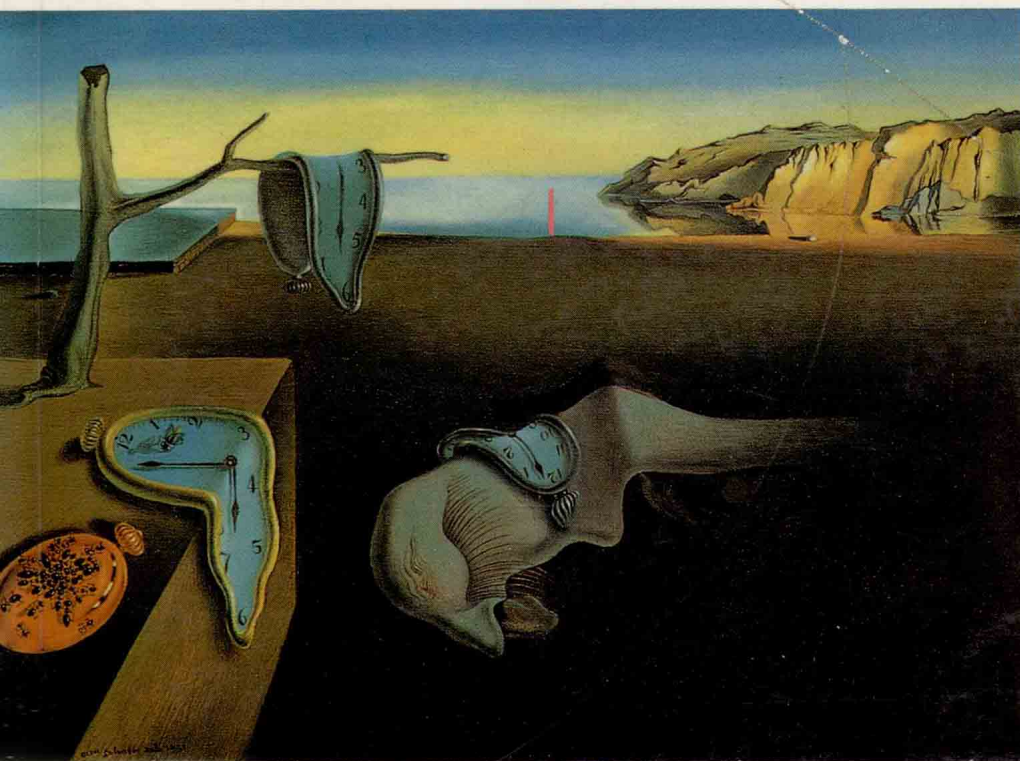


ROBERT J. BARRO

XAVIER SALA-I-MARTIN

ECONOMIC GROWTH



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ECONOMIC GROWTH

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Harvard University

Xavier Sala-i-Martin

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To My First Grandchild
—*Robert J. Barro*

A la Gubi i a la Schuxeta
—*Xavier Sala-i-Martin*

FOREWORD

The field of economic growth has reawakened. When I began studying economics almost two decades ago, the field of economic growth was dormant. The courses I took in macroeconomics included at most a brief section on long-run economic growth. And even that was at the end of the course. It was part of the material that the professor, always running behind schedule, never had time to cover in class.

Today, economic growth is central to the study of macroeconomics. Economists have come to understand that long-run growth is as important—perhaps even more important—than short-run fluctuations. The newspaper is filled with accounts of monthly changes in industrial production and retail sales. But these short-run changes have a relatively minor impact on economic well-being. Why GDP rose or fell a few percent over the last three months can be an intriguing question. Even more significant, however, is why the United States is so much richer than Nigeria or why growth in U.S. incomes over the past quarter-century has been slower than growth over the previous quarter-century.

Scholars choose the topics they study, however, based on more than the topics' importance. To a large extent, they choose topics based on their ability to say something novel. It is for this reason that the field of economic growth became dormant and then reawoke. Work on economic growth stopped in the 1960s because economists had nothing new to say. Twenty years later, a small group of economists began to explore alternative ways of explaining the large differences in income we observe across countries and over time. The new growth theory has highlighted ideas that played only a small role in the growth theory inherited from the past. Increasing returns, human capital, research and development, learning-by-doing, and externalities are now central to discussions of economic growth. At the same time, new data on economic growth have become available for a large sample of countries. These data have allowed the new research to include a healthy interplay between theory and empirics.

