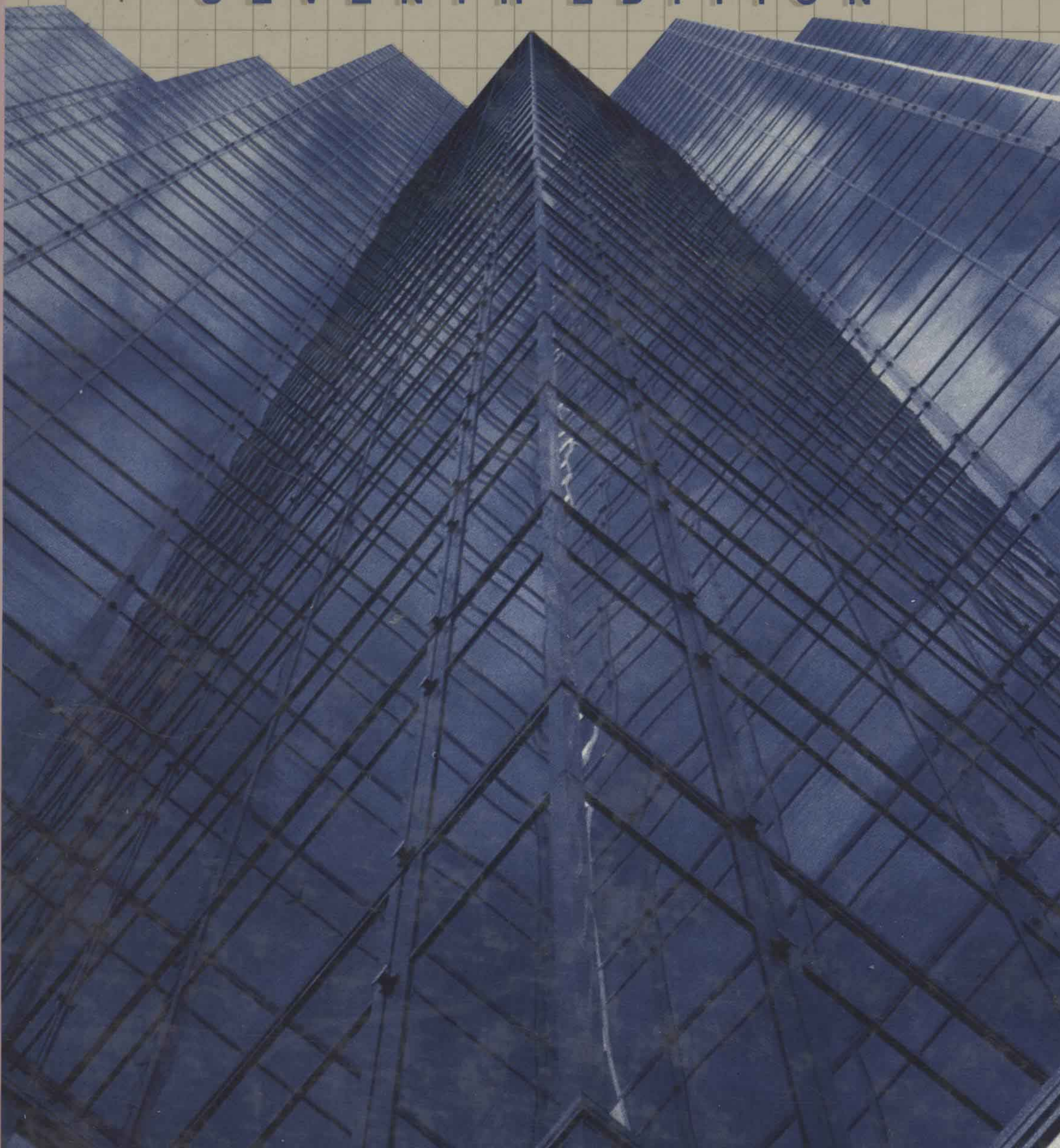
COLLEGE ALGEBRA

SEVENTH EDITION



Grady • Drooyan • Beckenbach

SEVENTH EDITION

College Algebra

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SYMBOLS

[1.1]	a^n	the nth power of a, or a to the nth power; $a \cdot a \cdot a \cdot \cdots \cdot a$ (n factors)
	$P(x)$, $D(y)$, etc.	P of x, D of y, etc.
[2.1]	a^0	$1 \quad (a \neq 0)$
	a^{-n}	the reciprocal of a^n, $\frac{1}{a^n} \quad (a \neq 0)$
[2.2]	$a^{1/n}$	the nth real root of a for $a \in R$, or the positive one if there are two
	$a^{m/n}$	$(a^{1/n})^m$
[2.3]	$\sqrt[n]{a}$	the nth real root of a for $a \in R$, or the positive one if there are two
[2.5]	C	the set of complex numbers
	i	the imaginary unit, $\sqrt{-1}$
	$\sqrt{-b}, \quad b > 0$	$i\sqrt{b}$
	z	a complex number
	$\bar{z}$	the conjugate of z
[3.7]	(a, b)	interval of real numbers between a and b
	$[a, b]$	interval of real numbers between a and b, and a and b
[4.1]	(a, b)	the ordered pair of numbers whose first component is a and whose second component is b
	$R \times R$, or R^2	the Cartesian product of R and R
	f, g, h, F, etc.	symbols denoting functions
	$f(x)$	f of x, or the value of f at x
	$f(x)\|_a^b$	$f(b) - f(a)$
	$f \circ g$	the composition of f and g

[4.2]	P_1P_2, or d	the distance between two points P_1 *and* P_2
	m	the slope of a line
[4.5]	$[x]$	the greatest integer not greater than x
[6.1]	b^x	the power with base $b > 0$ and exponent x
	e	an irrational number, approximately equal to 2.718281828
[6.2]	$\log_b x$	the logarithm to the base b of x, or the logarithm of x to the base b
	$\ln x$	$\log_e x$
[7.2]	(x, y, z)	the ordered triple of numbers whose first component is x, second component is y, and third component is z
[8.1]	$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$, etc.	matrix
	$A_{m \times n}$	m by n matrix
