Second Edition

BASIC TECHNICAL MATHEMATICS WITH CALCULUS

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Basic Technical Mathematics with Calculus

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HarperCollins*CollegePublishers*

This book is dedicated to my wife, Joyce, without whose patience, love, and understanding this book would not have been possible.

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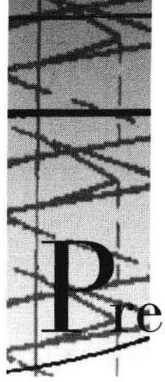


Preface

This second edition of *Basic Technical Mathematics with Calculus* is designed to provide students with the necessary mathematical skills to pursue a course of study in scientific or engineering technology or to prepare students for calculus and provide an introduction to differential and integral calculus. To use this book effectively, students should have completed two years of high-school mathematics (algebra and geometry). For students requiring a quick review, Chapters 1 through 3 offer a concise summary of essential topics in algebra and geometry. The text can be used in a one-, two- or three-semester program.

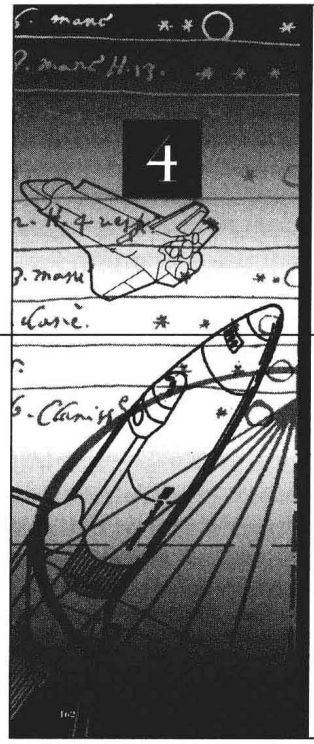
A Note on Graphics Calculators

This book has been written to accommodate the full range of attitudes toward graphics calculators. Instructors who rely on the graphics calculator as an essential teaching tool will appreciate the special boxed keystrokes, examples, and exercises. This material is designed both to reinforce mathematical concepts and to familiarize students with the most popular models of graphics calculators.

Instructors who do not use the graphics calculators can easily skip the graphics calculator material, since it is all clearly marked. In some cases, concepts are covered more than once to make the book effective for both graphics calculator users and nonusers. For example, the "Introduction to the Graphics Calculator" in Chapter 1 introduces basic calculator techniques and concepts. It also includes a discussion of the coordinate plane, which is repeated and expanded upon in Chapter 5 for the benefit of all students.

Features

Chapter Openers
Chapter openers present a real-world problem drawn from principles taught in *Basic Technical Mathematics with Calculus.* Students solve these problems later on in the chapter.



4

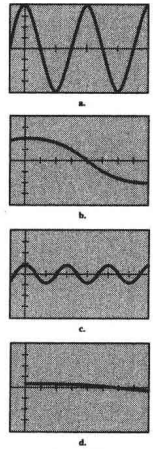
Linear Equations and Dimensional Analysis

The Saturn rocket often moves objects from the surface of earth to outer space. Though you may have seen many fiery blast-offs, have you thought about the forces needed to lift these metal giants into orbit?

The weight of the Saturn 5 before takeoff is 6,391,120 pounds! The thrust of the engine must at least exceed the weight of the rocket, or it will never take off. The first stage thrust of the Saturn 5 rocket is about 8 million pounds, more than enough to insure a lift-off.

The resultant acceleration of the rocket is computed by determining the difference between the downward pull of its weight and the upward push of the thrust. The equation is $F_{net} = ma$ (m stands for the mass of the object, which in this case is 198,000 slugs). What is the acceleration of the rocket lift-off? The solution is given in Section 4.4, Example 6.

Graphics Calculator Boxes, Examples, and Exercises
Throughout the text, boxes provide calculator solutions to section examples. These include keystrokes for the most popular models—the T1-82 and the Casio fx-7700G. Graphics calculator examples and exercises are indicated by an identifying symbol so they can be skipped if desired.



a.

b.

c.

d.

FIGURE 8.26

EXAMPLE 7

Use a graphics calculator to graph the functions in the interval $0° \le \theta \le 360°$.

a. $y = 4 \cos 2\theta$ **b.** $y = 2 \cos \frac{\theta}{2}$

c. $y = \frac{1}{2} \cos 3\theta$ **d.** $y = \frac{1}{4} \cos \frac{\theta}{3}$

For each of the functions determine the amplitude, the period, and the number of cycles in $0° \le \theta \le 360°$.

