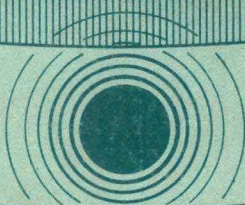


SYMPOSIUM on FLOW-INDUCED VIBRATIONS

VOLUME 2 VIBRATION OF ARRAYS OF CYLINDERS IN CROSS FLOW



Editors
M.P. PAIDOUSSIS
M.K. AU-YANG
S.S. CHEN



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VOLUME 2 VIBRATION OF ARRAYS OF CYLINDERS IN CROSS FLOW

Presented at

THE ASME WINTER ANNUAL MEETING
NEW ORLEANS, LOUISIANA
DECEMBER 9-14, 1984

Symposium co-sponsored by

Applied Mechanics, Fluids Engineering, Heat Transfer,
Noise Control and Acoustics, Nuclear Engineering,
and Pressure Vessels and Piping Divisions

Sessions in this Volume co-sponsored by

PRESSURE VESSELS AND PIPING AND
NUCLEAR ENGINEERING DIVISIONS

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Library of Congress Catalog Card Number 84-72467

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PREFACE

The 1984 ASME Symposium on Flow-Induced Vibration is a unique event in the annals of technical meetings organized by ASME. Apart from promising to be one of the most important symposia anywhere on this topic in recent memory (only time will tell exactly how important), it is the first time that such a large symposium on the subject has been organized by ASME. Furthermore, it is the first time that no less than six Divisions of the ASME have cooperated in co-sponsoring a symposium on any given subject, which surely bespeaks of the importance of the subject matter of this particular Symposium. The participating Divisions are:

Applied Mechanics, Fluids Engineering, Heat Transfer, Noise Control and Acoustics, Nuclear Engineering, and Pressure Vessels and Piping.

I should like to thank them all, for without their support this Symposium would not have been the success that it is promising to be.

The Proceedings of the Symposium are published in six bound volumes, containing sixty eight papers in all, as follows:

Volume 1 Excitation and Vibration of Bluff Bodies in Cross Flow

Volume 2 Vibration of Arrays of Cylinders in Cross Flow

Volume 3 Vibration in Heat Exchangers

Volume 4 Vibration Induced by Axial and Annular Flows

Volume 5 Turbulence-Induced Noise and Vibration of Rigid and Compliant Surfaces

Volume 6 Computational Aspects of Flow-Induced Vibration

The organization of a Symposium of this size, with world-wide participation (from 12 countries), has been both a challenging and rewarding experience. It entailed a great deal of work by many people: the session developers, the reviewers, ASME Headquarters' staff, the 1984 WAM Organizers and, of course, the authors. Of the many people involved, too numerous to mention by name here, I am specially indebted to the session developers and co-editors (O. M. Griffin, M. Sevik, M. K. Au-Yang, S. -S. Chen, J. M. Chenoweth, M. D. Bernstein and A. J. Kalinowski), and would like to single out two: Dr. M. K. Au-Yang and Dr. S. -S. Chen, whom I would like to thank for their unswerving support from the very beginning, when the possibility of a "multidivisional symposium" looked like a pie in the sky! I would also like to thank my secretary, Ruth Gray, for efficiently handling the enormous amount of paperwork involved in several passes of sixty-eight-plus papers across my desk.

Michael P. Paidoussis
Principal Symposium Coordinator
and Principal Editor

FOREWORD

By comparison to the study of the flow around a solitary cylinder in cross flow and of the associated flow-induced forces and vibration, the systematic and scientific study of the same in the case of clusters and arrays of cylinders (involving from two to thousands of cylinders) is relatively new. As recently as the early 1970's, understanding of the mechanisms involved in flow-induced vibration of arrays of cylinders was rudimentary, and design methods were mainly empirical. Indeed, one of the most important phenomena, the most troublesome from the practical point of view, namely the so-called fluidelastic instability in large cylinder arrays, was not "discovered" until then — theretofore being confused with vibration due to "vortex shedding". Roberts' work in 1962 was largely ignored, and it was Connors' work in 1970 that put fluidelastic instability "on the map", so to speak.

