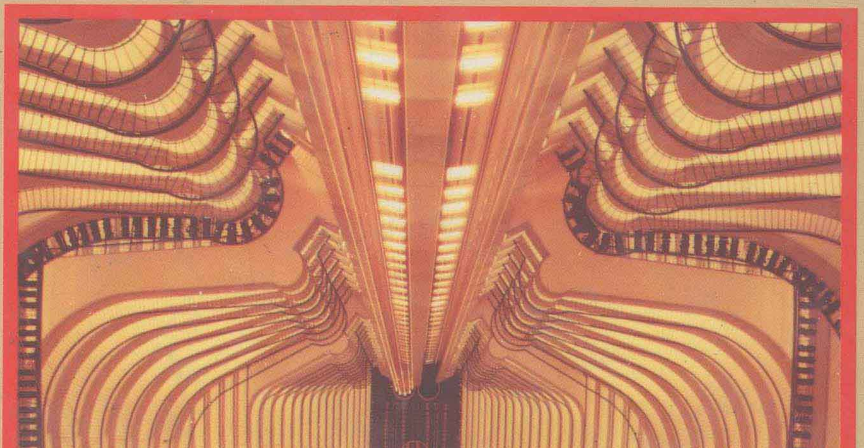
College Algebra

RALPH C. STEINLAGE



Second Edition

College Algebra

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Ralph C. Steinlage PROFESSOR OF MATHEMATICS UNIVERSITY OF DAYTON

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Preface

Overview

This text covers the traditional topics in a college algebra course; the primary emphasis is on development of skills in algebraic manipulation. Topics are presented in an intuitive manner, yet care is taken to insure that mathematical statements are precise. Word problems appear throughout the text to help students develop skills in translating applied problems into mathematics.

Revision Notes

In preparing the second edition of *College Algebra,* I have been influenced heavily by comments and suggestions received from users and reviewers of the first edition. I have especially tried to make the text more inviting to students. Much of the material is now presented less formally.

Some examples have been rewritten at a lower level of difficulty and more elementary examples and exercises have been included. Comments have been added to many of the solutions to examples and special attention has been paid to the coordination between examples and exercises. The exercise sets have been restructured to improve the grading in level of difficulty, and exercise instructions have been clarified. Some of the more intimidating examples and exercises have been removed. The more challenging exercises that remain are now numbered in red.

Caution boxes describing typical errors are a new feature in this edition. The number of Calculator Comments boxes and Historical Perspectives has also been increased. Discussions of rounding, significant digits, and the effects on computations have been added. Checking of solutions is also given increased emphasis.

Organization

Chapter 1 begins with a review of the real number system followed by the basic development of algebra. Complex numbers are introduced in preparation for the discussion of complex roots obtained by the quadratic formula (Chapter 2). Depending on the background of the class, Chapter 1, either partially or in its entirety, may be treated as an optional review. For a more basic review of sets, signed numbers, and fractional arithmetic, Appendix A has been added to the second edition. Further changes in this edition include the separation of radicals and fractional exponents into separate sections, significant expansion and reorganization of factoring into a separate section, combining of product expansions with polynomials, and placing of rational expressions in a separate section.

Chapter 2 is a thorough study of equations and inequalities; the techniques introduced here are used throughout the text. The first edition's section on word problems has been split. The first part (Section 2.2) describes a systematic approach to the solution of word problems, but is restricted essentially to linear word problems and expanded material on literal equations. Then a later word-problem section (Section 2.5) treats applications of quadratic equations. Additional non-science word problems have been included here and throughout the text. Scientific sophistication has been deemphasized throughout the text in both examples and exercises. Intervals are now emphasized as solutions to inequalities. Test points are now used to provide an expedient solution for higher-order inequalities and fractional inequalities.

Chapter 3 introduces the student to graphing in a Cartesian coordinate system. Circles, lines, and parabolas are studied extensively in the first part of the chapter. The remainder of the chapter treats the other conic sections: ellipses and hyperbolas. This latter material can be postponed or even omitted if the instructor chooses to do so. No later topics are dependent on this material. In this edition, the discussion of symmetry has been moved to Chapter 3 from Chapter 4.

Chapter 4 introduces the concept of function. The techniques of modifying a known graph by translation, reflection, and scaling are still emphasized in the general approach to graphing functions but there has been some rearrangement of topics for this edition. One-to-one functions are now included with the discussion of inverse functions.

