to solve the IVP.
- For certain parameter values it is advantageous to have either exact solutions or analytical approximations that can be compared with the numerical approximation.
- Efficiently and effectively presenting and displaying output data generated from numerical simulations of complex problems can be a challenge, but it is also an aspect of the problem where imagination and creativity may be effectively used.
- Common sense and a healthy degree of skepticism when viewing output results, such as the numbers or the graphs, are desirable traits to develop.
- The better you understand your computational tool—its capabilities and limitations—the better off you are.
- The ability to conduct high quality simulation experiments is a craft that requires knowledge of several technical skills, and is developed through education and experience.

Numerical Algorithms

IVP solvers available in the companion ODE Architect Tool, MATLAB, *Maple*, or *Mathematica* should be used to perform most of the numerical calculations required in section exercises and chapter projects. Nevertheless, we introduce Euler's method for a scalar first order equation in Section 1.3. We return in Sections 2.7 and 2.8 to a discussion of errors and of methods more efficient than Euler's for approximating solutions numerically. In Section 3.7 we extend these methods to systems of first order equations. There are several reasons why we include introductory material on numerical algorithms in the text.

- Euler's method, in particular, gives great insight into the meaning of a dynamical system (system of first order differential equations) in that it provides a simple algorithm for advancing the state of the dynamical system in discrete time.
- The Runge-Kutta method, while less intuitive than the Euler method, shows how a cleverly designed algorithm can yield great rewards in terms of accuracy and efficiency.
- Numerical methods are an important component of the body of knowledge of differential equations. It is conceivable that many instructors may wish to provide their students with an introduction to some of the algorithms and their analysis.
- There are situations where a slight modification of an elementary numerical method, such as Euler's method, may provide a satisfactory scheme for approximating the solution of a problem. Illustrations of this are contained in the projects presented in Section 2.P.4, Monte Carlo Option Pricing, and Section 4.P.4, Uniform Distribution of Points on a Sphere.

Relation of This Text to Boyce and DiPrima

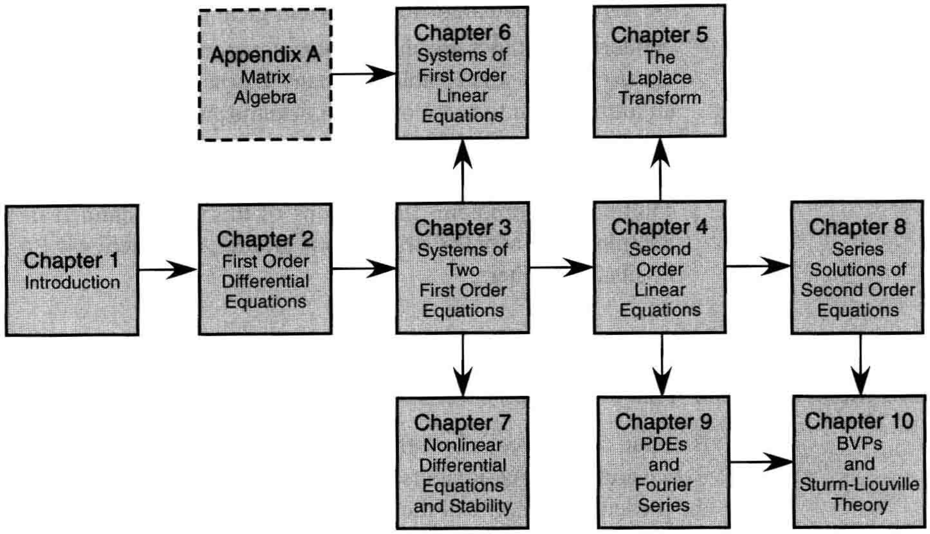
Brannan and Boyce is an offshoot, but not a new edition, of the well-known textbook by Boyce and DiPrima. Readers familiar with Boyce and DiPrima will doubtless recognize in the present book some of the hallmark features that distinguish that textbook.

To help avoid confusion among potential users of either text, the primary differences are described below:

- Boyce and DiPrima is more comprehensive and is laid out along fairly traditional lines. It includes all of the topics that are often included in a first course on differential equations. There are chapters on higher order linear equations, power series methods applied to both ordinary and regular singular points, two point boundary value problems, and partial differential equations using Fourier series methods. None of this material, except for brief references to higher order linear equations, appears in Brannan and Boyce.
- Brannan and Boyce is more sharply focused on the needs of students of engineering and science, whereas Boyce and DiPrima targets a somewhat more general audience, including engineers and scientists.
- Brannan and Boyce is intended to be more consistent with the way contemporary scientists and engineers actually use differential equations in the workplace.
- Brannan and Boyce emphasizes systems of first order equations, introducing them earlier, and also examining them in more detail than Boyce and DiPrima. Brannan and Boyce has an extensive appendix on matrix algebra to support the treatment of systems in n dimensions.
- Brannan and Boyce integrates the use of computers more thoroughly than Boyce and DiPrima. Brannan and Boyce introduces numerical approximation methods in Chapter 1, and assumes that most students will use computers to generate approximate solutions and graphs throughout the book.
- Brannan and Boyce emphasizes contemporary applications to a greater extent than Boyce and DiPrima, primarily through end of chapter projects.
- Brannan and Boyce makes somewhat more use of graphs, with more emphasis on phase plane displays, and uses engineering language (for example, state variables, transfer functions, gain functions, and poles) to a greater extent than Boyce and DiPrima.