Most of the studies in this volume are concerned with large arrays of cylinders, such as those commonly found in heat exchangers. However, of equal importance are, flow-induced vibration and instabilities of a smaller number of more widely spaced cylinders, e.g. in overhead electrical-conductor bundles and in underwater applications; it is gratifying that papers of this type may also be found in this volume.

It is now widely accepted that vibration in large arrays of cylinders may arise by the following mechanisms: (i) turbulent buffeting, (ii) resonance with periodicities in the interstitial flow, and (iii) fluidelastic instabilities. Interstitial-flow periodicity may also give rise to acoustical resonances in the case of confined gaseous flows, which may not cause vibration of the cylinders, but may nevertheless cause problems. All these aspects of flow, flow-induced vibration and noise are dealt with by the various papers in this volume, Volume 2 of the Symposium Proceedings, as well as Volume 3 which deals with the more practical aspects of problems arising specifically in heat exchangers. Most of the papers in this volume deal with the more fundamental aspects of this topic.

Casual reading of the table of contents will show that there is a heavy concentration of papers dealing with fluidelastic instability in large arrays of cylinders. This is not fortuitous: in the last fifteen years or so, the power generating and process industries have suffered cumulative damages and lost production of the order of \$100 million to \$1 billion around the world due to vibration-related problems in heat-exchange equipment; the most serious of these problems is fluidelastic instability. Despite the great efforts made in the last decade, it is only now that sufficient understanding is being gained, to permit sophisticated modelling of this phenomenon. Some of the papers in this volume make definitive advances in this area.

We would like to thank the authors for their cooperation in submitting papers of high quality to this Symposium and specifically on the topic of this volume of the proceedings, as well as for their willingness to participate and share their experience with others in the field. It is gratifying that most of the big "field" guns from around the world are contributors to this volume. We would also like to thank the reviewers for their thoughtful comments and for the experience they have brought to bear in the review process, which has ensured the selection of only worthy papers for the Symposium and contributed to the improvement of those finally accepted.

M. P. Paidoussis

M. K. Au-Yang

S.-S. Chen

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CLASSIFICATION OF FLOW-INDUCED OSCILLATIONS OF TWO PARALLEL CIRCULAR CYLINDERS IN VARIOUS ARRANGEMENTS

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ABSTRACT

A wide variety of flow-induced oscillations of two parallel circular cylinders have occurred in engineering applications. The flow interference between two cylinders, which depends on their spacing and orientation relative to the free stream, was responsible for a series of new kinds of excitation.

Three categories of interference were proposed based on the available experimental data: proximity, proximity-and-wake and wake interference. Each category of interference was discussed separately with relevant excitations in terms of the reduced velocity, mass ratio, structural damping and number of degrees of freedom of two cylinders.

The following excitations were described: forced symmetric vortex shedding, synchronized alternate vortex shedding, jet switch, gap flow switch, wake displacement and wake galloping. The excitations induced by the displacement of cylinders at high reduced velocities were linked to the interference effects. Finally mathematical modelling of wake interference and means for suppressing some excitations were reviewed briefly.

NOMENCLATURE

A	= amplitude of oscillation	Tu	= free stream turbulence
A/D	= relative amplitude of oscillation	V	= free stream velocity
AR	= H/D aspect ratio of cylinders	V _r	= resultant velocity
C	= mechanical damping term	V _w	= local velocity in wake
C _D	= drag coefficient	W	= V/nD reduced velocity
C _L	= lift coefficient	x	= streamwise coordinate
D	= diameter of cylinders	\dot{x}	= streamwise velocity of cylinder
DF	= degree of freedom	y	= transverse coordinate
f	= frequency of shedding	\dot{y}	= transverse velocity of cylinder
F	= fluid force	α	= induced angle of incidence
H	= height of cylinder	δ	= logarithmic decrement of damping
k	= V _w /V ratio of velocity scaling	γ	= kinematic viscosity of fluid
		ρ	= density of fluid
		ω	= circular natural frequency

K	= stiffness term	<u>Subscripts</u>	
L	= streamwise spacing of cylinders' axes	i	= 1,2 refers to two cylinders
L/D	= streamwise spacing	x	= refers to streamwise direction
M	= mass of cylinders	y	= refers to transverse direction
n	= natural frequency of cylinders		
Re	= VD/ν Reynolds number		
Sc	= $2M\delta/\rho D^2H$ Scruton number		
St	= fD/V Strouhal number		
SR	= surface roughness		
T	= transverse spacing of cylinders' axes		
T/D	= transverse spacing		