Polynomial and rational functions and their graphs are discussed in detail in Chapter 5. The chapter begins with an analysis of the roots of a polynomial and methods for finding these roots. The chapter concludes with the graphing of polynomial and rational functions. In graphing these functions, emphasis is placed on the importance of roots and on the graphing techniques developed in Chapter 4. In this edition, the summary boxes for graphing polynomial and rational functions have been redone and the range limitation test has been changed to standard form. This entire chapter is independent of the remainder of the text and may be postponed or omitted with no loss of continuity.

Chapter 6 emphasizes the uses of exponential and logarithmic functions, such as in the description of exponential growth, compound interest, and carbon dating. Many questions in these areas require finding a logarithm. Logarithms continue to be important even though their use as a computational tool has been usurped by inexpensive calculators. In this edition, more emphasis has been given to the inverse nature of the logarithmic and exponential functions.

The material on systems of equations (Chapter 7) has been reordered and restructured. Echelon form now receives more emphasis. More elementary examples have been added. Matrices are introduced here as a means to simplify the solution of a system of equations. A section on matrix algebra is included for those who desire it but this section (7.7) could be omitted with no loss of continuity. Systems of inequalities are also studied in Chapter 7, and these lead naturally into linear programming. No topics later in the text depend on either of these sections, however.

Finally, Chapter 8 introduces the topics of discrete mathematics: mathematical induction, progressions, the binomial theorem, permutations, combinations, and probability. The topic of odds has been added to this edition. The emphasis in this chapter is on a structured approach to problems rather than memorization of formulas.

Features

Instructional Aids

1. **Chapter Introductions** An informal discussion of the concepts to be studied paves the way into the chapter and motivates the material in the chapter.

2. **Historical Perspectives** These are included to give the student an appreciation of the development of mathematics.
3. **Emphasis on Graphing** An intuitive approach is taken to the development of most topics. Whenever possible, graphs are used to illustrate the solution of a problem. Many of the exercises also emphasize graphs and graphing techniques.
4. **Calculators** The text recognizes and encourages the use of calculators without becoming a "calculator text." The use and operation of a calculator are explained in general terms (without reference to a specific model). Calculator Comments boxes describe calculator usage and warn students of erroneous answers that could be obtained by haphazardly using a calculator. Students are encouraged to use calculators rather than tables for computations involving exponents and logarithms. For those who desire it, however, an optional section (6.4) detailing the use of tables is included. Separate groups of calculator exercises have been added to this edition.

Study Aids

1. **Exercises** The exercise sets are extensive and are graded from easy to challenging. Students need not be expected to work all the exercises in the text. Supplementary exercise sets included at the end of each chapter may be used by the students as review before an exam. These extensive exercise sets should also serve as an excellent source of test problems for instructors. Answers to all odd-numbered exercises are provided at the end of the book.
2. **Rules Boxes** Throughout the text, rules and formulas are displayed in boxes and examples of their use are provided. In many cases, specific examples appear alongside the rules in these boxes.
3. **Listing of Steps** Whenever several steps are involved in a given procedure, each step is identified and listed so that the student can readily reference steps when needed.
4. **Emphasis on Orderly Procedure to Solve Long Problems** Solving large systems of equations can sometimes be frustrating. This is especially true when a minor arithmetic mistake has been made in the middle of a problem and a maze of computations must be checked. A record-keeping procedure is used to simplify the task of retracing one's steps in order to find an error in solving a system of equations or evaluating a large-order determinant.
5. **Chapter Reviews** Reviews summarize important terms and concepts, rules and formulas, and techniques introduced in the chapter. In many cases, a simple illustration or example appears alongside the concept, rule, or technique.

Accuracy

In order to guarantee the accuracy of the answers provided, all of the exercises and examples have been checked by an independent problem solver.

Applications

Numerous real-life applications are included throughout the text as examples and as exercises. These applications introduce and reinforce the concepts being studied, and serve to motivate the student whose primary interest is in a discipline other than mathematics. Many exercise sets also include applications problems.

Supplements

1. The Study Guide to accompany *College Algebra* (by Paul Fileger and John Piccirillo) includes a checklist of key concepts, additional examples, and hints for problem solving. Complete worked solutions are provided for selected odd-numbered exercises.
2. The Instructor's Manual to accompany this text contains solutions to all exercises.
3. The Test Bank provides four sample tests (one a multiple choice test) for each chapter and four sample final examinations. Answer keys with solutions are also provided for these tests and examinations.
4. Transparency Masters for important illustrations and graphs are available for use with an overhead projector.