Options for Course Structure

Chapter dependencies are shown in the following block diagram.



The book has much built-in flexibility and allows instructors to choose from many options. Depending on the course goals of the instructor and background of the students, selected sections may be covered lightly or even omitted.

- Chapters 5, 6, and 7 are independent of each other, and Chapters 6 and 7 are also independent of Chapter 4. It is possible to spend much class time on one of these chapters, or class time can be spread over two or more of them.
- The amount of time devoted to projects is entirely up to the instructor.
- For an honors class, a class consisting of students who have already had a course in linear algebra, or a course in which linear algebra is to be emphasized, Chapter 6 may be taken up immediately following Chapter 2. In this case, material from Appendix A, as well as sections, examples, and problems from Chapters 3 and 4 may be selected on an as needed, or desired, basis. This offers the possibility of spending more class time on Chapters 5, 7 and/or selected projects.

Comments on the use of projects. There are several ways of using projects in the course.

- For some of the projects, the work may be done outside of the classroom on an individual basis.
- Some of the lengthier, more challenging projects might be more appropriately assigned to small groups of students.
- Enough exposition is provided in a few of the projects so that students can derive some benefit from simply reading about the application. For example, students may be interested in James Watt's clever use of a centrifugal pendulum for controlling the speed of a steam engine as described in Section 5.P.3, or how systems of ordinary differential equations are used to describe wave propagation as described in Section 3.P.3. Of course, more benefit may be derived by doing some or all of the project exercises.

- Some instructors may wish to incorporate one or two projects into the course, with a lecture devoted to each one.
- An optional strategy, say for a modeling course, is to design the course to be "project driven". In this approach, selected projects are used to drive the discussion of the mathematics required to solve the problems posed in the project. The discussion of mathematical methods and theory are then intertwined with the project. This might be appropriate for some of the projects that are highly integrative in nature. For example, virtually every topic in Chapter 6, and then some, is utilized to solve the problem posed in Section 6.P.3, Controlling a Spring-Mass System to Equilibrium. Similarly, most of the machinery of Chapter 5 is required to solve the problem in Section 5.P.3, The Watt Governor, Feedback Control, and Stability. The projects then serve to motivate the mathematics.
- Projects can be given as "extra credit" assignments.

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Clemson, South Carolina

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Grafton, New York

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Supplemental Resources for Instructors and Students

The *ODE Architect* tool is available in WileyPlus or via an access code provided with new copies of the text is included with every copy of the text. *ODE Architect* is a prize-winning, state-of-the-art NSF-sponsored learning software package, which is Windows-compatible. A solver tool allows you to build your own models with ODEs and study them in a truly interactive point-and-click environment. The *Architect* includes an interactive library of more than one hundred model differential equation systems with graphs of solutions. The *Architect* also has 14 interactive multimedia modules, which provide a range of models and phenomena, from a golf game to chaos.

An Instructor's Solutions Manual, ISBN 978-0-470-52654-5 includes solutions for all problems in the text.

A Student Solutions Manual, ISBN 978-0-470-41851-2 includes solutions for selected problems in the text.

A Companion Web site, www.wiley.com/college/brannan, provides a wealth of resources for students and instructors, including:

- PowerPoint slides of important ideas and graphics for study and note taking.
- Review and Study Outlines to help students prepare for quizzes and exams.
- Online Review Quizzes to enable students to test their knowledge of key concepts. For further review diagnostic feedback is provided that refers to pertinent sections in the text.
- Additional problems for use with Mathematica, Maple, and MATLAB, allowing opportunities for further exploration of important ideas in the course utilizing these computer algebra and numerical analysis packages.

WileyPLUS Expect More from Your Classroom Technology

This text is supported by *WileyPLUS*—a powerful and highly integrated suite of teaching and learning resources designed to bridge the gap between what happens in the classroom and what happens at home. *WileyPLUS* includes a complete online version of the text, algorithmically generated exercises, all of the text supplements, plus course and homework management tools, in one easy-to-use website.

Organized around the everyday activities you perform in class, *WileyPLUS* helps you:

- Prepare and Present: *WileyPLUS* lets you create class presentations quickly and easily using a wealth of Wiley-provided resources, including an online version of the textbook, PowerPoint slides, and more. You can adapt this content to meet the needs of your course.
- Create Assignments: *WileyPLUS* enables you to automate the process of assigning and grading homework or quizzes.
- Track Student Progress: An instructor's gradebook allows you to analyze individual and overall class results to determine students' progress and level of understanding.
- Promote Strong Problem-Solving Skills: *WileyPLUS* can link homework problems to the relevant section of the online text, providing students with context-sensitive help. *WileyPLUS* also features mastery problems that promote conceptual understanding of key topics and video walkthroughs of example problems.
- Provide numerous practice opportunities: Algorithmically generated problems provide unlimited self-practice opportunities for students, as well as problems for homework and testing.