When the editors at McGraw-Hill asked me to help them assemble a series of advanced textbooks in economics, I had no doubt that a book on economic growth

should be high on the agenda. Much had been learned and reported in academic journals. But no book was available to explain systematically all this material to the student. This book, the first in the McGraw-Hill series, fills that void. Moreover, this synthesis is presented by two of the most important scholars in this exploding field.

Economic growth comes largely from the accumulation of knowledge. This knowledge passes from one generation to the next in the form of textbooks. So, in a sense, this wonderful book by Robert Barro and Xavier Sala-i-Martin is not just about economic growth. It is itself part of the process of economic growth.

*N. Gregory Mankiw
Harvard University
July 1994*

PREFACE

Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, *what*, exactly? If not, what is it about the "nature of India" that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else. (Lucas [1988])

Economists have, in some sense, always known that growth is important. Yet at the core of the discipline, the study of economic growth languished after the late 1960s. Then, after a lapse of nearly two decades, this research became vigorous again in the mid-1980s. The impending tenth anniversary of this revival is a good time to assess the recent investigations and to place them in the context of earlier work. This unified approach brings out the contributions of the old and new research and also reveals areas in which knowledge is lacking. We attempt in some cases to fill the holes and in other cases to point out profitable directions for future work.

The research of the mid-1980s began with models of the determination of long-run growth, an area that is now called endogenous growth theory. Other recent research extended the older, neoclassical growth model, especially to bring out further the empirical implications of the theory. This book combines new results with expositions of the main research that appeared from the 1950s through the 1990s. The discussion stresses the empirical implications of the theories and the relation of these hypotheses to data and evidence. This combination of theory and empirical work is the most exciting aspect of the ongoing resurgence of work on economic growth.

The introduction motivates the study, brings out some key empirical regularities in the growth process, and provides a brief history of modern growth theory. Chapters 1–3 deal with the neoclassical growth model, from Solow–Swan in the 1950s, to Cass–Koopmans (and recollections of Ramsey) in the 1960s, to recent extensions. Chapters 4 and 5 cover the versions of endogenous growth theory that rely on forms of constant returns to reproducible factors. Chapters 6–8 explore recent models of technological change and R&D, including expansions in the variety and quality of products and the diffusion of knowledge. Chapter 9 allows for an

endogenous determination of labor supply and population, including models of migration, fertility, and labor/leisure choice. Chapter 10 details the nature and availability of applicable data, and Chapters 11 and 12 discuss some empirical findings.

The material is written as a text at the level of first-year graduate students in economics. It is especially suitable for courses in macroeconomics, economic growth, and economic development. The authors developed and used the manuscript in second-year elective courses on economic growth and have used parts of the material in first-year, core graduate courses in macroeconomics. Other professors have already successfully used the manuscript for classes in macroeconomics, growth, and development.

Most of the chapters include problems that guide the students from routine exercises through suggestive extensions of the models. The level of mathematics includes differential equations and dynamic optimization, topics that are discussed in the mathematical appendix at the end of the book. For undergraduates who are comfortable with this level of mathematics, the book would work well for an advanced, elective course.

The lively pace of theoretical and empirical research on growth means that this version of the book will not remain up to date for many years. We therefore plan to revise as needed to maintain currency with developments in the field. Suggestions from readers—including notices of omissions of important contributions—would be appreciated. We have benefited in the preparation of this first edition from comments on the text or on related papers of ours by Philippe Aghion, Minna S. Andersen, Gary Becker, Olivier Blanchard, Juan Braun, Paul Cashin, Daniel Cohen, Michelle Connolly, Oded Galor, Zvi Griliches, Gene Grossman, Elhanan Helpman, Dale Jorgenson, Ken Judd, Jinill Kim, Michael Kremer, Phil Lane, Norman Loayza, Greg Mankiw, Casey Mulligan, Kevin M. Murphy, Pietro Peretto, Torsten Persson, Jordan Rappaport, Sergio Rebelo, Paul Romer, Michael Sarel, Etsuro Shioji, Chris Sims, B. Anna Sjögren, Nancy Stokey, Robert Tamura, Merritt Tilney, Aaron Tornell, Jaume Ventura, and Alwyn Young.