Acknowledgments

I wish to thank the people who reviewed this text in its various stages of development. Their advice and comments regarding style, inclusion of topics, order of presentation, and level of difficulty were especially helpful. Many pertinent suggestions relating to both the details and the overall organization of the text were obtained from these evaluations. The list of reviewers for the second edition includes the following people:

Karen Baker   Indiana University
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Finally, I wish to thank Jeff Cole of Anoka-Ramsey Community College, who checked every exercise and example in the book.

Ralph C. Steinlage
January 1987

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Basic Algebra

1

Preceding even the most primitive attempts to record history, numbers and the counting process developed in a natural way to communicate essential ideas such as the size of a herd of animals. As number systems continued to be developed, they became an absolute necessity for commerce and communication. For example, our monetary system is based on numbers and the arithmetic operations of addition, subtraction, multiplication, and division; space flights would not be possible without algebra, trigonometry, calculus, and so on.

Scientists express the laws of nature in terms of letters that can represent different numbers in different situations. An example of this is the mass-energy equation $E = mc^2$ developed by Albert Einstein (1879–1955). In **algebra,** letters and symbols are used to represent numbers and to state rules by indicating that a property holds for all numbers. Since letters are simply names for numbers, the rules of arithmetic apply. We will review the elementary operations of arithmetic as they apply to the system of real numbers.

Section 1.1

The Real Number System

Natural Numbers

The counting numbers 1, 2, 3, . . . are called **natural numbers.** The sum and the product of two natural numbers is again a natural number. For example, $2 + 3 = 5$ and $3 \cdot 7 = 21$. Whenever one number is written as a product of other numbers as in $c = a \cdot b$, a and b are called **factors** or **divisors** of c; c is called a **multiple** of a and of b. Thus, 3 and 7 are factors of 21; we say that 21 can be **factored** as 3 times 7.

A natural number other than 1 whose only factors are 1 and itself is called a **prime number.** For example, 2, 3, 5, 7, 11, 13, 17, 19 are prime numbers, whereas $4 = 2 \cdot 2$, $9 = 3 \cdot 3$, and $12 = 3 \cdot 4$ are not. Every natural number that is not itself a prime number (these are called **composite numbers**) can be factored as a product of primes. For instance, $12 = 2 \cdot 2 \cdot 3$.

Factoring can facilitate finding the **greatest common divisor (GCD)** and **least common multiple (LCM)** (see Appendix A) of several numbers. For instance, we write

$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

and

$$84 = 2 \cdot 2 \cdot 3 \cdot 7$$

The factors 2, 2, and 3 divide both 60 and 84; thus, the GCD of 60 and 84 is $2 \cdot 2 \cdot 3 = 12$ since 12 is the largest natural number that divides evenly into both 60 and 84: $60 = 5 \cdot 12$ and $84 = 7 \cdot 12$. On the other hand, any multiple of 60 and 84 must have factors of 2, 2, 3, 5, and 7. The LCM of 60 and 84 is $2 \cdot 2 \cdot 3 \cdot 5 \cdot 7 = 420$ since 420 is the smallest natural number that is a multiple of both 60 and 84: $420 = 7 \cdot 60$ and $420 = 5 \cdot 84$. These techniques will be adapted to algebraic expressions later.

Integers

The **integers** are the natural numbers together with their negatives and 0:

$$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

The minus sign ($-$) is used to denote *below zero* or *negative*. For instance, $-10°$ represents 10° below zero. Death Valley has an elevation of -282 feet; it is 282 feet below sea level.

Rational Numbers

A number that can be expressed as a ratio of integers m/n with $n \neq 0$ is called a **rational number.** Every integer is also a rational number since $m = m/1$. Rational numbers are used to denote *fractional parts* or *fractions.* For instance, $\frac{2}{7}$ is used to indicate two pieces of something that has been divided into seven equal parts. In a fraction a/b, a is called the **numerator** and b is called the **denominator.**

Historical Perspective

One of the best-known theorems in mathematics is the **Pythagorean theorem,** which states that *the sum of the squares of the legs of a right triangle equals the square of the hypotenuse.* [The product of a number a with itself is denoted a^2 (read "a squared"). Thus, 7 squared is $7^2 = 7 \cdot 7 = 49$.]

