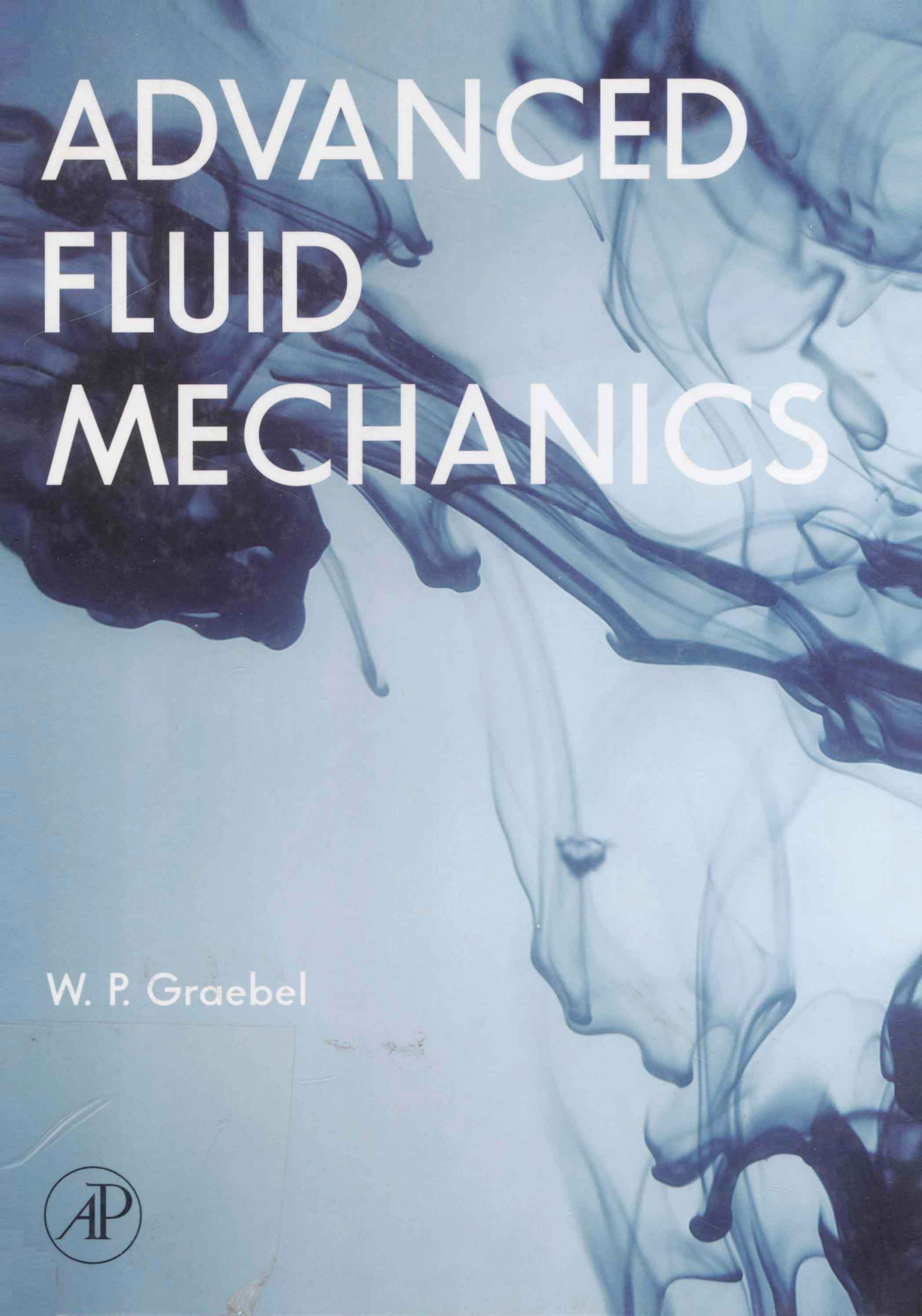

ADVANCED FLUID MECHANICS

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I maintained my edge by always being a student.
You will always have ideas, have something new to learn.
Jackie Joyner-Kersee

Education is not the filling of a pail, but the lighting of the fire.
William Butler Yeats

I have always believed that 98% of a student's progress is due to his own efforts, and 2% to his teacher.
John Philip Sousa

The one thing that matters is the effort.
Antoine de Saint-Exupery

Preface

This book covers material for second fluid dynamics courses at the senior/graduate level. Students are introduced to three-dimensional fluid mechanics and classical theory, with an introduction to modern computational methods. Problems discussed in the text are accompanied by examples and computer programs illustrating how classical theory can be applied to solve practical problems with techniques that are well within the capabilities of present-day personal computers.

