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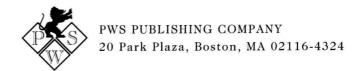
THIRD EDITION

S. T. TAN

Stonehill College



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PREFACE

Calculus for the Managerial, Life, and Social Sciences, Third Edition is a brief edition of the author's Applied Calculus, Third Edition. This book is suitable for use in a one-semester or two-quarter introductory calculus course for students in the managerial, life, and social sciences.

As with the previous editions, the objective of *Calculus for the Managerial*, *Life, and Social Sciences, Third Edition*, is twofold: (1) to write a textbook that is readable by students and (2) to make the book a useful teaching tool for instructors. We hope that with the present edition we have come one step closer to realizing our goal. The third edition of this text incorporates many suggestions by users of both earlier editions.

Features

The following list includes some of the many important features of the book:

- Coverage of Topics This book contains more than enough material for the usual applied calculus course. Optional sections have been marked with an asterisk in the table of contents, thereby allowing the instructor to be flexible in choosing the topics most suitable for his or her course.
- Approach The problem-solving approach is stressed throughout the book. Numerous examples and solved problems are used to amplify each new concept or result in order to facilitate students' comprehension of new material. Figures are used extensively to help students visualize concepts and ideas.
- Level of Presentation Our approach is intuitive and we state the results informally. However, we have taken special care to ensure that this approach does not compromise the mathematical content and accuracy. Proofs of certain results are given, but they may be omitted if desired.
- Applications The text is application oriented. Many interesting, relevant, and up-to-date applications are drawn from the fields of business, economics, social and behavioral sciences, life sciences, physical sciences, and other fields of general interest. Some of these applications have their source in newspapers, weekly periodicals, and other magazines. Applications are found in the illustrative examples in the main body of the text as well as in the exercise sets. In fact, one goal of the text is to include at least one real-life application in each section (whenever feasible).
- Exercises Each section of the text is accompanied by an extensive set of exercises containing an ample set of problems of a routine, computational nature that will help students master new techniques. The routine problems are followed by an extensive set of application-oriented problems that test students' mastery of the topics. Self-check exercises are also included at the end of each section. These exercises give the students a chance to test themselves on their understanding of the material.

Preface

■ Portfolios These interviews are designed to convey to the student the realworld experiences of professionals who have a background in mathematics and use it in their professions.

Changes in the Third Edition

- Chapters 1 and 2 have been streamlined; the precalculus review has been reorganized and condensed. Logarithms are now reviewed right before the material on logarithmic functions in Section 5.2. The sections on limits and continuity have been reorganized and condensed, allowing for an earlier introduction to the derivative.
- Section 2.6 has been rewritten and now gives a more concise introduction to the derivative along with a step-by-step procedure for finding the derivative.
- Computer/graphics calculator exercises have been added. Textual examples that make use of graphing utilities are marked with a computer/graphics calculator margin icon, and end-of-section exercises meant to be solved with the use of a computer or graphics calculator are identified by a small boxed "C" in the margin.
- More introductory real-life illustrations that illustrate the concepts being developed have been included at the beginning of sections. (See Sections 4.1 through 4.6.)
- In many sections, easier examples have been added to illustrate a new concept or the use of a new technique when it is first introduced. More explanatory side remarks have also been included to help the students. Many new problems have been added, including practice problems at the beginning of the exercise sets, to help students gain confidence and computational skill before they move on to the more difficult problems and applications.
- Optimization problems that require formulation are now covered in a separate section. New problem-solving guidelines are given with accompanying stepby-step worked examples.
- Topics on integration have been reorganized. The definite integral is now defined using Riemann sums.
- Chapter overviews with photographs relating to the applications to be covered have been added. The review exercises for each chapter have also been expanded.
- Exercises have been paired (evens and odds) where appropriate.
- Common errors and pitfalls have been highlighted by a caution symbol.
- Applications have been labeled within each exercise set.

