



# Sedimentation Coefficients, Diffusion Coefficients, Partial Specific Volumes, Frictional Ratios, and Second Virial Coefficients of Polymers in Solution<sup>\*,\*\*</sup>

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## A. SEDIMENTATION COEFFICIENTS, DIFFUSION COEFFICIENTS, PARTIAL SPECIFIC VOLUMES, AND FRICTIONAL RATIOS OF POLYMERS IN SOLUTION (2, 349–357, 799, 800)

### 1. Introduction

**1.1. Sedimentation Coefficient** The sedimentation coefficient  $s$  is defined as the sedimentation velocity in a unit force field

$$s = (dr/dt)/(\omega^2 r) \quad (\text{A1})$$

where  $r$  is the distance from center of rotation, and  $\omega$ , the angular velocity. For a given polymer–solvent system, the sedimentation coefficient is dependent on temperature  $T$ , pressure  $p$ , and polymer concentration  $C$ . Experimentally, sedimentation coefficients can be determined only with an ultracentrifuge.

#### Concentration Dependence

For polymer solutions studied by ultracentrifugation, the following concentration dependence of the sedimentation coefficient holds:

$$1/s = (1/s_0)(1 + k_s C + k'_s C^2 + \dots) \quad (\text{A2})$$

where  $s_0$  is the sedimentation coefficient at zero concentration. In Section B the concentration dependence of  $s$  according to Eq. (A2) is listed. In some cases, special extrapolation procedures are used. The tables in Section B also refer to the appropriate reference. For special treatments, see, e.g., Gehatia (358,359).

Sector shaped cells are normally used in an ultracentrifuge. In these cells, the square-dilution rule has to be taken into account (360):

$$C_t/C_0 = (r_0/r_t)^2 \quad (\text{A3})$$

where  $r_0$  and  $r_t$  are the distances of the meniscus and of the boundary from the axis of rotation and  $C_0$  and  $C_t$  the

concentrations at zero time and at time  $t$  at the boundary, respectively.

#### Averages of the Sedimentation Coefficient

The average of a function  $X(M)$  is defined as

$$X_\beta = \left( \int_0^\infty W(M) M^{\beta-1} X(M) dM \right) / \left( \int_0^\infty W(M) M^{\beta-1} dM \right) \quad (\text{A4})$$

with  $\int_0^\infty W(M) dM = 1$ , where  $W(M)$  denotes the probability density function (in our case the molar mass distribution). The average with  $\beta = 0$  is called the number average  $X_n$ , with  $\beta = 1$ , the mass average  $X_w$ , and with  $\beta = 2$ , the z-average  $X_z$ .

In principle it is possible to evaluate the various averages of the sedimentation coefficient from the distribution of the concentration gradient,  $\partial C/\partial r$ , in the cell:

$$s_n = M_n \int_0^\infty W(M) M^{-1} s(M) dM$$

$$s_w = \int_0^\infty W(M) s(M) dM$$

$$s_z = \int_0^\infty W(M) M^2 W(M) s(M) dM \quad (\text{A5})$$

where  $W(M) = C(M)/\int_0^\infty C(M) dM = C(M)/C_0$  is the mass distribution of the polydisperse polymer in the cell and  $s_n$ ,  $s_w$ , and  $s_z$  the number average, mass average and z-average of the sedimentation coefficient. In many cases the migration of the maximum of the quantity  $\partial C/\partial r$  is determined, which leads to a sedimentation coefficient  $s_t$ , that in turn is related, in a complicated manner, to the averages of  $s$  in Eq. (A5). Another method of evaluating the sedimentation coefficient is by observing the median, that is the line dividing the gradient curve in two equal areas, yielding a value  $s_m$  which is also a rather complicated average. Provided the skewness of the molar mass distribution is not very pronounced,  $s_w$  can be calculated by (Ref. 259)

$$s_w = (3s_m - s_t)/2 \quad (\text{A6})$$

Most ultracentrifuges measure the refractive index difference between polymer and solvent,  $dn$ , or the refractive index gradient  $\partial n/\partial r$  versus  $r$  (Schlieren optics). With certain assumptions  $\partial n/\partial r$  can be related to the concentration gradient (Ref. 14):

$$\partial n/\partial r = R(\partial C/\partial r) \quad (\text{A7})$$

where  $R$  is called the specific refractive index increment. The interference optics measure the deflection of the parallel interference lines in the solution. This deflection is directly proportional to the concentration of the polymer.

The absorption optics measure the optical density of the system as a function of the rotor distance, which, according to the Lambert–Beers law is proportional to the concentration of the polymer. The intensity is measured with a photoelectric scanner.

A multiplexer is used in order to measure several concentrations during one run (869–871).