Modern fluid dynamics covers a wide range of subject areas and facets—far too many to include in a single book. Therefore, this book concentrates on incompressible fluid dynamics. Because it is an introduction to basic computational fluid dynamics, it does not go into great depth on the various methods that exist today. Rather, it focuses on how theory and computation can be combined and applied to problems to demonstrate and give insight into how various describing parameters affect the behavior of the flow. Many large and expensive computer programs are used in industry today that serve as major tools in industrial design. In many cases the user does not have any information about the program developers' assumptions. This book shows students how to test various methods and ask the right questions when evaluating such programs.

The references in this book are quite extensive—for three reasons. First, the originator of the work deserves due credit. Many of the originators' names have become associated with their work, so referring to an equation as the Orr-Sommerfeld equation is common shorthand.

A more subversive reason for the number of references is to entice students to explore the history of the subject and how the world has been affected by the growth of science. Isaac Newton (1643–1747) is credited with providing the first solid footings of fluid dynamics. Newton, who applied algebra to geometry and established the fields of analytical geometry and the calculus, combined mathematical proof with physical observation. His treatise *Philosophiae Naturalis Principia Mathematica* not only firmly established the concept of the scientific method, but it led to what is called the Age of Enlightenment, which became the intellectual framework for the American and French Revolutions and led to the birth of the Industrial Revolution.

The Industrial Revolution, which started in Great Britain, produced a revolution in science (in those days called "natural philosophy" in reference to Newton's treatise) of gigantic magnitude. In just a few decades, theories of dynamics, solid mechanics, fluid dynamics, thermodynamics, electricity, magnetism, mathematics, medical science, and many other sciences were born, grew, and thrived with an intellectual verve never before found in the history of mankind. As a result, the world saw the invention of steam engines and locomotives, electric motors and light, automobiles, the telephone, manned flight, and other advances that had only existed in dreams before then. A chronologic and geographic study of the references would show how ideas jumped from country to country and how the time interval between the advances shortened dramatically in time. Truly, Newton's work was directly responsible for bringing civilization from the dark ages to the founding of democracy and the downfall of tyranny.

This book is the product of material covered in many classes over a period of five decades, mostly at The University of Michigan. I arrived there as a student at the same time as Professor Chia-Shun Yih, who over the years I was fortunate to have as a teacher, colleague, and good friend. His lively presentations lured many of us to the excitement of fluid dynamics. I can only hope that this book has a similar effect on its readers.

I give much credit for this book to my wife, June, who encouraged me greatly during this work—in fact, during all of our 50+ years of marriage! Her proofreading removed some of the most egregious errors. I take full credit for any that remain.

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Chapter 1

Fundamentals

1.1 Introduction

A few basic laws are fundamental to the subject of fluid mechanics: the law of conservation of mass, Newton's laws, and the laws of thermodynamics. These laws bear a remarkable similarity to one another in their structure. They all state that if a given volume of the fluid is investigated, quantities such as mass, momentum, and energy will change due to internal causes, net change in that quantity entering and leaving the volume, and action on the surface of the volume due to external agents. In fluid mechanics, these laws are best expressed in rate form.

Since these laws have to do with some quantities entering and leaving the volume and other quantities changing inside the volume, in applying these fundamental laws to a finite-size volume it can be expected that both terms involving surface and volume integrals would result. In some cases a global description is satisfactory for carrying out further analysis, but often a local statement of the laws in the form of differential equations is preferred to obtain more detailed information on the behavior of the quantity under investigation.

There is a way to convert certain types of surface integrals to volume integrals that is extremely useful for developing the derivations. This is the ***divergence theorem***, expressed in the form

$$\iint_S \mathbf{n} \cdot \mathbf{U} dS = \iiint_V \nabla \cdot \mathbf{U} dV. \tag{1.1.1}$$

In this expression $\mathbf{U}$[1] is an arbitrary vector and $\mathbf{n}$ is a unit normal to the surface S. The closed surface completely surrounding the volume V is S. The unit normal is positive if drawn outward from the volume.