INTRODUCTION

The complexity and diversity of flow-induced oscillations of a single circular cylinder have stimulated an enormous research effort, as summarised in the recent reviews (1-5). There are still substantial gaps in the fundamental understanding of the mechanism of interaction between an oscillating cylinder and the surrounding flow. The latter is dominated by a near-wake instability which leads to a more or less periodic formation and shedding of vortices behind the base of the cylinder. Vortex shedding is not the only cause of excitation and a large number of displacement-excited mechanisms have been discovered, particularly for two interfering cylinders.

The susceptibility of an elastic or flexibly mounted rigid cylinder to flow-induced oscillation depends on the following dimensionless parameters:

$$\frac{A(x,y)}{D} = F(W, Sc, Df, Re, Ar, SR, Tu) \quad (1)$$

where $\frac{A}{D}$ is the relative amplitude with components, usually, in both streamwise and transverse directions,

$W = V/n$ D is the reduced velocity based on the natural frequency n of the elastic cylinder,

$Sc = 2M\delta/\rho D^2H$ is the Scruton number (6), which is a product of the mass ratio and damping expressed through the logarithmic decrement δ ,

Df is the number of degrees of freedom of the flexibly mounted cylinder

Re is the Reynolds number based on the diameter of the cylinder,

Ar is the aspect ratio expressed as the height to diameter ratio of the cylinder

SR is the surface roughness described by the texture and relative-size of roughness elements, and

Tu is the free stream turbulence characterized by the intensity, scale and frequency spectrum.

The amplitude of flow-induced oscillations depends strongly on Df , Sc and W , and weakly on the other four parameters. The restriction to one degree of freedom confines displacement to that direction only and also changes the flow field. Various excitations can be fully suppressed if Sc is kept above certain values. The excitations of a single cylinder are usually confined within a range of reduced velocities. The existence and extent of each range is governed by the value of Sc .

It is convenient to describe all the observed excitations of a single cylinder in terms of the reduced velocity W as follows:

1) Low W ; two excitations have been observed, leading to oscillations in a streamwise direction,

a) $1 < W < 3$ and $Sc < 1.2$. The initial displacement of the cylinder in the streamwise direction disrupts the natural vortex shedding. A subsequent symmetric formation of two vortices terminates with a forced symmetric shedding when the cylinder reaches the maximum downstream amplitude (7).

There is a significant difference in drag force when the cylinder moves upstream and the vortices are small, compared with the downstream stroke when the vortices are more developed. The variation of the drag force maintains the self-excitation and produces amplitudes up to 0.2 diameters at very low values of Sc .

b) $2.5 < W < 4$ and $Sc \leq 1.0$. The flow instability in the form of alternate vortex shedding imposes a fluctuating drag force at twice the frequency of vortex shedding from one side of the cylinder. This excites oscillations in the streamwise direction and the oscillating cylinder synchronizes vortex shedding at its natural frequency within the range of W given above. The maximum amplitude at $W = 3$ to 3.5 does not exceed 0.15 diameters at low Sc .

2) Medium W ; $4 < W < 8$ to 11 and $Sc \leq 15$ to 20 for a finite cylinder with a free end or $Sc \leq 60-80$ for a nominally two-dimensional cylinder between end plates. The excitation is mostly into the transverse direction for amplitudes up to 0.6 diameters and the response forms a number of eight pattern normal to the free-stream direction for large amplitude. The interaction between elastic or flexibly mounted rigid cylinder and the vortex shedding instability passes through three phases as W is increased:

a) When the frequency of vortex shedding approaches the natural frequency of the cylinder, the fluctuating transverse lift force excites the cylinder to oscillate near the resonance condition.

b) As W increases the amplitude of oscillations continues to rise beyond the resonance condition because the shedding frequency becomes captured by, and synchronized to, the oscillation frequency. Significant changes in the near-wake are observed during synchronized vortex shedding such as continuous decrease of Strouhal number, considerable shortening of the length of the vortex formation region (8), an improvement in the spanwise correlation of pressure, velocity and sectional lift forces and an increase in strength of the vortices (8). All these changes contribute to an amplification of the lift and drag forces, which in turn increase the amplitude of oscillations.

c) In the middle of the synchronization range an amplitude is reached at which a rapid variation of the phase angle between the exciting force and the displacement takes place. The self-excited oscillations becomes self-limiting as fluid damping is produced in the near-wake by different timing of the vortex formation and shedding (9).