#### Sedimentation Coefficient – Molar Mass Relationship

The dependence of  $s$  on the molar mass can be given by a power law expression:

$$s = K_s M^{a_s} \quad (\text{A8})$$

where  $K_s$  and  $a_s$  are empirical constants determined for each polymer–solvent system at given values of temperature and pressure. In Section B, relationship (A8) is listed whenever quoted in the literature, in preference to single  $s$  values,

#### Temperature Dependence, Pressure Dependence

Svedberg and Pedersen (2) and Mosimann and Signer (361) derived expressions for the temperature and pressure dependence of the sedimentation coefficient with restricted validity:

$$s/s^0 = (\eta_1^\theta/\eta_1)(1 - v_2^\theta\rho_1^\theta)/(1 - v_2\rho_1) \quad (\text{A9})$$

where  $s^0$ ,  $\eta_1^\theta$ ,  $v_2^\theta$  and  $\rho_1^\theta$  denotes the values at the reference temperature and/or pressure. Equation (A9) holds in the case of an incompressible medium and if no changes of size, shape and solvation state of the dissolved molecules in the temperature and pressure region take place. From these assumptions it may be deduced that Eq. (A9) can be applied with sufficient accuracy only on proteins in aqueous solvents.

Due to the high centrifugal fields applied in the ultracentrifuge, pressure gradients with pressures up to 200 bars are encountered in the cell and change appreciably the viscosity and the density of the solvent and the partial specific volume of the polymer. Thus the sedimentation coefficient,  $s$ , measured at a pressure  $p$  differs from the sedimentation coefficient,  $s^0$ , measured at 0 bar. Application of Eq. (A9) for the calculation of  $s$  requires the knowledge of  $\eta_1$  (362),  $\rho_1$  (given in a number of Handbooks) and  $v_2$  (148). More precise equations for the calculation of the pressure influence on the sedimentation have been worked out by several authors (363,364,365,800). The equation of Oth and Desreux (Ref. 363)

$$s = s^0(1 - \mu p) \quad (\text{A10})$$

has very often been used for the calculation of  $s^0$  (800). The

coefficient  $\mu$  depends on the polymer–solvent system and varies only slightly with pressure.

In Section B, the intrinsic sedimentation coefficient,  $[s^0]$ , instead of  $s^0$ , has been occasionally listed.  $[s^0]$  is related to Eq. (9):  $[s_0] = s_0\eta_1/(1 - v_2\rho_1)$ .

**1.2. Diffusion Coefficient** The translational diffusion coefficient  $D$  is defined by Ficks' first law:

$$J = -D\nabla C \quad (\text{A11})$$

where  $J$  is the flow of substance (total number of particles transported in unit time across unit surface) and  $\nabla C$  the concentration gradient. For one dimensional diffusion, Eq. (A11) reduces to

$$(1/A)(\partial n/\partial t) = -D(\partial C/\partial r) \quad (\text{A12})$$

where  $A$  denotes the area and  $n$  the number of moles. The negative sign in Eqs. (A11) and (A12) indicate that diffusion takes place in the direction of decreasing concentration. For practical reasons, Ficks second law is preferred

$$(\partial C/\partial t) = D\Delta C \quad (\text{A13})$$

where  $\Delta$  denotes the Laplace operator. Eq. (A13) reduces to

$$(\partial C/\partial t) = D(\partial^2 C/\partial r^2) \quad (\text{A14})$$

for the one dimensional case. Eqs. (A13) and (A14) allow the determination of  $D$  when solved with consideration to certain initial and boundary conditions.

Experimentally translational diffusion coefficients can be determined with an ultracentrifuge (see Section 1.3) in special diffusion cells using Eqs. (A11)–(A14) (800) and with dynamic light scattering (DLS). Since 1970, when the dynamic light scattering became available, nearly all diffusion coefficients have been measured by this method. Based on the theory of Pecora (853,854), the experimentally measured autocorrelation function  $g_2(t)$  is linked with the diffusion coefficient in polymer solutions (Refs. 730,855):

$$\lim_{c \rightarrow 0} g_2(t) = A + B \left( M_w^{-1} \int_0^\infty MW(M)p(q, M) \exp(-q^2 D(M)t) dM \right)^n \quad (\text{A15})$$

where  $n = 1$  for the heterodyne procedure and  $n = 2$  for the homodyne one. Eq. (A15) reduces, in the case of a monodisperse polymer solution, to

$$\lim_{c \rightarrow 0} g_2(t) = A + B(p(q, M) \exp(-q^2 D(M)t))^n \quad (\text{A16})$$

$A$  and  $B$  are measurable constants. Extrapolation of  $q$  against zero yields  $P(q, M) = 1$ . Note that  $g_2(t)$  is the z-average of the function,  $\exp(-q_2 D(M)t)$ . In this way one obtains the direct measurable quantity  $D_z$  from dynamic light scattering on polydisperse polymer solutions.

For polydisperse polymer solutions, Eq. (A16) may be expanded in a series (Ref. 856):

$$\lim_{\substack{C \rightarrow 0 \\ q \rightarrow 0}} g(t) = \exp(-q^2 D_z t) (1 + (\mu_2/2!)t^2 - (\mu_3/3!)t^3 + \dots) \quad (A17)$$

where the moments,  $\mu_i$ , are related to the molar mass distribution of the polymer (671).

More general methods for the determination of the distribution of the diffusion coefficient and of the molar mass are given elsewhere (857,858).

As has been pointed out, only the translational diffusion coefficient is listed in the tables. For higher modes of the autocorrelation function (e.g. rotation, vibration) see the appropriate literature (853,854).