The theorem assumes that the scalar and vector quantities are finite and continuous within V and on S. It is sometimes useful to add appropriate singularities in these functions, generating additional terms that can be simply evaluated. This will be discussed in later chapters in connection with inviscid flows.

In studying fluid mechanics three of the preceding laws lead to differential equations—namely, the law of conservation of mass, Newton's law of momentum, and the first law of thermodynamics. They can be expressed in the following descriptive form:

$$\text{Rate at which} \begin{bmatrix} \text{mass} \\ \text{momentum} \\ \text{energy} \end{bmatrix} \text{accumulates within the volume}$$

$$+ \text{ rate at which} \begin{bmatrix} \text{mass} \\ \text{momentum} \\ \text{energy} \end{bmatrix} \text{enters the volume}$$

$$- \text{ rate at which} \begin{bmatrix} \text{mass} \\ \text{momentum} \\ \text{energy} \end{bmatrix} \text{leaves the volume}$$

$$= \begin{bmatrix} 0 \\ \text{net force acting on volume} \\ \text{rate of heat addition + rate of work done on volume} \end{bmatrix}.$$

These can be looked at as a balance sheet for mass, momentum, and energy, accounting for rate changes on the left side of the equation by describing on the right side the external events that cause the changes. As simple as these laws may appear to us today, it took many centuries to arrive at these fundamental results.

Some of the quantities we will see in the following pages, like pressure and temperature, have magnitude but zero directional property. Others have various degrees of directional properties. Quantities like velocity have magnitude and one direction associated with them, whereas others, like stress, have magnitude and two directions associated with them: the direction of the force and the direction of the area on which it acts.

The general term used to classify these quantities is ***tensor***. The ***order*** of a tensor refers to the number of directions associated with them. Thus, pressure and temperature are tensors of order zero (also referred to as scalars), velocity is a tensor of order one (also referred to as a vector), and stress is a tensor of order two. Tensors of order higher

[1] Vectors are denoted by boldface.

than two usually are derivatives or products of lower-order tensors. A famous one is the fourth-order Einstein curvature tensor of relativity theory.

To qualify as a tensor, a quantity must have more than just magnitude and directionality. When the components of the tensor are compared in two coordinate systems that have origins at the same point, the components must relate to one another in a specific manner. In the case of a tensor of order zero, the transformation law is simply that the magnitudes are the same in both coordinate systems. Components of tensors of order one must transform according to the parallelogram law, which is another way of stating that the components in one coordinate system are the sum of products of direction cosines of the angles between the two sets of axes and the components in the second system. For components of second-order tensors, the transformation law involves the sum of products of two of the direction cosines between the axes and the components in the second system. (You may already be familiar with Mohr's circle, which is a graphical representation of this law in two dimensions.) In general, the transformation law for a tensor of order N then will involve the sum of N products of direction cosines and the components in the second system. More detail on this is given in the Appendix.

One example of a quantity that has both directionality and magnitude but is not a tensor is finite angle rotations. A branch of mathematics called ***quaternions*** was invented by the Irish mathematician Sir William Rowan Hamilton in 1843 to deal with these and other problems in spherical trigonometry and body rotations. Information about quaternions can be found on the Internet.

In dealing with the general equations of fluid mechanics, the equations are easiest to understand when written in their most compact form—that is, in vector form. This makes it easy to see the grouping of terms, the physical interpretation of them, and subsequent manipulation of the equations to obtain other interpretations. This general form, however, is usually not the form best suited to solving particular problems. For such applications the component form is better. This, of course, involves the selection of an appropriate coordinate system, which is dictated by the geometry of the problem.

When dealing with flows that involve flat surfaces, the proper choice of a coordinate system is ***Cartesian coordinates***. Boundary conditions are most easily satisfied, manipulations are easiest, and equations generally have the fewest number of terms when expressed in these coordinates. Trigonometric, exponential, and logarithmic functions are often encountered. The conventions used to represent the components of a vector, for example, are typically (v_x, v_y, v_z), (v_1, v_2, v_3), and (u, v, w). The first of these conventions use (x, y, z) to refer to the coordinate system, while the second convention uses (x_1, x_2, x_3). This is referred to either as ***index notation*** or as ***indicial notation***, and it is used extensively in tensor analysis, matrix theory, and computer programming. It frequently is more compact than the x, y, z notation.