The so-called Pythagorean school of mathematics, science, and philosophy was so well disciplined and organized that it survived for over 100 years after the death of Pythagoras (ca. 575–500 B.C.). All the works of the school were attributed to Pythagoras. However, there is no direct evidence that Pythagoras, or even the Pythagorean school, proved the Pythagorean theorem. In fact, the Babylonians knew of the "Pythagorean property" more than 1000 years before the time of Pythagoras, but they were unable to prove it.

The Pythagorean school evolved into a cult. Philosophy and mathematics became so intertwined that its members believed everything depended on the whole numbers. This is perhaps the only example in history of a religion based in mathematics. Their belief in the omnipotence of the whole numbers encompassed the fractional or rational numbers also; $\frac{1}{3}$ could be considered as being obtained by dividing a whole into three equal parts.

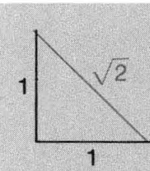


Figure 1.1
THE LENGTH $\sqrt{2}$

The Pythagorean theorem indicates the existence of a line segment whose length is $\sqrt{2}$. (The symbol $\sqrt{a}$ is used to denote a number that when multiplied by itself yields a; it is called the **square root** of a. Thus, $\sqrt{2} \cdot \sqrt{2} = 2$.) Here the $\sqrt{2}$ is the hypotenuse of a right triangle with legs of length 1 as shown in Figure 1.1. One of the greatest contributions of the Pythagoreans to the development of mathematics was their discovery that $\sqrt{2}$ could not be a rational number; it cannot be written as a fraction or ratio of integers m/n. Such numbers are said to be **irrational.** However, this discovery shook the Pythagorean school to its very core. It rendered invalid their basic belief that "everything depends on the whole numbers." Legend has it that Hippasus (ca. 500 B.C.) was set adrift at sea for revealing to outsiders the irrationality of $\sqrt{2}$.

Real Numbers

The real number line is used to describe numbers like $\sqrt{2}$, which arise from elementary geometry as lengths of line segments. (See the Historical Perspective.) Choosing a unit of length and marking an origin O on some line l, we assign to each point P on l a number equal to the length of the segment $\overline{OP}$. Points to the right of the origin are assigned positive numbers, and points to the left are assigned negative numbers. The number zero is assigned to the origin O. Each such number is called a **real number.** The line is called the **real number line.** The locations of several numbers on the real line are illustrated in Figure 1.2.

Figure 1.2
THE REAL NUMBER LINE

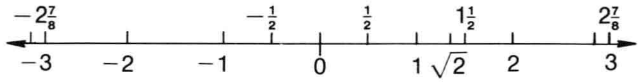
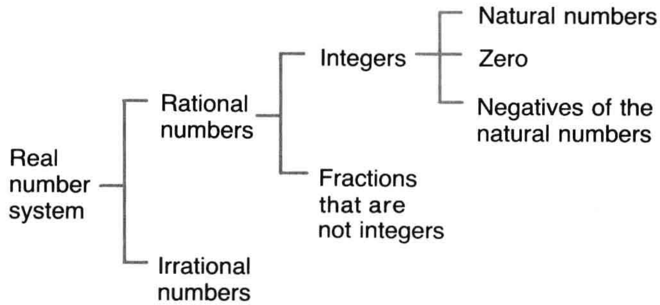


Figure 1.3 shows a breakdown of the number systems that we have discussed up to this point.

Figure 1.3
NUMBER SYSTEMS



One of the fundamental properties of the real number line is that it contains within itself the square roots, cube roots, and so on of each of its positive numbers. Later we shall describe the extension of the real numbers to a larger system, the complex numbers, which will include the square roots of all negative numbers as well.

The two basic arithmetic operations in the real number system are **addition** $(+)$ and **multiplication** $(\cdot)$. These operations are commutative and associative, and the distributive property holds.