The entire description of the translational diffusion of a polymer molecule in solution requires the determination of the mutual or cooperative diffusion coefficient,  $D_m$ , which characterizes the relaxation of a concentration gradient and the self-diffusion coefficient,  $D_s$ , which describes the Brownian or random motions of the polymer molecule (860–863).

$$D_m = (RT/f)(1 - v_2\rho_1)(1 + 2A_2MC + 3A_3MC^2 + \dots)$$

$$D_s = RT/f$$

At infinite dilution,  $D_m = D_s \times D_m$  is normally measured with classical diffusion cells or dynamic light scattering whereas  $D_s$  can be studied by measuring the migration of labeled solute molecules (864), by pulsed field gradient NMR (865), and by forced Rayleigh scattering (866). Normally,  $D_m$  is given in the tables. In some cases, especially where  $D_s$  is extrapolated to zero concentration,  $D_s = D_m$  is given in the tables.

#### Concentration Dependence

The concentration dependence of the diffusion coefficient may be described as

$$D = D_0(1 + k_D C + k'_D C^2 + \dots) \quad (A18)$$

where  $D_0$  is the diffusion coefficient at zero concentration. In Section B a concentration dependence of  $D$  according to Eq. (A18) is listed. In some cases, special extrapolation procedures are used. In these cases Section B refers to the appropriate references.

#### Averages of the Diffusion Coefficient

Similar considerations as made on sedimentation coefficients hold for diffusion coefficients. From measurements

in a diffusion cell, one obtains the various averages of the diffusion coefficient from the distribution of the concentration gradient,  $\partial C/\partial r$ , in the cell:

$$D_n = M_n \int_0^\infty W(M) M^{-1} D(M) dM$$

$$D_w = \int_0^\infty W(M) D(M) dM$$

$$D_z = (1/M_w) \int_0^\infty W(M) M D(M) dM \quad (A19)$$

where  $W(M) = C(M)/C_0$ .

The diffusion cell may be combined in the same manner as in the case of the ultracentrifuge with Schlieren optics, interference optics, and absorption optics. In most cases, the mass average and the area average of the diffusion coefficient are determined. They can be calculated with the help of Eqs. (A14) and (A19):

$$D_w = \left( \int_0^\infty r^2 (\partial C/\partial r) dr \right) / \left( 2t \int_0^\infty (\partial C/\partial r) dr \right) \quad (A20)$$

$$D_A = \int_0^\infty ((\partial C/\partial r) dr)^2 / (4\pi t (\partial C/\partial r)_{\max})^2 \quad (A21)$$

$(\partial C/\partial r)_{\max}$  is the maximum height of  $\partial C/\partial r$ . The integral in Eq. (A21) is the area of the curve.

As has been pointed out, dynamic light scattering measurements normally yield the z-average of  $D$ . Nevertheless it might be possible to determine averages other than  $D_z$  with the help of Eqs. (A19) and (A21). In Section B the different averages of  $D$  are quoted. In some cases unusual averages are listed. In these cases Section B refers to the appropriate references.

#### Diffusion Coefficient – Molar Mass Relationship

The diffusion coefficient – molar mass dependence frequently takes the form

$$D = K_D M^{-a_D} \quad (A22)$$

where  $K_D$  and  $a_D$  are constants for each polymer–solvent system at given values of temperature and pressure. In Section B relationship A(22) is listed whenever quoted in the literature, in preference to single  $D$  values.

#### Temperature Dependence, Pressure Dependence

A similar expression as Eq. (A9) holds under the same restricted conditions for the temperature and pressure dependence of the diffusion coefficient:

$$(D/T)/(D^\theta/T_0) = \eta_1^\theta/\eta_1 \quad (A23)$$

where  $D^\theta$  and  $\eta_1^\theta$  denote the values at the reference temperature  $T_0$  and/or pressure  $p_0$ . As has been pointed out in Eq. (A9), Eq. (A23) can be applied with sufficient accuracy on proteins in aqueous solvents only.

The temperature dependence of the diffusion coefficient is often described by the following exponential function

$$D = D_\infty \exp(-E_D/RT) \quad (\text{A24})$$

where  $E_D$  denotes the apparent activation energy of the diffusion and  $D_\infty$  the diffusion coefficient at the limit  $T = \infty$ . In Section B we have listed the temperature and pressure dependence of  $D$  whenever quoted in the literature.

### 1.3. Molar Mass Averages Determined from Sedimentation and Diffusion Coefficients

For the experimental determination of the molar mass from sedimentation and diffusion measurements, the Svedberg Eq. (A2) is used:

$$M = (s_0/(1/v_2\rho_1))/(D_0/RT) \quad (\text{A25})$$

Here  $s_0$  and  $D_0$  are the corrected and standardized coefficients for zero polymer concentration. For polydisperse polymers the various averages of the sedimentation and diffusion coefficient  $s$  and  $D$  are inserted, obtaining certain molar mass averages  $M_{\beta,\gamma}$ . Thus Eq. (A25) acquires the more general form:

$$M_{\beta,\gamma} = (s_{0,\beta}/(1 - v_2\rho_1))/(D_{0,\gamma}/RT) \quad (\text{A26})$$