For geometries that involve either circular cylinders, ellipses, spheres, or ellipsoids, cylindrical polar, spherical polar, or ellipsoidal coordinates are the appropriate choice, since they make satisfaction of boundary conditions easiest. The mathematical functions and the length and complexity of equations become more complicated than in Cartesian coordinates.

Beyond these systems, general tensor analysis must be used to obtain governing equations, particularly if nonorthogonal coordinates are used. While it is easy to write the general equations in tensor form, breaking down these equations into component form in a specific non-Cartesian coordinate frame frequently involves a fair amount of work. This is discussed in more detail in the Appendix.

1.2 Velocity, Acceleration, and the Material Derivative

A fluid is defined as a material that will undergo sustained motion when shearing forces are applied, the motion continuing as long as the shearing forces are maintained. The general study of fluid mechanics considers a fluid to be a ***continuum***. That is, the fact that the fluid is made up of molecules is ignored but rather the fluid is taken to be a continuous media.

In solid and rigid body mechanics, it is convenient to start the geometric discussion of motion and deformation by considering the continuum to be made up of a collection of particles and consider their subsequent displacement. This is called a ***Lagrangian***, or ***material, description***, named after Joseph Louis Lagrange (1736–1836). To illustrate its usage, let $(X(X_0, Y_0, Z_0, t), Y(X_0, Y_0, Z_0, t), Z(X_0, Y_0, Z_0, t))$ be the position at time t of a particle initially at the point (X_0, Y_0, Z_0). Then the velocity and acceleration of that particle is given by

$$v_x(X_0, Y_0, Z_0, t) = \frac{\partial X(X_0, Y_0, Z_0, t)}{\partial t}$$
$$v_y(X_0, Y_0, Z_0, t) = \frac{\partial X(X_0, Y_0, Z_0, t)}{\partial t}$$
$$v_z(X_0, Y_0, Z_0, t) = \frac{\partial X(X_0, Y_0, Z_0, t)}{\partial t}$$

and (1.2.1)

$$a_x(X_0, Y_0, Z_0, t) = \frac{\partial v_x(X_0, Y_0, Z_0, t)}{\partial t} = \frac{\partial^2 X(X_0, Y_0, Z_0, t)}{\partial t^2},$$
$$a_y(X_0, Y_0, Z_0, t) = \frac{\partial v_y(X_0, Y_0, Z_0, t)}{\partial t} = \frac{\partial^2 Y(X_0, Y_0, Z_0, t)}{\partial t^2},$$
$$a_z(X_0, Y_0, Z_0, t) = \frac{\partial v_z(X_0, Y_0, Z_0, t)}{\partial t} = \frac{\partial^2 Z(X_0, Y_0, Z_0, t)}{\partial t^2}.$$

The partial derivatives signify that differentiation is performed holding X_0, Y_0 and Z_0 fixed.

This description works well for particle dynamics, but since fluids consist of an infinite number of flowing particles in the continuum hypothesis, it is not convenient to label the various fluid particles and then follow each particle as it moves. Experimental techniques certainly would be hard pressed to perform measurements that are suited to such a description. Also, since displacement itself does not enter into stress-geometric relations for fluids, there is seldom a need to consider using this descriptive method.

Instead, an ***Eularian***, or ***spatial, description***, named after Leonard Euler (1707–1783), is used. This description starts with velocity, written as $\mathbf{v} = \mathbf{v}(\mathbf{x}, t)$, where $\mathbf{x}$ refers to the position of a fixed point in space, as the basic descriptor rather than displacement. To find acceleration, recognize that *acceleration* means the rate of change of the velocity of a particular fluid particle at a position while noting that the particle is in the process of moving from that position at the time it is being studied. Thus, for instance, the acceleration component in the x direction is defined as

$$a_x = \lim_{\Delta t \to 0} \frac{v_x(x + v_x\Delta t, y + v_y\Delta t, z + v_z\Delta t, t + \Delta t) - v_x(x, y, z, t)}{\Delta t}$$
$$= \frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z},$$