SOME PROPERTIES OF REAL NUMBERS

	PROPERTY	EXAMPLE
Commutative	$a + b = b + a$	$5 + 4 = 4 + 5 = 9$
	$ab = ba$	$5 \cdot 4 = 4 \cdot 5 = 20$
Associative	$a + (b + c) = (a + b) + c$	$5 + (4 + 3) = (5 + 4) + 3 = 12$
	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$	$5 \cdot (4 \cdot 3) = (5 \cdot 4) \cdot 3 = 60$
Distributive	$a \cdot (b + c) = ab + ac$	$5 \cdot (4 + 3) = 5 \cdot 4 + 5 \cdot 3 = 35$
	$(a + b) \cdot c = ac + bc$	$(5 + 4) \cdot 3 = 5 \cdot 3 + 4 \cdot 3 = 27$
Identities	$a + 0 = a$	$5 + 0 = 5$
	$1 \cdot a = a$	$1 \cdot 3 = 3$
Inverses	$a + (-a) = 0$	$3 + (-3) = 0$
	$a \cdot \left(\frac{1}{a}\right) = 1, \quad a \neq 0$	$3 \cdot \left(\frac{1}{3}\right) = 1$

The number 0 is called the **additive identity** and 1 is called the **multiplicative identity;** $-a$ is called the **additive inverse** or **negative** of a, and $1/a$ is called the **multiplicative inverse** or **reciprocal** of a (when $a \neq 0$).

Multiplication by a natural number can be interpreted as repeated addition. For instance,

$$\begin{aligned} 2a &= (1 + 1) \cdot a \\ &= 1 \cdot a + 1 \cdot a && \text{(Distributive law)} \\ &= a + a && \text{(Multiplicative identity)} \end{aligned}$$

$$\begin{aligned} 3a &= (2 + 1)a \\ &= 2a + 1 \cdot a && \text{(Distributive law)} \\ &= (a + a) + a \\ &= a + a + a && \text{(Associative law)} \end{aligned}$$

Since $(a + a) + a = a + (a + a)$, we simply write this as $a + a + a$; the parentheses are unnecessary. Similarly, $(a + b) + (c + d)$ and $a + (b + c + d)$ can be written as $a + b + c + d$; $(ab)(cd)$ and $a(bc)d$ can be written as $abcd$ by virtue of the associative properties.

We note also that $0 \cdot a = 0$ for every real number a. For a, $1 \cdot a = (1 + 0) \cdot a = 1 \cdot a + 0 \cdot a = a + 0 \cdot a$. Thus $0 \cdot a$ must be the additive identity 0.

Subtraction $(-)$ undoes the operation of addition. If b is added to something to obtain a, then subtracting b from a yields this other number.

> DEFINITION
>
> For any two numbers a and b, the **difference** $a - b$ is that number c which when added to b yields a:
>
> $$a - b = c \qquad \text{means} \qquad a = b + c$$

For example,

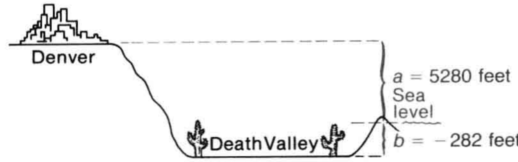
$$5 - 3 = 2 \qquad \text{since} \qquad 5 = 3 + 2$$

This property of subtraction is sometimes used when making change for a purchase. The excess amount can be returned by counting from the purchase price up to the tendered amount.

The rules governing subtraction and negative numbers are summarized as follows.

RULES FOR NEGATIVE NUMBERS

RULE	EXAMPLE
1. $a + (-b) = a - b$	**1.** A loss is a negative profit. Thus, a profit of \$10,000 plus a loss of \$2000 yields a net profit of \$8000: $\$10{,}000 + (-\$2000) = \$8000 = \$10{,}000 - \$2000$

RULE	EXAMPLE
2. $a - (-b) = a + b$	**2.**  5280 − (−282) = 5280 + 282 = 5562 Denver's elevation is 5562 feet greater than Death Valley's.
3. $-(-a) = a$	**3.** $-(-2) = 2$; the opposite of a \$2 loss is a \$2 gain.
4. $a(-b) = -(ab) = (-a)b$ In particular $(-b) = (-1)b$	**4.** $\left.\begin{array}{l} 2(-3) = (-3) + (-3) \qquad\qquad\quad = -6 \\ (-2)(3) = (-2) + (-2) + (-2) = -6 \end{array}\right\}$ $= -(2 \cdot 3)$ $(-1)b = -(1 \cdot b) = -b$
5. $(-a)(-b) = ab$	**5.** $(-2)(-7) = -[2(-7)] = -[-14]$ $= 14 = 2 \cdot 7$
6. $-(a + b) = -a - b$	**6.** $-(2 + 3) = -5 = -2 - 3$
7. a. $a(b - c) = ab - ac$	**7. a.** $3(6 - 2) = 3 \cdot 4 = 12$ and $3 \cdot 6 - 3 \cdot 2 = 18 - 6 = 12$
b. $(a - b)c = ac - bc$	**b.** $(7 - 4)5 = 3 \cdot 5 = 15$ and $7 \cdot 5 - 4 \cdot 5 = 35 - 20 = 15$