For  $\beta = n, w, z, \dots$  and  $\gamma = n, w, z, \dots$  Eq. (A26) defines several different molar mass averages  $M_{n,n}$ ,  $M_{w,n}$ ,  $M_{w,w}$  etc. The averages,  $M_{n,n}$ , and  $M_{w,w}$ , are different from  $M_n$  and  $M_w$  respectively (859). The coefficients with  $\beta = n$  and  $z$  and  $\gamma = n$  are determinable only with large error. Instead, the coefficients with  $\beta = t$  and  $m$  (see under Section 1.1.) and  $\gamma = A$  and  $w$  (see under Section 1.2.) are usually evaluated. Thus, rather peculiar averages, e.g.  $M_{t,A}$ ,  $M_{m,w}$ , and  $M_{m,A}$ , result. The more straightforward molar mass averages  $M_{n,w}$ ,  $M_{w,w}$ ,  $M_{z,w}$ , etc., in relation to the familiar averages,  $M_n$ ,  $M_w$ , and  $M_z$ , are found elsewhere in the literature (369–371,859).

### 1.4. Partial Specific Volumes

The volume,  $V^{\text{id}}$ , of an ideal two component system can be expressed in terms of the masses  $m_1$  and  $m_2$  and the specific volumes,  $v_1^0$  and  $v_2^0$ , of the two components by the equation

$$V^{\text{id}} = m_1 v_1^0 + m_2 v_2^0 \quad (\text{A27})$$

Most components do not behave ideally upon mixing; i.e. they react with each other in a way so that the total volume deviates from  $V^{\text{id}}$ . The total volume can then be written as

$$V = m_1 v_1 + m_2 v_2; \quad v_i = (\partial V/\partial m_i)_{p,T,m_j \neq i} \quad (\text{A28})$$

where  $v_i$  ( $i = 1, 2$ ) is the partial specific volume of component  $i$ . When component 2 is polymer and component 1 solvent, then, for practical reasons it is convenient to introduce the so-called apparent partial specific volume  $v_2^*$  which is defined by

$$V = m_1 v_1^0 + m_2 v_2^* \quad (\text{A29})$$

where  $v_1^0$  denotes the specific volume of the solvent. The quantity,  $v_2^*$ , now contains the parameter of nonideal mixing of both the solvent and the polymer. In practice, however  $v_2^*$  differs not much from  $v_2$  if the polymer concentration is kept low (up to 1%). Dividing Eq. (A29) by the total mass  $m_1 + m_2$  leads to

$$v = w_1 v_1^0 + w_2 v_2^* \quad (\text{A30})$$

where  $v$  is the specific volume of the solution and  $w_i = m_i / \sum m_i$  are the mass fractions of the solvent ( $i = 1$ ) and the polymer ( $i = 2$ ).

With  $v = 1/\rho$  and  $v_1^0 = 1/\rho_1$ ,  $\rho$  and  $\rho_1$  being the densities of the solution and the solvent, respectively, it is readily found that

$$v_2^* = (1/w_2)(\rho_1 - w_1\rho)/(\rho\rho_1) \quad (\text{A31})$$

Eq. (A31) demonstrates that  $v_2^*$  can be determined by measuring  $\rho_1$  and  $\rho$ . Numerous methods for determining densities are described in the literature (268,372–379).

In order to determine the partial specific volume from the apparent specific volume, Eq. (A29) yields

$$(\partial(m_2 v_2^*)/\partial m_2)_{m_1} = m_2 (\partial v_2^*/\partial m_2)_{m_1} + v_2^* = (\partial V/\partial m_2)_{m_1} \quad (\text{A32})$$

where  $(\partial V/\partial m_2)_{m_1}$  is, according to definition, equal to  $v_2$ . In terms of mass fractions, Eq. (A32) can finally be written as

$$v_2 = v_2^* + w_1 w_2 (\partial v_2^*/\partial w_2) \quad (\text{A33})$$

Most values reported in the literature are  $v_2^*$ -values rather than  $v_2$ -values, since extrapolation according to Eq. (A33) is usually omitted. The differences between  $v_2^*$  and  $v_2$  are, however, often small.

### 1.5. Frictional Ratios

The molar frictional coefficient,  $f_{\text{sp}}$ , of an unsolvated spherical molecule may be computed by the formula based on Stokes law:

$$f_{\text{sp}} = 6\pi\eta R_H N_A = 6\pi\eta[3N_A M v_2/(4\pi)]^{1/3} \quad (\text{A34})$$

where  $\eta$  is the viscosity of the solvent and  $N_A$  is Avogadro's number. When the shape of a molecule deviates from a sphere, or when it is solvated, then the frictional coefficient,  $f_0$ , of such a molecule is larger than that of the spherical molecule. The frictional ratio,  $f_0/f_{\text{sp}}$ , thus permits to draw conclusions concerning either solvation or shape of the

molecule. It is possible to calculate the dimensions of the nonspherical molecule, provided a particular model (ellipsoid, cylinder, etc.) for the molecule is adopted and either the degree of solvation is known or assumed to be negligible. The molar frictional coefficient can be determined either from sedimentation velocity data, provided the molar mass is known from independent measurements according to the Svedberg equation

$$f_0 s_0 = M(1 - v_2 \rho_1) \quad (\text{A35})$$

or from diffusion measurements, using the relation

$$f_0 = RT/D_0 \quad (\text{A36})$$

Eqs. (A34)–(A36) are most frequently used for calculating the frictional ratio.  $f_0/f_{\text{sp}}$  is listed in the tables. Other relationships for the determination of  $f_0/f_{\text{sp}}$  are quoted in the literature (2). In these cases special reference to the literature is made in the tables.