Solution The graphs of the functions are given in Figure 8.26a, b, c, and d, respectively.

a. The amplitude is 4. The period $= \frac{360°}{2}$ or 180°. There are two complete cycles of the curve in the interval $0° \le \theta \le 360°$, as shown in Figure 8.26a.

b. The amplitude is 2. The period $= \frac{360°}{\frac{1}{2}}$ or 720°. There is one-half of a complete cycle of the curve in 360°, as shown in Figure 8.26b.

c. The amplitude is $\frac{1}{2}$. The period $= \frac{360°}{3}$ or 120°. There are three complete cycles of the curve in 360°, see Figure 8.26c.

d. The amplitude is $\frac{1}{4}$. The period $= \frac{360°}{\frac{1}{3}}$ or 1080°. There is one-third of a cycle in 360°, see Figure 8.26d. ◆

8.3 Exercises

In Exercises 1–12, determine the following. **a.** The amplitude **b.** The period of each function **c.** The number of cycles in $0 \le \theta \le 2\pi$.

1. $y = 3 \sin \theta$
2. $y = \frac{1}{2} \cos \theta$
3. $y = 0.75 \cos \theta$
4. $y = 1.3 \sin \theta$
5. $y = \sin 3\theta$
6. $y = \cos \frac{\theta}{2}$
7. $y = -6 \cos \frac{\theta}{4}$
8. $y = 2 \sin \pi\theta$
9. $y = \frac{1}{3} \sin \frac{\theta}{3}$
10. $y = -\pi \cos \frac{\pi}{2}\theta$
11. $y = -\frac{1}{3} \cos \frac{1}{3}\theta$
12. $y = \frac{5}{3} \sin \frac{5}{3}\theta$

Examples
Approximately 1300 examples include detailed, well-explained solutions. Approximately 220 of these examples illustrate technical applications. Where appropriate, examples include a brief descriptive title to help students understand the purpose of the example and to aid in studying for examinations.

Cautionary Remarks
Common student errors and difficulties are highlighted and identified with the heading "Caution."

EXAMPLE 1 Determining the Period

For the function in Figure 8.2, determine the period p.

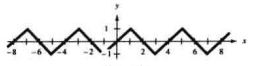


FIGURE 8.2

Solution The function goes through one complete cycle for each of the intervals from -8 to -4, -4 to 0, 0 to 4, and 4 to 8. Thus, the period is four units. Using the definition, we see that:

$$f(\boxed{-4}) = f(-4 + 4) = f(0) = 0$$
$$f(\boxed{1}) = f(1 + 4) = f(5) = 1.$$
$$f(\boxed{-1}) = f(-1 + 4) = f(3) = -1 \quad \blacklozenge$$

CAUTION We may be tempted to let $p = 2$ in Example 1. To see that this is incorrect, let $x = 1$ and substitute for x and p in the equality $f(x + p) = f(x)$. Thus, $f(1 + 2) \neq f(1)$ since $f(1 + 2) = -1$ and $f(1) = 1$. Clearly the function has only traveled through one-half of its full cycle. To determine the period p of the function, use the statement $f(x + p) = f(x)$ for all x. If we select $x = -4$, then $f(-4 + 2) = f(-2) = 0$ and $f(-4) = 0$, which gives the wrong impression—that 2 is the period. Therefore, always try more than one value to check. ■

As shown in Figure 8.2, the graph goes through one complete cycle from 0 to 4, completing four cycles from -8 to 8.

DEFINITION 8.2
A cycle of a periodic function is a portion of the graph of the function from any point on the graph to the first point at which the graph starts repeating itself. (See Figure 8.3.)

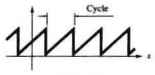


FIGURE 8.3

Key Terms
Essential terms are highlighted in boldface type within the explanations and examples.

Definitions, Formulas, Rules, and Procedures
These items are outlined in boxes to stress the importance of the material.

10.1

Introduction to Linear Equations

In Chapter 4, we learned how to solve linear equations with one variable, and in Chapter 5, we graphed linear equations in two variables. In technical fields, people commonly work with linear equations with two variables. The general form of the linear equation in two variables is $ax + by + c = 0$. A specific example is:

$$3x + 4y - 12 = 0, \quad \text{where } a = 3, b = 4 \quad \text{and} \quad c = -12.$$

We can tell that the equation is linear since the exponents of both the variables x and y are 1. For example, $3x^2 - 5y + 2 = 0$ is not a linear equation because the exponent of the variable x is 2. Another way of recognizing a linear equation is by its graph, which is always a straight line (as illustrated in Chapter 5).