The resonance, self-excited and self-limiting phases proceed in reverse order when the free-stream velocity is decreased. The initial oscillations are caused by the displacement of the cylinder and not by the resonance, as before, and a considerable hysteretic effect is found. There are other synchronizations at higher harmonics (10).

3) High W ; $W > 20$ to 40 . Far beyond the synchronization range, the cylinder starts to oscillate again and gradually builds up considerable amplitude, mostly in the transverse direction. The excitation seems to be related to the synchronization of displacement of the separation lines with the displacements of the oscillating cylinder. The separation of the free shear layer is delayed on the upper side when the cylinder moves upwards and the separation is promoted earlier on the lower side. The displacement of separation is limited and depends on Reynolds number.

INTERFERENCE FLOW REGIMES

When there is no interference the flow pattern around each of two parallel circular cylinders is the same as that around a single cylinder. Interference takes place when the cylinders are sufficiently close to each other, the downstream cylinder is adjacent to or submerged in the wake of the upstream cylinder, or, a combination of both. These three categories of interference are observed for the following arrangements of the two cylinders:

1) 'Proximity' interference occurs in both side-by-side and slightly staggered arrangements.

2) 'Wake' interference happens in both tandem and staggered arrangements at spacings for which the near-wake development of the upstream cylinder is unaffected by the downstream cylinder ($L/D > 4$).

3) 'Proximity-and-wake' interference is characterised at small spacings by significant disruption of the near-wake behind the upstream cylinder, for tandem and staggered arrangements ($L/D < 4$).

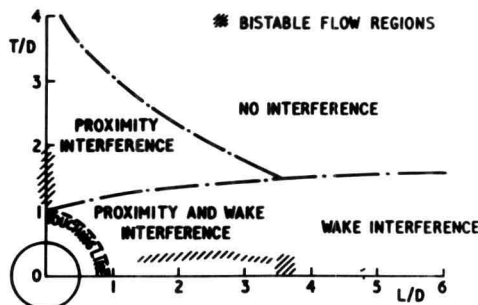


Fig. 1. Sketch of Interference Regions

Fig. 1 shows a sketch of the domains of the three interference regions. Wake interference starts well before the downstream cylinder reaches the wake boundary of the upstream cylinder¹. Proximity interference commences when vortex shedding becomes coupled behind both cylinders.

Three different flow regimes have been observed around two stationary cylinders in various side-by-side arrangements (11):

a) $1 < T/D < 1.2$; a single vortex street is formed behind the two cylinders, with a base bleed through the gap.

b) $1.2 < T/D < 2$ to 2.2 ; two asymmetric wakes are formed behind the cylinders. The narrow and wide wakes are divided by a biased jet flow through the gap. The jet flow is bistable and the narrow and wide wake can interchange behind the cylinders at irregular time intervals.

c) $2.2 < T/D < 4$; both wakes are equal in size but the two vortex streets are coupled and mirror each other along the gap axis. The vortex formation is synchronized behind both cylinders, as sketched in Table 1.

The 'staggered proximity' interference region has attracted the least research interest. The strong coupling of two vortex streets is expected to be similar to that found in regime (c). The bistable asymmetric flow (b) is found to extend to $L/D = \pm 0.15$ (12).

Wake interference is characterised by a decrease in drag force on the downstream cylinder with decreasing transverse T/D and streamwise spacing L/D . Fig. 2a shows typical drag coefficient distribution, measured in the upper subcritical range, with a significant wake effect even for $L/D = 50$ (13). The 'lift' force is always directed towards the wake-axis of the upstream cylinder and reaches a maximum near the wake boundary. The interference boundary in Fig. 2b is determined by $C_L = 0$, and not by a constant value of C_D .

¹It has been assumed incorrectly by many researchers that the wake and interference boundaries are the same.