In few cases, only the molar frictional coefficient  $f_0$ , rather than  $f_0/f_{\text{sp}}$ , was quoted in the literature. These values are inserted in the same column as the values for the frictional ratio.

### 1.6. List of Symbols and Abbreviations

#### Symbols

$C$	Concentration
$C_0$	Initial polymer concentration
$C_t$	Polymer concentration at time $t$
$D$	Diffusion coefficient
$D_A$	Area average of the diffusion coefficient
$D_n, D_w, D_z$	Number average, mass average, and z-average of the diffusion coefficient
$D$	Diffusion coefficient at reference temperature and/or pressure
$D_0$	Diffusion coefficient at zero concentration
$\eta_1$	Viscosity of solvent
$\eta_1^\theta$	Viscosity of solvent at reference temperature and/or pressure
$f_{\text{sp}}$	Molar frictional coefficient of a spherical molecule
$f_0$	Molar frictional coefficient at zero concentration
$k_s, k_s$	Concentration coefficients defined by Eq. (A2)
$k_D, k_D$	Concentration coefficients defined by Eq. (A18)
$M$	Molar mass of the polymer
$M_n, M_w, M_z$	Number average, mass average and z-average of the molar mass of the polymer
$M$	Molar mass determined by Mark–Houwink equation
$M_{\text{MW}}$	Molar mass determined by equations from Mandelkern and Flory, or Wales and Van Holde (see Section 1.7)

$M_{\beta,\gamma}$	Molar mass average determined by Eq. (A26) from $s_{0,\beta}$ and $D_{0,\gamma}$
$M_{s,w}, M_{s,A}, M_{s,D}$	Molar mass average determined from undefined sedimentation coefficient $s$ and $D_w, D_A$ and undefined diffusion coefficient $D$ , respectively
$n$	Refractive index or number of moles
$\omega$	Angular velocity
$r$	Distance from center of rotation
$r_0$	Distance of meniscus from center of rotation
$r_t$	Distance of boundary from center of rotation at time $t$
$\rho$	Density of solution
$\rho_1$	Density of the solvent
$R$	Gas constant
$s$	Sedimentation coefficient
$s_n, s_w, s_z$	Number average, mass average, and z-average of the sedimentation coefficient
$s_\theta$	Sedimentation coefficient at reference temperature and/or pressure
$s^0$	Sedimentation coefficient at pressure $p = 0$ bar
$s_0$	Sedimentation coefficient at zero concentration
$[s_0]$	Intrinsic sedimentation coefficient (see under Section 1.1)
$s_m$	Sedimentation coefficient determined from migration of gradient curve median
$s_t$	Sedimentation coefficient determined from migration of gradient curve maximum
$T$	Temperature
$\theta$	Theta temperature ( $A_2 = 0$ )
$t$	Time
$v_i$	Partial specific volume of component $i$ ( $i = 1$ , solvent; $i = 2$ , solute)
$v_1^0$	Specific volume of component $i$
$v_2^*$	Apparent specific volume of solute

#### Abbreviations

A	Archibald method
approx.	approximately
CLS	Classical light scattering method
DLS	Dynamic light scattering method
NBS	National Bureau of Standards, USA (now National Institute of Standard and Technology, NIST)
OS	Osmometry
PCC	Pressure Chemical Co., Pittsburgh, PA, USA
RT	Room temperature
SE	Sedimentation equilibrium
SRM	Standard reference material
SV	Sedimentation velocity
TS	Toyo Soda Co., Japan
V	Number of single values given in Ref. cited

**1.7. Miscellaneous** With certain assumptions it is possible to determine the molar mass from a combination of the intrinsic viscosity  $[\eta]$  and the limiting sedimentation coefficient  $s_0$ . In this way, Mandelkern and Flory (128) and Wales and Van Holde (134) derived an expression. An

expression similar to the equation between  $M$ ,  $[\eta]$ , and  $s_0$  was derived between  $M$ ,  $[\eta]$ , and  $D_0$  (128). The molar masses determined from either one of these relationships are referred to as MW.

TABLE 1. POLY(ALKENES)



TABLE 1. cont'd

Polymer	Solvent	T (°C)	M ( $\times 10^{-3}$ ) (g/mol)	$s_0 (\times 10^{13})$ (s)	$k_s$ (cm <sup>3</sup> /g)	$D_0 (\times 10^7)$ (cm <sup>2</sup> /s)	$k_D$ (cm <sup>3</sup> /g)	$f_0/f_{sp}$	$v_2$ (cm <sup>3</sup> /g)	Remarks	Refs.
Poly(l-hexene)	Acetone	20	46.8 222	11.5 21.5	16.2 6.4					$s_i; D_A; M_{i,A}$	53
			571	33.8	3.0						
Poly(tetrafluoroethylene) Perfluorotetracosane	20 325		260 2100		3.66 1.21	-35.2 -130.0		0.793		54 1004	

<sup>a</sup> See Ref. for various methods applied and values obtained.<sup>b</sup> Corrected for pressure influence.<sup>c</sup> See Ref. for def. of  $s$ .