A linear equation may take the form $y = mx + b$, where m and b are constants and x and y are the independent and dependent variables, respectively. This form of the equation of a line is called the **slope-intercept form** because m is the slope of the line, and b is the ordinate of the y-intercept. The **slope** of the line is a ratio of two numbers that indicates the slant of the line with respect to the positive x-axis. The slope is also defined as the angle the line forms with respect to the positive x-axis. The **y-intercept** is the point where the line crosses the y-axis.

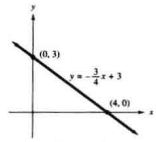


FIGURE 10.1

DEFINITION 10.1
The slope-intercept form of an equation of a line is:
$$y = mx + b.$$

The equation $3x + 4y - 12 = 0$ can be written in the form $y = mx + b$.

$4y = -3x + 12$ **Subtracting 3x and adding 12 to both sides of the equation**

$y = \frac{-3}{4}x + 3$ **Dividing both sides of the equation by 4.**

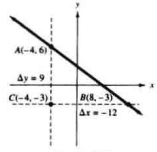


FIGURE 10.2

By rewriting the equation in this form, we see that $m = \frac{-3}{4}$ and $b = 3$. The graph is sketched in Figure 10.1. The line crosses the y-axis at the point (0, 3), the y-intercept.

Now we will look at the slope. As shown in Figure 10.2, draw a line parallel to the y-axis through the point $A(-4, 6)$ and another line parallel to the x-axis through the point $B(8, -3)$. These lines intersect at the point $C(-4, -3)$. The resulting figure is right triangle ABC, with a right angle at $C(-4, -3)$. The length of the line segment $\overline{BC}$ is:

$$\overline{BC} = -4 - 8 = \boxed{-12}.$$

49. From a land-based radar station it is determined that the bearing of ship A is 70° and it is a distance of 3.5 km from the station, while ship B has a bearing of 110° and is a distance of 5.2 km from the station. Determine the distance between the two ships.

50.* One side of a triangle is 3.472 times as long as another side x. The angle between them is 32° 01′.
- **a.** Determine the other two angles.
- **b.** Determine the third side in terms of the side x.

51.* A local carnival has a ferris wheel. There are 18 seats equally spaced on the wheel. If the radius of the wheel is 18 ft, determine the distance between any two adjacent seats on the wheel. (See the illustration.)

Writing About Mathematics

1. Two landscape designers were asked to design a triangular-shaped park. The city council does not understand why each design is shaped differently. Each designer claims that, with the information given, the shape of the triangle is correct. The only information given the designers was "two sides and an angle opposite one of the sides." Write a letter explaining why two solutions are possible.

2. You and a friend, having chopped down a tree, now need to move its trunk 25 ft to the edge of the road so that it can be trucked to the sawmill. You have attached two ropes to the log, and each of you is pulling on a rope. Using the sum of two vectors, explain how anyone can determine the force exerted by pulling on the two ropes.

Chapter Test

In Exercises 1 and 2, sketch the vectors on a coordinate system.

1. $\mathbf{A} = 4\angle 30°$ **2.** $\mathbf{B} = 6\angle 150°$

In Exercises 3 and 4, use the fact that $\mathbf{A} = 3.5\angle 137°$ to determine the following.

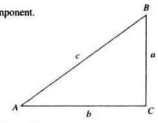
3. The horizontal component **4.** The vertical component.

In Exercises 5 and 6, use the fact that $\mathbf{A} = 5.0\angle 45°$ and $\mathbf{B} = 7.0\angle 215°$ to determine the following.

5. $\mathbf{A} + \mathbf{B}$ **6.** $\mathbf{B} - \mathbf{A}$

7. Determine the resultant of $5.0\angle -45° - 7.0\angle 115° + 3.0\angle 210°$.

In Exercises 8–13, determine the measure of the three other parts of the triangle, if possible. See the diagram for proper labeling.