	a_{ij}	the element in the ith row and jth column of the matrix A
	A^t	the transpose of the matrix A
	$0_{m \times n}$	the m by n zero matrix
	$-A_{m \times n}$	the negative of $A_{m \times n}$
[8.2]	$I_{n \times n}$	the identity matrix for all n by n matrices
[8.3]	$A \sim B$	A is row-equivalent to B (for matrices)

[8.4]	$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, etc.	determinant
	$\delta(A)$	the determinant of A
	M_{ij}	the minor of the element a_{ij}
	A_{ij}	the cofactor of a_{ij}
[8.6]	A^{-1}	the inverse of A
[9.1]	$s(n)$, or s_n	the nth term of a sequence
[9.2]	S_n	the sum of the first n terms in a sequence
	$\sum$	the sum
	S_∞	the sum of an infinite sequence
[9.3]	$\lim_{n \to \infty} s_n$	the limit of a sequence
[9.4]	$n!$	n factorial, or factorial n
	$0!$	zero factorial
	$\binom{n}{r}$	$\frac{n!}{r!(n-r)!}$
[10.1]	$n(A)$	the number of elements in the set A
	$A \times B$	the Cartesian product of A and B
	$P_{n,n}$	the number of permutations of n things taken n at a time
	$P_{n,r}$	the number of permutations of n thing taken r at a time

[10.2]	$C_{n,r}$ or $\binom{n}{r}$	the number of combinations of n things taken r at a time
[10.3]	E	event
	$P(E)$	the probability of E
[10.5]	$P(E_2 \mid E_1)$	the conditional probability of E_2, given the occurrence of E_1
[A]	$\text{antilog}_b\, x$	the antilogarithm to the base b of x, or the antilogarithm of x to the base b for $b = 10$ and $b = \text{e}$

SEVENTH EDITION

College Algebra

Preface

The seventh edition of *College Algebra,* like its predecessors, is designed to provide a contemporary mathematics course for college students who have one to two years of high school algebra or its community college equivalent. In this edition, as in previous editions, close attention has been given to pedagogical detail and the organization provides for maximum flexibility.