TABLE 2. POLY(DIENES)

Polymer	Solvent	T (°C)	M ( $\times 10^{-3}$ ) (g/mol)	$s_0 (\times 10^{13})$ (s)	$k_s$ (cm <sup>3</sup> /g)	$D_0 (\times 10^7)$ (cm <sup>2</sup> /s)	$k_D$ (cm <sup>3</sup> /g)	$f_0/f_{sp}$	$v_2$ (cm <sup>3</sup> /g)	Remarks	Refs.
Poly(butadiene)	Diethyl ketone	10.3	60	1.76						$M_{MW}$	590
	Hexafluorobenzene	80	187	2.76							
	Diethylketone	10.3 (T = $\theta$ )	350	3.45							
			436	4.28							
			778	4.52							
			1380	5.15							
<i>cis</i> -4	Dodecane	80		23.8							62.5
<i>cis</i> -4	Hexafluorobenzene	80		16.3							62.5
95%, 1,4 <i>cis</i>	Diethylketone	10.3 (T = $\theta$ )		$s_0 = 0.53 \times 10^{-15} M^{0.5}$							
90%, 1,4 <i>cis</i> (TiCl <sub>4</sub> )	Hexane/heptane (1/1)	20	55-1080	$s_0 = 2.80 \times 10^{-15} M^{0.48}$							
90%, 1,4 <i>cis</i> (CoCl <sub>2</sub> )	Hexane/heptane (1/1)		34.7-1040	$s_0 = 2.33 \times 10^{-15} M^{0.50}$							
<i>cis</i> -4	Hexaiacontan	80			4.78						
linear	Cyclohexane	25			$D_0 = 1.45 \times 10^{-4} M_w^{-0.561}$						
					$11000 \leq M_w \leq 760000 \text{ g/mol}$						
<i>cis</i> -4	Dioxane	26.5 (T = $\theta$ )			$D_0 = 6.34 \times 10^{-5} M_w^{-0.496}$						
18-armed star	Cyclohexane	25			$11000 \leq M_w \leq 760000 \text{ g/mol}$						
					$D_0 = 2.39 \times 10^{-4} M_w^{-0.570}$						
	Dioxane	26.5 (T = $\theta$ )			$99000 \leq M_w \leq 1900000 \text{ g/mol}$						
					$D_0 = 8.87 \times 10^{-5} M_w^{-0.497}$						
					$99000 \leq M_w \leq 1900000 \text{ g/mol}$						
					$D_0 = 1.38 \times 10^{-4} M^{-0.5}$						
Poly(butadiene- <i>star</i> -acrylonitrile)	Methyl ethyl ketone/ cyclohexane (47.5/52.5)	22 (T = $\theta$ )	34.8-780	$s_0 = 1.28 \times 10^{-2} M^{0.5}$							552
	Methyl ethyl ketone/ isopropanol (60/40)	20 (T = $\theta$ )	475	7.58							
			205	4.65							
			101	3.43							
					16.0						
					20.6						
Poly(butadiene- <i>co</i> -styrene) branched	Benzene	25	81.9-168	1.21-4.63							406
	Methyl <i>n</i> -propyl ketone	21 (T = $\theta$ )	49-514	$s_0 = 1.04 \times 10^{-15} M^{0.5}$							573
21 mass% styrene	Methyl <i>n</i> -propyl ketone	21 (T = $\theta$ )	49-514	$s_0 = 0.83 \times 10^{-15} M^{0.5}$							344
Poly(chloroprene)	Methyl ethyl ketone	25	56-380	$s_0 = 3.1 \times 10^{-15} M^{0.5}$							444
					0.773						
					0.995 ± 0.005						
					See Ref.						
					$M_n$ ; OS; SE						