8. $A = 42°, B = 60°, b = 15$ **9.** $B = 35°, b = 12, a = 16$

10. $A = 75°, b = 10, c = 20$ **11.** $B = 35°, b = 16, a = 12$

12. $C = 160°, a = 8, b = 12$ **13.** $a = 12, b = 25, c = 24$

***35.** Write a computer program to determine the mean of a set of data.

***36.** Write a computer program to determine the standard deviation of a set of data.

***37.** The following formulas can both be used to calculate the standard deviation of ungrouped data. Show that these equations are equal.

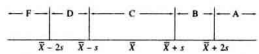
$$s = \sqrt{\frac{\Sigma (x_i - \bar{X}_i)^2}{n-1}} \qquad s = \sqrt{\frac{n(\Sigma\, x_i^2) - (\Sigma\, x_i)^2}{n(n-1)}}$$

Writing About Mathematics

1. Your friend does not understand how your school determines grade point averages. You recognize that the grade point average is the same as a weighted mean. Write a few paragraphs, including examples, to explain how to calculate a grade point average so that your friend does understand.

2. In labor negotiations the union states that the average pay of company employees is $35,000 per year. On the other hand, management claims that the average pay of employees is $42,000 per year. Both claims are correct because one group is using the median and the other is using the mean. Discuss why the difference exists and why each used a different average to make their point.

3. A psychology instructor assumes that her test grades are normally distributed. That is, she assigns letter grades by the method shown in the diagram. On a particular exam a student scores 116 out of a possible 120. The letter grade the teacher assigns is a C. Explain how that grade is possible using her technique.



4. Using the grading technique shown in Exercise 3, discuss the following questions. Is it possible that no student will receive an A? Is is possible that no student will fail?

Chapter Test

In Exercises 1–5, use the data set 2, 2, 3, 4, 4, 4, 5, 6, 8, 8, 9.

1. Determine the mean. **2.** Determine the median.

3. Determine the mode. **4.** Determine the range.

5. Determine the standard deviation.

Writing about Mathematics

The writing exercises at the end of each chapter encourage a deeper understanding of concepts. They also can be used as topics for group discussion. Answers are not given for these exercises because they are open-ended and instructors may use them in different ways.

Content Highlights

- Review material is collected in Chapters 1 through 3. Sections 1.5, 1.6, and 3.1 are important in understanding other parts of the text. The graphics calculator is introduced in Section 1.5, and the discussion provides basic rules for working with the calculator. The rules for expressing answers for problems containing approximate numbers are discussed in Section 1.6. These rules are used throughout the text. Section 3.1 is a review of plane figures and emphasizes angles, providing helpful background material for trigonometry in Chapter 6.
- In Chapter 4, Sections 4.1 and 4.2 provide the rules for solving simple linear equations. These rules are then applied in Sections 4.3 and 4.4 in working with dimensional units. Ratio, proportion, and variation are discussed early in the text because they are important tools in technical courses.
- Chapter 5 gives a brief introduction to the concept of function, providing the student with the basic terminology that can be used to build on the concept of function as we move throughout the text. Strong emphasis is given to graphing and to the use of graphs in understanding how functions behave.
- Chapters 6, 7, and 8 provide an early introduction to trigonometry as is required by some technical areas. In Chapter 6 the trigonometric functions are developed using only degree measurement; radian measure is then introduced. In Chapter 8 conventional graphing techniques for the trigonometric functions are discussed as well as graphics calculator techniques.
- Chapter 9 gives an introduction to imaginary and complex numbers. The applications in this chapter are directed primarily toward the field of electronics. Sections 9.1 and 9.2 are beneficial to the student in understanding imaginary roots of nonlinear equations.
- Chapters 10, 11, and 12 deal with the techniques of solving linear equations, quadratic equations, and equations of higher degree. The calculator is introduced as a tool in solving systems of equations using determinants and matrices. Additional graphs are used to help provide a better understanding of the techniques for determining roots of equations.
- In Chapter 13 logarithmic and exponential functions and the graphing of these functions is discussed. Techniques of solving equations, and logarithms to other bases, in addition to base 10 and base e are stressed in this chapter.
- In Chapter 14 the techniques of solving linear, quadratic, and higher-degree inequalities are discussed. The sign method is used to solve inequalities of higher degree.
- In Chapter 15 trigonometric identities, such as sum and difference of two angles, double angles, and half-angles, are developed. Graphing is introduced as a method of checking the equality of trigonometric statements. Methods of solving trigonometric equations and inverse trigonometric functions also are discussed.
- Chapter 16 provides the student with an introduction to arithmetic and geometric series. An important topic in this chapter is the binomial theorem. It is perhaps the only time the student will see how we calculate all the terms or how a specific term of the binomial expression is determined.