Review and Reference Material

The *Preliminary Concepts* section provides a review of basic facts from prerequisite courses and serves as a reference throughout the text. In addition, the first three chapters provide an extensive review for students who need it. If students are prepared in the prerequisite algebra, these chapters can be omitted entirely, or a review can be limited to the chapter reviews which consist of a list of the important terms and concepts as well as review problems. These reviews help students focus on concepts as well as on mechanical techniques.

The *Reference Outline* in the appendix provides an overview of all the important ideas that are introduced in the text. This provides an excellent reference for the student while studying for chapter tests and the final examination.

Use of Calculators

Procedures for approximating solutions to polynomial equations using a calculator are introduced along with the traditional results in the theory of equations, thus giving instructors the flexibility to choose the approach that is best suited for their objectives.

In addition, decimal approximations to powers and logarithms are obtained using a calculator, but the methods for obtaining these results from the tables are provided in the appendix in case this approach better meets the needs of the instructor.

Improved Problem Sets

The exercises in most sections have been expanded to provide more variety and more drill exercises on which the student can practice. This includes many more applied problems, particularly in Chapter 6 on logarithms and exponentials.

The exercises have been carefully graded beginning with the A exercises, which are designed to give the students confidence, and proceeding through the B exercises, which are designed to stretch the students' abilities.

A set of B exercises which are not keyed to specific sections has been added to most chapter reviews. These exercises will provide students the opportunity to test and improve their skills while studying for chapter tests.

Organization and Format

Examples that were included in the exercise sets in the sixth edition are now included in the textual discussions and are numbered. The exercises for each section are keyed to these examples. This organization allows students to develop their problem solving skills with less dependence on examples, while at the same time providing easy access to help should they need it.

Chapters 6 through 11 are sufficiently independent to allow one or more to be omitted in the interest of time. This organization of the material allows instructors to design the course according to their particular curriculum needs.

The format of the seventh edition is one which will make it easier for the student to focus on the important facts and procedures and make the text easier to read.

Supplemental Material

A *Student Manual* that includes solutions to the even-numbered exercises and other helpful study aids is available for student use. Additional ancillary materials that relate to the text are available to instructors.

Acknowledgments

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This edition is dedicated to the memory of Professor Edwin F. Beckenbach, an eminent mathematician, an outstanding teacher, and an inspirational coauthor.

We sincerely thank Mary-Margaret M. Grady for her excellent work in preparing the manuscript for publication, and Pamela Christopherson for her excellent help in proofreading the examples and exercises.

Michael D. Grady
Irving Drooyan

Preliminary Concepts

The following definitions, notations, algebraic properties, formulas, and geometric facts have been introduced in previous courses. They are presented here to provide you with a convenient reference for your work in this text.

Lowercase letters, such as a, b, x, and y, will be used to denote real numbers both in the following lists and in the text.

GLOSSARY

Prime and Composite Numbers

If a is an element of the set N of positive integers, and $a \neq 1$, then a is a **prime number** if and only if a has no factor in N other than itself and 1; otherwise, a is a **composite number.** For example, 2 and 3 are prime numbers; $6 = 2 \cdot 3$ is composite. The number 1 is considered to be neither prime nor composite.

Two integers a and b are said to be **relatively prime** if and only if they have no prime factors in common. For example, 6 and 35 are relatively prime, since $6 = 2 \cdot 3$ and $35 = 5 \cdot 7$; but 6 and 8 are not, since they have the prime factor 2 in common.

Set

A **set** is a collection of objects. Any one of the objects of a collection is called a **member** or an **element** of the set. For example, the collection of numbers 2, 4, 6, 8 is a set denoted by {2, 4, 6, 8} and the number 2 is a member or an element of that set. We denote this by

$$2 \in \{2, 4, 6, 8\}.$$

Set Equality

Two sets A and B are **equal,** $A = B$, if and only if they consist of exactly the same elements. For example,

$$\{3, 1\} = \{3, 2 - 1\}.$$

Subset If every element of a set A is an element of a set B, then A is a **subset** of B (denoted $A \subset B$). For example,