TABLE 3. ACRYLIC POLYMERS

			CLS, DLS	1006
Poly( <i>N</i> -isopropylmethacrylamide- <i>co</i> -acrylamide)				
IPAM-100	Deionized water		0.490	
IPAM-85			0.439	
IPAM-70			0.394	
IPAM-55			0.421	
IPAM-40			0.524	
IPAM			0.335	
Poly(acrylamide- <i>co</i> -sodium-3-acrylamido-3-methylbutanoate- <i>co</i> -(2-(acrylamido)-2-methylpropyl) trimethylammonium chloride)				
AM	NaAMB	AMPTAC	1023	
ATABAM 0.5-0.5	Water/1 M NaCl (1 M)	25	3430	0.498
ATABAM 2.5-2.5			13900	0.313
ATABAM 5.0-5.0			3450	0.476
ATABAM 10-10			10800	0.264
ATABAM 15-15			12200	0.313
ATABAM 5.0-10			11200	0.361
ATABAM 10-5.0			19400	0.389
Poly(acrylamide- <i>co</i> -3-(2-acrylamido-2-methylpropyl) propane dimethylammonio-1-propane-sulfonate) (AM- <i>co</i> -AMPDAPS)				
DAPSAM-1	Water/1 M NaCl	25	3200	0.505
DAPSAM-5			12400	0.316
DAPSAM-10-2			700	0.401
DAPSAM-10-3			21500	0.320
DAPSAM-25-2			8200	0.397
DAPSAM-40-2			15100	0.362
DAPSAM-40-3			17500	0.324
DAPSAM-60-2			5400	0.360
DAPSAM-75-2			6200	0.490
DAPSAM-100			3000	0.628
Poly(acrylamide- <i>co</i> -NaAMPS- <i>co</i> -AMPTAC)				
ATASAM 0.5-0.5	Water/1 M NaCl	25	6030	0.423
ATASAM 2.5-2.5			2780	0.577
ATASAM 5-5			5070	0.381
ATASAM 10-10			5870	0.449
ATASAM 15-15			6770	0.433
ATASAM 10-5			3880	0.413
Poly(NaAMPS- <i>co</i> -AMPTAC)				
ATAS-0	Water/1 M NaCl	25	6120	0.387
ATAS-10-2			8220	0.419
ATAS-25-2			6200	0.473
ATAS-40-2			7920	0.385
ATAS-50-2			7680	0.463
ATAS-70-2			4980	0.406
ATAS-100			1470	0.637
Poly(1,4-benzamide)	Dimethylacetamide/ 3% (g/mol) LiCl	25	55	0.387
Poly(1,4-phenylene terephthalamide)	96 w.t. % H <sub>2</sub> SO <sub>4</sub>	30	48.3 38.4	0.072 0.088
			22.7 15.9	0.113 0.159
			$D_0 = 4.38 \times 10^{-4} M_w^{0.42}$	1007



Dimethylacetamide/ 0.3% LiCl	2.5	79.3 60.6 41.1 29.3	1.45 1.33 1.12 0.88	900 890 720 460	
Poly(benzyl acrylate- <i>co</i> -methyl acrylate) different comp.	20	46	17.7	0.94	<i>v</i> <sub>2</sub> and <i>M</i> given
Methyl ethyl ketone	23.3	306	0.207	<i>M</i> <sub>v</sub>	
Methyl isobutyl ketone	25		0.53	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Methyl ethyl ketone	25	237	0.160	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Acetone			0.194	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Methanol			0.230	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Pentane			0.244	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Acetone	25	180	0.182	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Methanol			0.203	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Pentane			0.228	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Methyl ethyl ketone	25	149	0.139	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Acetone			0.162	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Methanol			0.191	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Pentane			0.191	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Methyl ethyl ketone	25	75	0.109	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Acetone			0.117	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Methanol			0.127	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Pentane			0.137	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Methyl isobutyl ketone	21.2	820	0.13	<i>M</i> <sub>w</sub> ; <i>s</i> <sub>0</sub>	
Poly( <i>n</i> -butyl methacrylate)			0.192	<i>M</i> <sub>v</sub>	148
Propan-2-ol	21.5	40-8000	6.3 × 10 <sup>-5</sup> <i>M</i> <sup>-0.50</sup>	<i>M</i> <sub>w</sub> , <i>C</i> = 0.1-0.5 g/dm <sup>3</sup> ;	399
Ethyl acetate	20	254	0.174	see Ref.	646
			0.145	<i>M</i> <sub>w</sub> , <i>C</i> = 0.1-0.5 g/dm <sup>3</sup> ;	
			0.181	see Ref.	
Tetrahydrofuran		340	0.181	<i>M</i> <sub>w</sub> , <i>C</i> = 0.1-0.5 g/dm <sup>3</sup> ;	
			0.191	see Ref.	
		380	0.191	<i>M</i> <sub>w</sub> , <i>C</i> = 0.1-0.5 g/dm <sup>3</sup> ;	
				see Ref.	
Poly( <i>tert</i> -butyl methacrylate)	25		<i>s</i> <sub>0</sub> = 0.12 <i>M</i> <sup>0.31</sup>	<i>C</i> = 0.5 g/dm <sup>3</sup>	1052
Butyl acetate	20		<i>s</i> <sub>0</sub> = 3.02 × 10 <sup>-5</sup> <i>M</i> <sub>w</sub> <sup>0.41</sup>	<i>D</i> <sub>A</sub> ; <i>M</i> <sub>s,D</sub> given in Ref.	551
	21			<i>M</i> <sub>s,D</sub> ; <i>D</i> <sub>A</sub>	560
Poly(cetyl methacrylate)	Heptane	21	<i>s</i> <sub>0</sub> = 1.42 × 10 <sup>-14</sup> × <i>M</i> <sub>w</sub> <sup>0.36</sup>	<i>M</i> <sub>s,D</sub> ; <i>D</i> <sub>A</sub>	580
Benzene	21	140-780	<i>s</i> <sub>0</sub> = 1.9 × 10 <sup>-15</sup> <i>M</i> <sub>w</sub> <sup>0.46</sup>	<i>M</i> <sub>s,D</sub>	
Poly(cholesteryl acrylate)					
Toluene	25				
Poly(2,6-diisopropyl-phenylmethacrylate)					
Poly(2,6-dimethyl-phenylmethacrylate)	Toluene	25			
Poly(acrylic acid)	Water/NaCl	20-22 <sup>c</sup>	1100	0.789	0.894
				0.73	0.73
(0.2 mol/dm <sup>3</sup> )					
(0.02 mol/dm <sup>3</sup> )					
(0.012 mol/dm <sup>3</sup> )					
(0.006 mol/dm <sup>3</sup> )					
7.3 ± 0.5					
81.0				35.0	20.0
114.0				0.65 ± 0.02 <sup>d</sup>	28.2
					49.0