- Chapter 17 on statistics can be taught any time after Chapter 4.
- Chapter 18, analytic geometry, is a study of the conic sections starting with the center (vertex) at the origin. The concept of translation has been developed for those who want to go beyond the basics.
- Chapters 19 through 24 provide a development of differentiation and integration of algebraic and transcendental functions. The calculus is presented with many examples and graphs to help provide an intuitive as well as a formal understanding of limit, continuity, differentiation, and integration.
- Chapters 25 and 26 provide an introduction to first-order and high-order linear differential equations. Different methods of solving differential equations are discussed, including Laplace transforms.
- Appendix A contains a brief discussion of the International System of Units (SI) and the United States Customary System (USCS) of measurements.
- In Appendix B the techniques of using a scientific calculator in problem solving are discussed. This material is discussed in the appendix so it will not be confused with the material presented on the graphics calculator.
- Appendix C is a table of the areas of a standard normal distribution.
- Appendix D contains a short table of integrals.

Answers to Odd-numbered Exercises

Answers to half the exercises in the book allow students to check their work quickly.

Exercises

Section Exercises More than 7000 exercises are provided, covering the full range of concepts. Approximately 2200 of these exercises are technical applications.

Chapter Review Exercises These end-of-chapter exercises tie together chapter material, encouraging students to review concepts covered in the section exercises. More than 1900 are included.

Chapter Tests The comprehensive chapter tests allow students to practice for in-class examinations.

Supplements

This edition is accompanied by an extensive supplemental package that includes answers, solutions, and testing materials for both students and instructors.

For the Instructor

Instructor's Manual This manual includes answers to all even-numbered exercises, solutions to selected exercises, and two test forms per chapter. In addition, it includes "Tips for Using Graphics Calculators in Class" and copies of log and semi-log paper that can be reproduced for homework assignments.

HarperCollins Test Generator/Editor for Mathematics with QuizMaster is available in IBM and Macintosh versions and is fully networkable. The Test Generator enables instructors to select questions by objective, section, or chapter, or to use a ready-made test for each chapter. The Editor enables instructors to edit any preexisting data or to easily create their own questions. The software is algorithm driven, allowing the instructor to regenerate constants while maintaining problem type, providing a nearly unlimited number of available test or quiz items. Instructors may generate tests in multiple-choice or open-response formats, scramble the order of questions while printing, and produce up to 25 versions of each test. The system features printed graphics and accurate mathematical symbols. It also features a preview option that allows instructors to view questions before printing and to replace or skip questions if desired. QuizMaster enables instructors to create tests and quizzes using the Test Generator/Editor and save them to disk so that students can take the test or quiz on a stand-alone computer or network. QuizMaster then grades the test or quiz and allows the instructor to create reports on individual students or classes.

For the Student

Student's Solution Manual Prepared by Bill Ferguson and Ken Seidel of Columbus State Community College, this manual contains solutions to every other odd-numbered section and review exercise. (ISBN 0-673-46376-1)

Interactive Tutorial Software with Management System This innovative package is available in IBM or Macintosh versions and is fully networkable. As with the Test Generator/Editor, this software is algorithm driven, automatically regenerating constants so students will not see the numbers repeat in a problem type if they return to any particular section. The tutorial is self-paced and provides unlimited opportunities to review lessons and practice problem solving. When students give a wrong answer, they can request to see a problem worked out. The program is menu driven for ease of use, and on-screen help can be obtained at any time with a single keystroke. Students' scores are automatically recorded and can be printed out for a permanent record. The optional Management System lets instructors record student scores on disk and print diagnostic reports for individual students or classes. In addition, there is a specific set of computer programs, with examples, that students can use when working text exercises.

GraphExplorer With this sophisticated software, available in IBM and Macintosh versions, students can graph rectangular, conic, polar, and parametric equations; zoom; transform functions; and experiment with families of equations quickly and easily.

College Algebra with Trigonometry: Graphing Calculator Investigations This supplemental text, written by Dennis Ebersole of Northampton County Area Community College, provides investigations that help students visualize and explore key concepts, generalize and apply concepts, and identify patterns. (ISBN 0-06-500888-X)

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Stuart R. Porter

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