$$\{1, 2, 3\} \subset \{1, 2, 3, 4\}.$$

Union of Two Sets The **union** of two sets A and B (denoted $A \cup B$) is the set of all elements that belong either to A or B or to both. For example, for sets $A = \{2, 4, 6\}$ and $B = \{1, 3, 4, 5\}$, "A union B" is

$$A \cup B = \{1, 2, 3, 4, 5, 6\}.$$

Intersection of Two Sets The **intersection** of two sets A and B (denoted $A \cap B$) is the set of all elements common to both A and B. For example, for sets $A = \{1, 2, 3, 4\}$ and $B = \{2, 3, 4, 5\}$, "A intersect B" is

$$A \cap B = \{2, 3, 4\}.$$

NOTATION

$\{a, b\}$	the set whose elements are a and b
$A \subset B$	A is a subset of B
$\varnothing$	the null, or empty, set
$A \cup B$	the union of sets A and B
$A \cap B$	the intersection of sets A and B
$\in$	is an element of
$\{x \mid \ldots\}$	the set of all x such that ...
J	the set of integers; $\ldots, -2, -1, 0, 1, 2, \ldots$
N	the set of positive integers; $1, 2, 3, \ldots$
Q	the set of rational numbers
H	the set of irrational numbers
R	the set of real numbers
C	the set of complex numbers
$a + b$	the sum of a and b
$a \cdot b$	the product of a and b (also written ab)
$-a$	the negative of a; $(-1)a = -a$
$a - b$	the difference b subtracted from a; $a - b = a + (-b)$
$\frac{a}{b}$	the quotient a divided by b; $\frac{a}{b} = a \cdot \frac{1}{b}$
$a < b$	a is less than b
$a > b$	a is greater than b
$a \leqslant b$	a is less than or equal to b
$a \geqslant b$	a is greater than or equal to b
$\lvert a\rvert$	the absolute value of a; $\lvert a\rvert = \begin{cases} a \text{ if } a \geqslant 0 \\ -a \text{ if } a < 0 \end{cases}$
$P(x)$	algebraic expression in the variable x

ALGEBRAIC AXIOMS FOR THE REAL NUMBERS

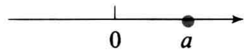
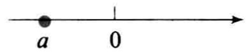
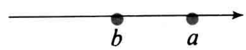
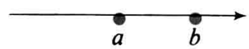
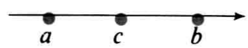
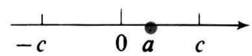
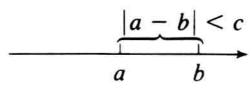
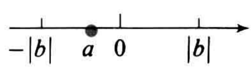
1. $a + b$ is a unique real number.	*Closure law for addition.*
2. $(a + b) + c = a + (b + c)$	*Associative law for addition.*
3. For each $a \in R$, $a + 0 = a$. 0 is called the *additive identity for R.*	*Additive-identity law.*
4. For each $a \in R$, $a + (-a) = 0$. $-a$ is called the *additive inverse of a.*	*Additive-inverse law.*
5. $a + b = b + a$	*Commutative law for addition.*
6. $a \cdot b$ is a unique real number.	*Closure law for multiplication.*
7. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$	*Associative law for multiplication.*
8. $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$	*Distributive law.*
9. For each $a \in R$, $a \cdot 1 = a$. 1 is called the *multiplicative identity in R.*	*Multiplicative-identity law.*
10. $a \cdot b = b \cdot a$	*Commutative law for multiplication.*
11. For each $a \in R$, $a \neq 0$, $a \cdot \frac{1}{a} = 1$. $1/a$ is called the *multiplicative inverse of a.*	*Multiplicative-inverse law.*

ORDER AXIOMS OF REAL NUMBERS

O-1. If a is a real number, then exactly one of the following statements is true: a is positive, a is zero, or a is negative.	*Trichotomy law.*
O-2. If a and b are positive real numbers, then $a + b$ is positive and ab is positive.	*Closure law for positive numbers.*
O-3. There is a one-to-one correspondence between the real numbers and the points on a geometric line.	*Completeness property.*

GEOMETRIC INTERPRETATION OF ORDER

Certain algebraic statements concerning the order of real numbers can be interpreted geometrically. We summarize some of the more common correspondences in the table, where in each case $a, b, c \in R$.