TABLE 3. *cont'd*

Polymer	Solvent	T (°C)	M ( $\times 10^{-3}$ ) (g/mol)	$s_0$ ( $\times 10^{13}$ ) (s)	$k_t$ (cm <sup>3</sup> /g)	$D_0$ ( $\times 10^7$ ) (cm <sup>2</sup> /s)	$k_D$ (cm <sup>3</sup> /g)	$f_0/f_{sp}$	$v_2$ (cm <sup>3</sup> /g)	Remarks	Refs.
(1.0 mol/dm <sup>3</sup> ) (0.2 mol/dm <sup>3</sup> ) (0.012 mol/dm <sup>3</sup> ) (0.006 mol/dm <sup>3</sup> ) (0.0012 mol/dm <sup>3</sup> ) (1.0 mol/dm <sup>3</sup> ) (0.2 mol/dm <sup>3</sup> ) (0.012 mol/dm <sup>3</sup> ) (0.006 mol/dm <sup>3</sup> )	420	6.5 ± 0.5	2.4 4.2 32.0 60.0	0.2 1.0 10.0 27.4	$M_{v,D}$	555					
Methyl isobutyl ketone	70	2.4 ± 0.1	0.0 2.75 8.0	146 22.3 22.3 150	$M_{v,D}$	~					
<i>n</i> -Propanol	28	18-155	$s_0 = 1.05 \times 10^{-2} M_n^{0.47}$	286	0.951	$M_v/M_w$	148 456				
Poly(ethyl acrylate- <i>cis</i> -acrylic-acid) sodium salt	Water/NaCl (0.05 mol/dm <sup>3</sup> ) (0.1 mol/dm <sup>3</sup> ) (0.3 mol/dm <sup>3</sup> ) (0.5 mol/dm <sup>3</sup> ) (0.7 mol/dm <sup>3</sup> ) (1.0 mol/dm <sup>3</sup> ) (1.2 mol/dm <sup>3</sup> ) 2-Heptanone	185-1300	$D_{0,z} = 5.76 \times 10^{-4} M^{-0.65}$ $D_{0,z} = 8.55 \times 10^{-4} M^{-0.67}$ $D_{0,z} = 3.73 \times 10^{-4} M^{-0.59}$ $D_{0,z} = 1.54 \times 10^{-4} M^{-0.52}$ $D_{0,z} = 1.41 \times 10^{-4} M^{-0.51}$ $D_{0,z} = 1.24 \times 10^{-4} M^{-0.50}$ $D_{0,z} = 1.10 \times 10^{-4} M^{-0.49}$ $D_{0,z} = 1.35 \times 10^{-4} M^{-0.51}$	$M_w/M_n = 1.8-2.3$	637						
Poly(ethyl acrylate- <i>cis</i> -methylacrylate) different comp.	Methyl ethyl ketone	20	77-7440 34.1-9350	$s_0 \sim M^{0.43}$ 8.95-120	$D_0 \sim M^{-0.57}$ 17.4-0.85	$f_{sp} \sim M^{0.57}$	0.798 0.798 0.798	$v_2$ and $M$ given $M_{t,A}$ ; $s_i$ ; See Ref. extrapolated value	506 59 18		
Poly(ethyl methacrylate)	Acetone	20	74.6-7440 410 1036	7.45-107 63.0-67.5	23-60 20.0-0.95	0-63 0.863 0.900	$M_{t,w}; s_i; D_A$ , See Ref. $M_v$ $A'$	268 52 148 427			
Methyl isobutyl ketone (theta-solvent)*	22.9 25	875				$C = 2.5 \text{ g/cm}^3; s_w$ (Trautmann)	0.789	1053			
Poly(isooctyl methacrylate)	Toluene	25									
Poly(phenyl methacrylate)	alkyl : hexyl	25	306 494	1.22 1.02	$1.54 \times 10^{-4}$ 1.72 × 10 <sup>-4</sup>	$M_w/M_n = 1.5; dn/dC =$ $0.135 \text{ cm}^3/\text{g}$	100				
Poly(sulfonfalkyl) methacrylate)	0.1 M NaCl octyl decyl ethyl	701 734	1.00 0.975	$1.99 \times 10^{-4}$ 2.07 × 10 <sup>-4</sup>	$M_w/M_n = 2.0; dn/dC =$ $0.140 \text{ cm}^3/\text{g}$	~					
Dimethylsulfoxide	25	34 130 260 429 432	0.46 1.02 1.27 1.66 1.85	$M_w/M_n = 1.8; dn/dC =$ $0.149 \text{ cm}^3/\text{g}$	638						
Poly(methacrylic acid)					$M_w/M_n = 2.1; dn/dC =$ $0.170 \text{ cm}^3/\text{g}$						