Supplements

- A Student's Solutions Manual, available to both students and instructors, includes the solutions to odd-numbered exercises.
- An Instructor's Complete Solutions Manual includes solutions to all exercises.
- A Test Bank with Chapter Tests, free to adopters of the book, contains sample tests for each chapter.

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■ EXPTest and ExamBuilder With these computerized testing systems for IBM PCs and Macintosh, respectively, instructors can select or modify questions from prepared test banks or add their own test items to create any number of tests. These tests can be viewed on screen and printed with typeset-quality mathematical symbols, notation, graphs, and diagrams.

- A *Graphing Calculator Supplement*, written by Robert E. Seaver of Lorain County Community College, available to both students and instructors, further develops selected examples and exercises from the text and includes additional problems for reinforcement. It is specifically geared for use with the TI-line of programmable graphic calculators.
- Derive and Theorist Notebooks, packaged with the text at additional nominal cost to students, provide more than 1000 book-specific and manipulatable electronic problems to be used in conjunction with popularly used and distributed computer algebra systems.

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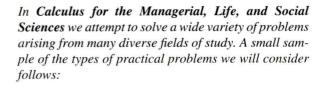
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S. T. Tan Stonehill College

APPLICATIONS





POPULATION GROWTH A study prepared for a Sunbelt town's Chamber of Commerce projected that the town's population in the next three years would grow according to the rule

$$P(x) = 50,000 + 30x^{3/2} + 20x$$

where P(x) denotes the population x months from now. How fast will the population be increasing nine months from now? Sixteen months from now?



AIR POLLUTION According to the South Coast Air Quality Management District, the level of nitrogen dioxide, a brown gas that impairs breathing, present in the atmosphere on a certain May day in downtown Los Angeles is approximated by

$$A(t) = 0.03t^3(t-7)^4 + 60.2 \quad (0 \le t \le 7)$$

where A(t) is measured in pollutant standard index and t is measured in hours, with t = 0 corresponding to 7 A.M. How fast is the level of nitrogen dioxide increasing at 11 A.M.?



OPTIMAL DRIVING SPEED A truck gets 400/x miles per gallon when driven at a constant speed of x miles per hour (between 50 and 70 miles per hour). If the price of fuel is \$1 per gallon and the driver is paid \$8 an hour, at what speed between 50 and 70 miles per hour is it most economical to drive?







LEARNING CURVES The Eastman Optical Company produces a 35-mm single-lens reflex camera. Eastman's training department determined that after completing the basic training program, a new, previously inexperienced employee would be able to assemble

$$Q(t) = 50 - 30e^{-0.5t}$$

model F cameras per day, t months after the employee began work on the assembly line. How many model F cameras can a new employee assemble per day after basic training? How many model F cameras can the average experienced employee assemble per day?

SUBWAY FARES A city's Metropolitan Transit Authority (MTA) operates a subway line for commuters from a certain suburb to the downtown metropolitan area. Currently, an average of 6000 passengers a day take the trains, paying a fare of \$1.50 per ride. The Board of the MTA, contemplating raising the fare to \$1.75 per ride in order to generate a larger revenue, engaged the services of a consulting firm. The firm's study revealed that for each 25-cent increase in fare, the ridership would be reduced by an average of 1000 passengers a day. The consulting firm recommended that MTA stick to the current fare of \$1.50 per ride, which already yields a maximum revenue. Show that the consultants' recommendations were correct.

ADVERTISING EXPENDITURES The Ross-Simons Company has a monthly advertising budget of \$60,000. Their marketing department estimates that if they spend *x* dollars on newspaper advertising and *y* dollars on television advertising, then the monthly sales will be given by

$$z = f(x, y) = 90x^{1/4}y^{3/4}$$

dollars. Determine how much money Ross-Simons should spend on newspaper ads and on television ads per month to maximize its monthly sales.

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