Robert J. Barro
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CONTENTS

Foreword	xv
Preface	xvii
Introduction	1
1.1 The Importance of Growth	1
1.2 Empirical Regularities about Economic Growth	5
1.3 A Brief History of Modern Growth Theory	9
1 Growth Models with Exogenous Saving Rates (The Solow–Swan Model)	14
1.1 The Basic Structure	14
1.2 The Neoclassical Model of Solow and Swan	16
1.2.1 The Neoclassical Production Function	16
1.2.2 The Fundamental Dynamic Equation for the Capital Stock	17
1.2.3 The Steady State	19
1.2.4 The Golden Rule of Capital Accumulation and Dynamic Inefficiency	19
1.2.5 Transitional Dynamics	22
1.2.6 Policy Experiments	24
1.2.7 An Example: Cobb–Douglas Technology	25
1.2.8 Absolute and Conditional Convergence	26
1.2.9 Convergence and the Dispersion of Per Capita Income	31
1.2.10 Technological Progress	32
1.2.11 A Quantitative Measure of the Speed of Convergence	36
1.3 Models of Endogenous Growth	38
1.3.1 The AK Model	39
1.3.2 Endogenous Growth with Transitional Dynamics	41
1.3.3 Constant-Elasticity-of-Substitution Production Functions	42
1.3.4 The Leontief Production Function and the Harrod–Domar Controversy	46
1.3.5 Growth Models with Poverty Traps	49

Appendix	Proofs of Various Propositions	52
	Proof That Each Input Is Essential for Production with a Neoclassical Production Function	52
	Properties of the Convergence Coefficient in the Solow–Swan Model	53
	Proof That Technological Progress Must Be Labor Augmenting	54
	Properties of the CES Production Function	55
	Problems	56
2	Growth Models with Consumer Optimization (The Ramsey Model)	59
2.1	Households	60
2.1.1	Setup of the Model	60
2.1.2	First-Order Conditions	63
2.2	Firms	67
2.3	Equilibrium	70
2.4	Alternative Environments	71
2.5	The Steady State	72
2.6	Transitional Dynamics	74
2.6.1	The Phase Diagram	74
2.6.2	The Shape of the Stable Arm	76
2.6.3	Behavior of the Saving Rate	77
2.6.4	The Paths of the Capital Stock and Output	79
2.6.5	Speeds of Convergence	80
Appendix 2A	Log-Linearization of the Ramsey Model	87
Appendix 2B	Behavior of the Saving Rate	89
Appendix 2C	Proof That $\gamma_{\hat{k}}$ Declines Monotonically If the Economy Starts from $\hat{k}(0) < \hat{k}^*$	90
	Problems	92
3	The Open Economy, Finite Horizons, and Adjustment Costs	96
3.1	An Open-Economy Version of the Ramsey Model	96
3.1.1	Setup of the Model	96
3.1.2	Behavior of a Small Economy’s Capital Stock and Output	98
3.1.3	Behavior of a Small Economy’s Consumption and Assets	99
3.1.4	The World Equilibrium	100
3.2	The World Economy with a Constraint on International Credit	101
3.2.1	Setup of a Model with Physical and Human Capital	101
3.2.2	The Closed Economy	102
3.2.3	The Open Economy	103
3.3	Variations in Preference Parameters	108
3.4	Economic Growth in a Model with Finite Horizons	110
3.4.1	Choices in a Model with Finite Horizons	110
3.4.2	The Finite-Horizon Model of a Closed Economy	114
3.4.3	The Finite-Horizon Model of an Open Economy	116

3.5	Adjustment Costs for Investment	119
3.5.1	The Behavior of Firms	119
3.5.2	Equilibrium with a Given Interest Rate	122
3.5.3	Equilibrium for a Closed Economy with a Fixed Saving Rate	125
3.6	Some Conclusions	127
	Appendix Overlapping-Generations Models	128
	Households	128
	Firms	130
	Equilibrium	130
	The Steady State	131
	The Golden Rule and Dynamic Efficiency	133
	Dynamics	134
	Altruism, Bequests, and Infinite Horizons	135
	Problems	137
4	One-Sector Models of Endogenous Growth	140
4.1	The AK Model	141
4.1.1	Behavior of Households	141
4.1.2	Behavior of Firms	141
4.1.3	Equilibrium	142
4.1.4	Transitional Dynamics	142
4.1.5	Determinants of the Growth Rate	143
4.2	A One-Sector Model with Physical and Human Capital	144
4.3	Models with Learning-By-Doing and Knowledge Spillovers	146
4.3.1	Technology	146
4.3.2	Equilibrium	148
4.3.3	Pareto Nonoptimality and Policy Implications	149
4.3.4	A Cobb–Douglas Example	150
4.3.5	Scale Effects	151
4.4	Government and Growth	152
4.4.1	The Public-Goods Model of Productive Government Services	152
4.4.2	The Congestion Model of Productive Government Services	158
4.5	Transitional Dynamics in an Endogenous Growth Model	161
4.5.1	A Cobb–Douglas Example	161
4.5.2	A CES Example	164
4.6	Concluding Observations	166
	Appendix Conditions for Endogenous Growth in the One-Sector Model	167
	Problems	169
5	Two-Sector Models of Endogenous Growth (With Special Attention to the Role of Human Capital)	171
5.1	A One-Sector Model with Physical and Human Capital	172
5.1.1	The Basic Setup	172
5.1.2	The Constraint of Nonnegative Gross Investment	175