Division is the operation that undoes multiplication. If b is multiplied by another number c to yield a ($a = bc$), then dividing a by b yields this other number c. But $0 = 0 \cdot c$ for every real number c. Thus we cannot expect to recover the original number uniquely when $b = 0$. Hence, division by 0 is not defined.

DEFINITION

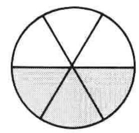
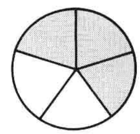
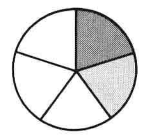
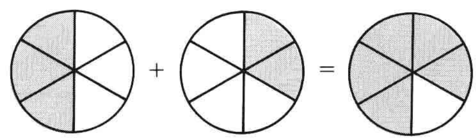
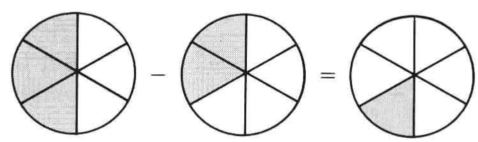
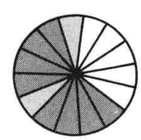
Division of a real number a by b, $b \neq 0$, $\left(\text{denoted by } a \div b,\ a/b, \text{ or } \dfrac{a}{b}\right)$ is that number c which when multiplied by b yields a:

$$\frac{a}{b} = c \qquad \text{means} \qquad a = bc, \qquad (b \neq 0)$$

c is called the **quotient** of the **dividend** a and the **divisor** b.

For example, $8 \div 4 = 2$, since $8 = 4 \cdot 2$; 8 divided by 4 yields the quotient 2.
The rules governing division and rational numbers are summarized in the following table.

RULES FOR FRACTIONS

RULE	EXAMPLE
1. $\frac{k \cdot m}{k \cdot n} = \frac{m}{n}$, $k \neq 0$, $n \neq 0$	**1.** $\frac{3}{6} = \frac{3 \cdot 1}{3 \cdot 2} = \frac{1}{2}$ 
2. a. $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$	**2. a.** $\frac{2}{5} + \frac{1}{5} = \frac{2 + 1}{5} = \frac{3}{5}$ 
b. $\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$	**b.** $\frac{2}{5} - \frac{1}{5} = \frac{2 - 1}{5} = \frac{1}{5}$ 
3. a. $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$	**3. a.** $\frac{1}{2} + \frac{1}{3} = \frac{1 \cdot 3}{2 \cdot 3} + \frac{2 \cdot 1}{2 \cdot 3} = \frac{1 \cdot 3 + 2 \cdot 1}{2 \cdot 3} = \frac{5}{6}$ 
b. $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$	**b.** $\frac{1}{2} - \frac{1}{3} = \frac{1 \cdot 3}{2 \cdot 3} - \frac{2 \cdot 1}{2 \cdot 3} = \frac{1 \cdot 3 - 2 \cdot 1}{2 \cdot 3} = \frac{1}{6}$ 
4. $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$	**4.** $-\left(\frac{2}{3}\right) = \frac{-2}{3} = \frac{2}{-3}$ since when any of these is added to $\frac{2}{3}$, the result is 0
5. $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$	**5.** $\frac{1}{5} \cdot \frac{2}{3} = \frac{1}{5} \cdot \frac{2 \cdot 5}{3 \cdot 5} = \frac{1}{5} \cdot \frac{10}{15} = \frac{2}{15}$ 
6. $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$	**6.** $\dfrac{\frac{2}{3}}{\frac{5}{7}} = \dfrac{\frac{2}{3} \cdot \frac{7}{5}}{\frac{5}{7} \cdot \frac{7}{5}} = \dfrac{\frac{2}{3} \cdot \frac{7}{5}}{1} = \frac{2}{3} \cdot \frac{7}{5}$