Algebraic Statement	*Geometric Statement*	*Graph*
1. a is positive	**1.** The graph of a lies to the right of the origin.	**1.** 
2. a is negative	**2.** The graph of a lies to the left of the origin.	**2.** 
3. $a > b$	**3.** The graph of a lies to the right of the graph of b.	**3.** 
4. $a < b$	**4.** The graph of a lies to the left of the graph of b.	**4.** 
5. $a < c < b$	**5.** The graph of c is to the right of the graph of a and to the left of the graph of b.	**5.** 
6. $\|a\| < c$	**6.** The graph of a is less than c units from the origin.	**6.** 
7. $\|a - b\| < c$	**7.** The graphs of a and b are less than c units from each other.	**7.** 
8. $\|a\| < \|b\|$	**8.** The graph of a is closer to the origin than the graph of b.	**8.** 

Notice that Statement 5, $a < c < b$, is a contraction of the two inequalities $a < c$ and $c < b$ and is read "a is less than c and c is less than b" or "c is between a and b." Similarly, $a < c$ and $c \leqslant b$ can be written as $a < c \leqslant b$.

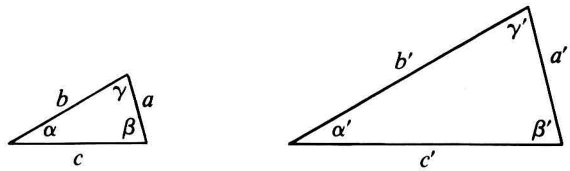
PROPERTIES OF REAL NUMBERS

1. If $a + b = 0$, then $b = -a$ and $a = -b$.
2. If $a \cdot b = 1$, then $a = 1/b$ and $b = 1/a$.
3. $a \cdot b = 0$ if and only if either $a = 0$ or $b = 0$, or both.
4. $-(-a) = a$
5. $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b} = -\frac{-a}{-b}$, $b \neq 0$
6. $\frac{a}{b} = \frac{-a}{-b} = -\frac{a}{-b} = -\frac{-a}{b}$, $b \neq 0$
7. $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$, $b, d \neq 0$
8. $\frac{ac}{bc} = \frac{a}{b}$, $b, c \neq 0$
9. $\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab}$, $a, b \neq 0$
10. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, $b, d \neq 0$
11. $\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}$, $c \neq 0$
12. $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$, $b, d \neq 0$
13. $\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}$, $c \neq 0$
14. $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$, $b, d \neq 0$
15. $\frac{1}{a/b} = \frac{b}{a}$, $a, b \neq 0$
16. $\frac{a/b}{c/d} = \frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$, $b, c, d \neq 0$
17. If $a = b$, then $a + c = b + c$.
18. If $a = b$, then $ac = bc$.
19. If $a < b$, then $a + c < b + c$.
20. If $a < b$, then $a - c < b - c$.
21. If $a < b$ and $c > 0$, then $a \cdot c < b \cdot c$.
22. If $a < b$ and $c < 0$, then $a \cdot c > b \cdot c$.

FACTS FROM GEOMETRY

Similar Triangles

Two triangles are said to be **similar** if corresponding angles are equal. If two triangles are similar, then the ratios of corresponding sides are equal. For example, the two triangles below are similar.



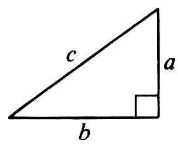
In this case $\alpha = \alpha'$, $\beta = \beta'$, and $\gamma = \gamma'$; further,

$$\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}.$$

Pythagorean Theorem

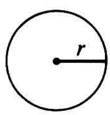
If a, b, and c denote the lengths of the sides of a right triangle, and c is the length of the side opposite the right angle, then

$$a^2 + b^2 = c^2.$$



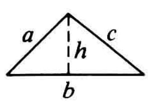
PERIMETER AND AREA FORMULAS

Circle



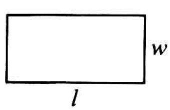
Circumference: $C = 2\pi r$
Area: $A = \pi r^2$

Triangle



Perimeter: $P = a + b + c$
Area: $A = \frac{1}{2}bh$

Rectangle



Perimeter: $P = 2l + 2w$
Area: $A = lw$

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