5.2	Different Technologies for Production and Education	179
5.2.1	The Model with Two Sectors of Production	179
5.2.2	The Uzawa–Lucas Model	182
5.2.3	The Generalized Uzawa–Lucas Model	196
5.2.4	The Model with Reversed Factor Intensities	197
5.3	Conditions for Endogenous Growth	198
5.4	Summary Observations	200
Appendix 5A	Transitional Dynamics with Inequality Restrictions on Gross Investment in the One-Sector Model	201
Appendix 5B	Solution of the Uzawa–Lucas Model	204
Appendix 5C	The Model with Reversed Factor Intensities	208
	Problems	210
6	Technological Change: Models with an Expanding Variety of Products	212
6.1	Models with a Variety of Producer Products	213
6.1.1	Production with a Fixed Number of Products	213
6.1.2	Expansions in the Variety of Products	215
6.1.3	Households and Market Equilibrium	218
6.1.4	Determinants of the Growth Rate	220
6.1.5	Pareto Optimality	220
6.1.6	Erosion of Monopoly Power	223
6.1.7	Romer’s Model of Technological Change	226
6.2	Models with a Variety of Consumer Products	231
6.2.1	Varieties of Consumer Goods	231
6.2.2	A Comparison of Consumer Variety with Producer Variety	236
6.3	Concluding Observations	237
	Problems	238
7	Technological Change: Models with Improvements in the Quality of Products	240
7.1	Sketch of the Model	241
7.2	Behavior of Firms	242
7.2.1	Levels of Quality in the Production Technology	242
7.2.2	The Incentive to Innovate	246
7.2.3	The Behavior of the Aggregate Quality Index	251
7.2.4	The Market Value of Firms	252
7.3	Households and Market Equilibrium	252
7.4	Innovation by the Leader	254
7.4.1	The Leader as a Monopoly Researcher	255
7.4.2	Research by Outsiders	257
7.5	Pareto Optimality	259
7.6	Summary Observations about Growth	262
	Problems	263

8	The Diffusion of Technology	265
8.1	A Leader-Follower Model	266
8.1.1	Behavior of Innovators in the Leading Country	267
8.1.2	Behavior of Imitators in the Follower Country	268
8.1.3	Variations in the Cost of Imitation	272
8.1.4	Empirical Implications for Convergence	274
8.2	Mutual Invention and Imitation	276
8.3	Foreign Investment	276
8.4	Leapfrogging	279
8.5	Summary Observations about Diffusion and Growth	281
	Problems	281
9	Labor Supply and Population	285
9.1	Migration in Models of Economic Growth	285
9.1.1	Migration in the Solow–Swan Model	286
9.1.2	Migration in the Ramsey Model	294
9.1.3	The Braun Model of Migration and Growth	300
9.2	Fertility Choice	308
9.2.1	An Overlapping-Generations Setup	309
9.2.2	The Model in Continuous Time	311
9.3	Labor/Leisure Choice	321
	Appendix 9A The Form of the Utility Function with Consumption and Work Effort	326
	Problems	328
10	Data on Economic Growth, Growth Accounting	330
10.1	Panel Data for Countries	330
10.2	Long-term Data on GDP	332
10.3	Regional Data Sets	341
10.3.1	Data for U.S. States	341
10.3.2	Data for European Regions	342
10.3.3	Data for Canadian Provinces	344
10.3.4	Data for Japanese Prefectures	345
10.4	Growth Accounting	346
10.4.1	General Setup	346
10.4.2	Discrete Time and Variable Shares	347
10.4.3	Measuring Input Shares and the Growth Rates of Inputs	348
10.4.4	Results from Growth Accounting	350
10.4.5	Extensions to Include R&D	351
10.4.6	Limitations of Growth Accounting	352
11	Empirical Analysis of Regional Data Sets	382
11.1	Two Concepts of Convergence	383
11.2	Convergence across the U.S. States	387
11.2.1	β Convergence	387
11.2.2	Measurement Error	392
11.2.3	σ Convergence	392

