SHELDON M. ROSS

STOCHASTIC PROCESSES

SECOND EDITION

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AND MATHEMATICAL STATISTICS

STOCHASTIC PROCESSES

Second Edition

Sheldon M. Ross
University of California, Berkeley

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STOCHASTIC PROCESSES

Second Edition

On March 30, 1980, a beautiful six-year-old girl died.
This book is dedicated to the memory of

Nichole Pornaras

Preface to the First Edition

This text is a nonmeasure theoretic introduction to stochastic processes, and as such assumes a knowledge of calculus and elementary probability. In it we attempt to present some of the theory of stochastic processes, to indicate its diverse range of applications, and also to give the student some probabilistic intuition and insight in thinking about problems. We have attempted, wherever possible, to view processes from a probabilistic instead of an analytic point of view. This attempt, for instance, has led us to study most processes from a sample path point of view.

I would like to thank Mark Brown, Cyrus Derman, Shun-Chen Niu, Michael Pinedo, and Zvi Schechner for their helpful comments.

SHELDON M. ROSS

Preface to the Second Edition

The second edition of *Stochastic Processes* includes the following changes:

(i) Additional material in Chapter 2 on compound Poisson random variables, including an identity that can be used to efficiently compute moments, and which leads to an elegant recursive equation for the probability mass function of a nonnegative integer valued compound Poisson random variable;

(ii) A separate chapter (Chapter 6) on martingales, including sections on the Azuma inequality; and

(iii) A new chapter (Chapter 10) on Poisson approximations, including both the Stein-Chen method for bounding the error of these approximations and a method for improving the approximation itself.

In addition, we have added numerous exercises and problems throughout the text. Additions to individual chapters follow:

In Chapter 1, we have new examples on the probabilistic method, the multivariate normal distribution, random walks on graphs, and the complete match problem. Also, we have new sections on probability inequalities (including Chernoff bounds) and on Bayes estimators (showing that they are almost never unbiased). A proof of the strong law of large numbers is given in the Appendix to this chapter.

New examples on patterns and on memoryless optimal coin tossing strategies are given in Chapter 3.

There is new material in Chapter 4 covering the mean time spent in transient states, as well as examples relating to the Gibb's sampler, the Metropolis algorithm, and the mean cover time in star graphs.

Chapter 5 includes an example on a two-sex population growth model.

Chapter 6 has additional examples illustrating the use of the martingale stopping theorem.

Chapter 7 includes new material on Spitzer's identity and uses it to compute mean delays in single-server queues with gamma-distributed interarrival and service times.

Chapter 8 on Brownian motion has been moved to follow the chapter on martingales to allow us to utilize martingales to analyze Brownian motion.

Chapter 9 on stochastic order relations now includes a section on associated random variables, as well as new examples utilizing coupling in coupon collecting and bin packing problems.

We would like to thank all those who were kind enough to write and send comments about the first edition, with particular thanks to He Sheng-wu, Stephen Herschkorn, Robert Kertz, James Matis, Erol Pekoz, Maria Rieders, and Tomasz Rolski for their many helpful comments.

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Contents