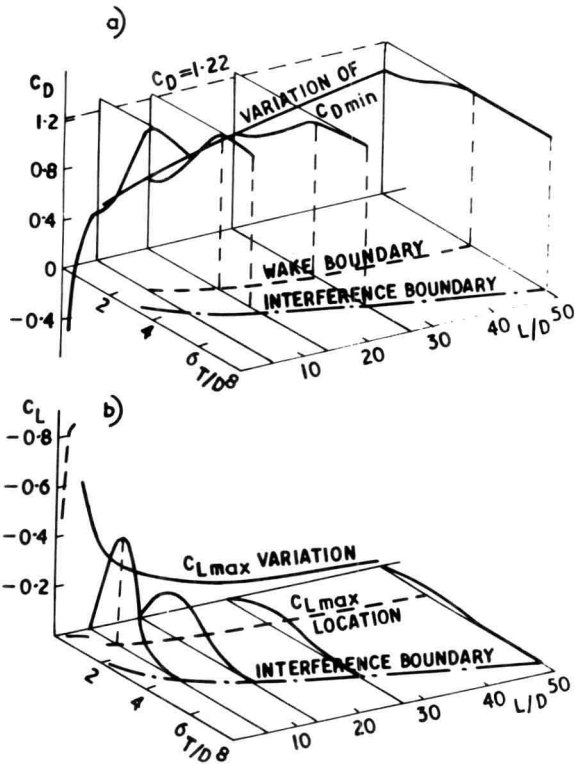


Fig. 2. Measured Aerodynamic Force for Large Spacings at $Re=50 \times 10^4$ (13); a) Drag coefficient b) Lift coefficient

The 'proximity-and-wake' interference is related to a significant disruption of the near-wake of the upstream cylinder by the presence of the downstream one (14). Fig. 3 shows the measured distribution of drag and lift coefficients with a remarkable coincidence of C_{Lmax} and C_{Dmin} . They both occurred near to the wake axis when an intense gap flow was fully established. The gap flow abruptly ceased as the downstream cylinder was brought near to the wake axis. The two flow regimes were bistable for a certain range of T/D when C_{Lmax} could collapse intermittently as reported in (14).

The third bistable region was found in the tandem arrangement for $3.5 < L/D < 3.8$ where intermittent vortex shedding behind the upstream cylinder was established. The two flow regimes with and without vortex shedding affected the pressure distribution and drag coefficient of both cylinders (14)

All three bistable regions showed a considerable hysteretic effect (11,15). One of the two flow regimes persisted longer when the velocity was increased or, at the same velocity when the cylinder was displaced in one direction, and the other regime lasted longer when the opposite conditions were imposed (15). The hysteresis produced an apparent overlap of the two flow regimes although only one flow regime existed at a time.

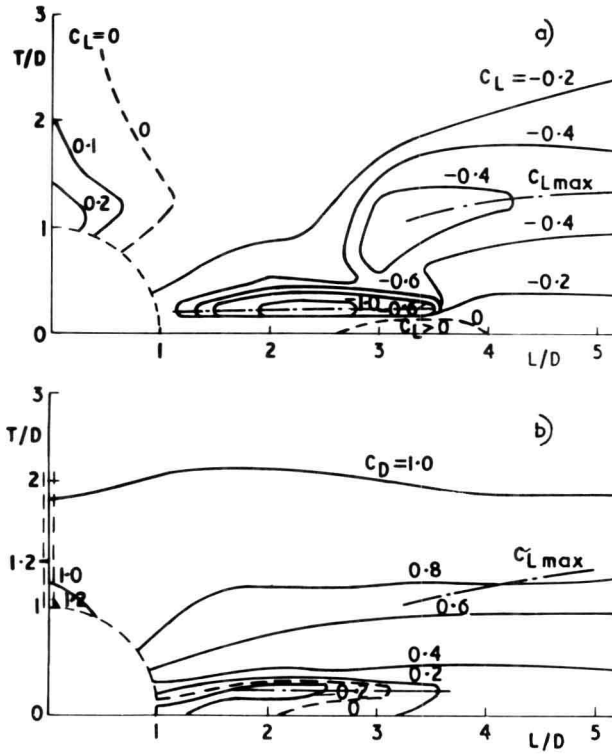


Fig. 3. Measured Aerodynamic Force for Small Spacings at $Re=6 \times 10^4$ (11); a) Drag coefficient b) Lift coefficient