11.3	Convergence across Japanese Prefectures	393
11.3.1	β Convergence	393
11.3.2	σ Convergence across Prefectures	397
11.4	Convergence across European Regions	398
11.4.1	β Convergence	398
11.4.2	σ Convergence	400
11.5	Migration across the U.S. States	401
11.6	Migration across Japanese Prefectures	404
11.7	Migration across European Regions	407
11.8	Migration and Convergence	410
11.9	Conclusions	413
12	Empirical Analysis of a Cross Section of Countries	414
12.1	Losers and Winners from 1965 to 1985	415
12.2	The Empirical Analysis of Growth Rates	420
12.2.1	Effects from State Variables	421
12.2.2	Control and Environmental Variables	422
12.3	Regression Results for Growth Rates	424
12.3.1	A Basic Regression	424
12.3.2	Tests of Stability of Coefficients	436
12.3.3	Additional Explanatory Variables	436
12.3.4	World Bank Data on GDP	444
12.3.5	Results from a Single Cross Section	445
12.4	Sources of Growth for Slow and Fast Growers	446
12.5	Empirical Analysis of the Investment Ratio	451
12.6	Empirical Analysis of Fertility and Health	452
12.6.1	Results for Fertility	453
12.6.2	Results on Health	454
12.7	Summary and Conclusions about Growth	455
	Appendix on Mathematical Methods	462
1.1	Differential Equations	463
1.1.1	Introduction	463
1.1.2	First-Order Ordinary Differential Equations	464
1.1.3	Systems of Linear Ordinary Differential Equations	471
1.2	Static Optimization	491
1.2.1	Unconstrained Maxima	491
1.2.2	Classical Nonlinear Programming: Equality Constraints	492
1.2.3	Inequality Constraints: The Kuhn–Tucker Conditions	494
1.3	Dynamic Optimization in Continuous Time	498
1.3.1	Introduction	498
1.3.2	The Typical Problem	499
1.3.3	Heuristic Derivation of the First-Order Conditions	500
1.3.4	Transversality Conditions	503
1.3.5	The Behavior of the Hamiltonian over Time	503
1.3.6	Sufficient Conditions	503
1.3.7	Infinite Horizons	504
1.3.8	Example: The Neoclassical Growth Model	505

1.3.9	Transversality Conditions in Infinite-Horizon Problems	507
1.3.10	Summary of the Procedure to Find the First-Order Conditions	508
1.3.11	Present-Value and Current-Value Hamiltonians	509
1.3.12	Multiple Variables	510
1.4	Useful Results in Matrix Algebra: Eigenvalues, Eigenvectors, and Diagonalization of Matrices	510
1.5	Useful Results in Calculus	512
1.5.1	<i>Implicit Function Theorem</i>	512
1.5.2	Taylor's Theorem	513
1.5.3	L'Hôpital's Rule	514
1.5.4	Integration by Parts	515
1.5.5	Fundamental Theorem of Calculus	515
1.5.6	Rules of Differentiation of Integrals	516
	References	518
	Index	529

INTRODUCTION

1.1 THE IMPORTANCE OF GROWTH

The real per capita gross domestic product (GDP) in the United States grew by a factor of 8.1 from \$2244 in 1870 to \$18,258 in 1990, all measured in 1985 dollars. The increase in real per capita GDP corresponds to a growth rate of 1.75 percent per year. This performance gave the United States the highest level of real per capita GDP in the world in 1990 (with the possible exception of the United Arab Emirates, an oil producer with a small population).¹

To appreciate the consequences of apparently small differentials in growth rates when compounded over long periods of time, we can calculate where the United States would have been in 1990 if it had grown since 1870 at 0.75 percent per year, one percentage point per year below its actual rate. A growth rate of 0.75 percent per year is close to the rate experienced in the long run—from 1900 to 1987—by India (0.64 percent per year), Pakistan (0.88 percent per year), and the Philippines (0.86 percent per year). If the United States had begun in 1870 at a real per capita GDP of \$2244 and had then grown at a rate of 0.75 percent per year over the next 120 years, then its real per capita GDP in 1990 would have been \$5519, only 2.5 times the value in 1870 and 30 percent of the actual value in 1990 of \$18,258. Then, instead of ranking first in the world in 1990, the United States would have ranked 37th out of 127 countries with data. To put it another way, if the growth rate had been lower by just 1 percentage point per year, then the U.S. real per capita GDP in 1990 would have been close to that in Mexico and Hungary and would have been about \$1000 less than that in Portugal and Greece.

¹The long-term data on GDP are in Tables 10.2 and 10.3 of Chapter 10. The cross-country information for recent years is in Table 10.1. See Chapter 10 for sources and definitions.