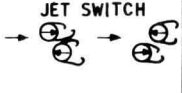


























All three categories of interference with sketches of the corresponding flow regimes are shown in Table 1. The existence of bistable flow regimes behind stationary cylinders provides the exciting force for an initial displacement. The oscillating cylinders can synchronize at their natural frequencies the change-over of flow regimes and excite large amplitude oscillations. The always present hysteresis of the change-over of flow regimes prolongs the period during which the exciting force and velocity are in phase, as will be discussed in next sections.

PARAMETERS INFLUENCING EXCITATIONS OF TWO CYLINDERS

The susceptibility of two cylinders to flow induced oscillations depends not only on their arrangement ($L/D, T/D$) but also on other physical parameters. The relative amplitude of oscillation A/D is generally a function of the following dimensionless parameters.

$$\frac{A_i(x, y)}{D} = f_i \left(\frac{L}{D}, \frac{T}{D}, W_i, Sc_i, (DF)_i, (AR)_i, Re_i, (SR)_i, T_u \right) \quad (2)$$

TABLE I. CLASSIFICATION OF INTERFERING FLOW INDUCED OSCILLATIONS

CATEGORY	INITIAL ARRANGEMENT	STREAM WISE L/D	TRANS VERSE T/D	FLOW REGIMES FOR STATIONARY CYLINDERS		TYPE OF EXCITATION			
						LOW W	MED. W	HIGH W	
P R O X I M I T Y	SIDE BY SIDE	0	1.0 1.2	SINGLE VORTEX STREET		?	?		
		0	1.1 2.2	BIASED JET					
		0	2.0 4.0	COUPLED		?	?		
	STAGGERED	<3.8	<I _B	VORTEX STREETS		?	?		
P R O X I M I T Y A N D W A K E	TANDEM	1.1 1.5	0	SYNCHRONIZED REATTACHMENT					
		1.5 3.8	0	QUASI-STEADY REATTACHMENT					
	STAGGERED	1.1 3.2	0.2 0.3	GAP FLOW		?	?		
W A K E	TANDEM	>3.8	0	COUPLED VORTEX STREETS		?			NONE
		>8	0	DISTURBED VORTEX STREET			?		
	STAGGERED	>3.2	>C _{Lmax}	DISPLACED WAKE			?		WAKE DISPLACEMENT
		>8	<I _B	OUTSIDE WAKE BOUNDARY		NO INTERFERENCE		WAKE GALLOPING	

I_B - INTERFERENCE BOUNDARY

where $i = 1, 2$ designates the two cylinders, which may have different natural frequencies (W_1, W_2), different equivalent mass or structural damping $Sc_1 \neq Sc_2$ and different numbers of degrees of freedom $(DF)_1 \neq (DF)_2$ ¹. A small aspect ratio affects strongly the amplitude of oscillations particularly if the cylinders are finite with one free-end. The other influencing parameters are presumably less important, except in the transitional regimes.

The number of influencing parameters can be reduced considerably if the two cylinders are the same in size, smooth, nominally two-dimensional, elastically identical, with the same number of degrees of freedom and submerged in a non-turbulent free-stream. Regrettably these criteria were not satisfied in all experimental studies and disagreement between the available data should be expected.

Most researchers restricted the response of the downstream cylinder, allowing its displacement in only one direction and/or eliminating the response of the upstream cylinder by mounting it rigidly. The mode of self-excitation of the downstream cylinder, having one degree of freedom, might be considerably different in some arrangements from the response of the same cylinder, having two degrees of freedom and interfering with the oscillating

¹ A lot of research has been carried out with a fixed upstream cylinder.

upstream cylinder (15). The experimentally observed oscillations of two interfering cylinders will be described separately for each interference category in terms of low, medium and high reduced velocity W .

PROXIMITY INTERFERENCE

Low W . When two cylinders are placed side by side in close proximity, $T/D < 1.2$, the small gap between them should inhibit the formation of a symmetric vortex pair behind each cylinder. The latter is essential for the streamwise self-excitation of a single cylinder by symmetric vortex shedding. At present there are no experimental data in the range $1 < W < 3$ for cylinders in close proximity.

The next stage of spacings, characterised by two asymmetric wakes, has been tested in water in the subcritical regime (16). Both cylinders had two degrees of freedom, two natural frequencies ($f_1/f_2 = 0.93$) and were arranged side by side at $T/D = 1.75$. The oscillations in the streamwise direction started in the range $1.3 < W < 1.5$ and reached maximum amplitude in the range $2.25 < W < 3$. The mode of oscillations of the two cylinders was in phase up to $W = 3$ but beyond changed to an out-of-phase mode. However, the second peak in amplitude in the stream-wise direction did not appear, despite the change in the mode of oscillations (16).

The coupled vortex street regime has not yet attracted research. The destructive vibration of trash racks (17) in water might have been caused by coupled symmetric vortex shedding because $W = 1.2$ at the start of oscillations for all three natural frequencies of the rods.

Medium W . The initial excitation of a single cylinder is caused by vortex shedding at the Strouhal frequency. When two cylinders had a single degree of freedom in transverse direction and were arranged side-by-side at $T/D = 1.1$ the in-phase oscillation of both was found at $W = 10$ (18). This confirmed that only one vortex street existed behind both cylinders.

The asymmetric wakes at larger spacings had different associated Strouhal numbers and two separate excitations should be expected. When two cylinders having two degrees of freedom were placed in water the first excitation was found at $W = 3.8$ for $T/D = 1.5$ (16) with an out-of-phase mode of oscillation. The same mode of oscillation was found in air when two cylinders had only a transverse degree of freedom (18), see Fig. 4a, but the excitation started at $W = 2.8$ for $T/D = 1.5$, and the maximum amplitude occurred at $W = 3.6$. This considerable discrepancy in the critical reduced velocities indicates the influence of the differing degrees of freedom. The second excitation started at $W = 5$, see Fig. 4a, for cylinders having one degree of freedom (18). The maximum amplitude was reached at $W = 5.4$ and ceased at $W = 6.2$. The mode of oscillation was in-phase.

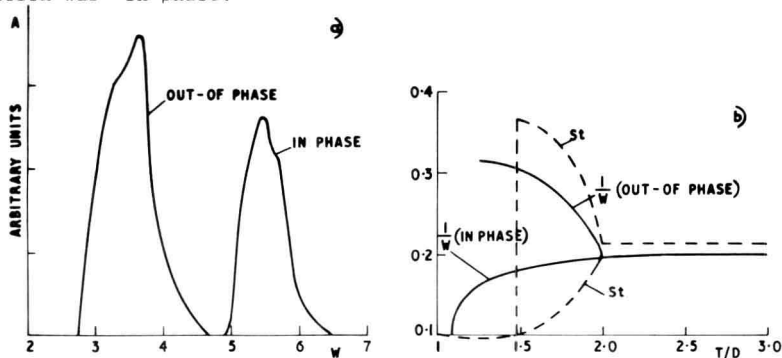


Fig. 4. Excitation of two side-by-side cylinders for $T/D=1.5$ (18); a) Synchronization ranges b) Strouhal number and $1/W$ versus T/D

Fig. 4b shows a comparison of measured Strouhal numbers, in the asymmetric wakes behind two stationary cylinders and reciprocal values of reduced velocities at which the maximum amplitude of oscillation was found for each of the two modes of oscillation. The first, out-of-phase, mode was excited by vortex shedding in the narrow wake (high St) and the maximum amplitude occurred at upper W^{-1} due to the synchronization phenomenon. The second in-phase mode was not excited by vortex shedding in the wide wake, which would occur at higher value of W than 5. The oscillations seemed to be self-excited and synchronization produced two identical wakes behind both cylinders, as documented by the smoke visualization photographs (18). The in-phase mode of oscillations disappeared for $T/D > 2$ and only the out-of-phase mode was found up to $T/D = 2$ (18) as should be expected.

It has been a common engineering practice to increase the stiffness of two closely spaced chimney stacks by a rigid spacer placed near the free end. The response of such rigidly coupled finite cylinders ($H/D = 16$) having $T/D = 2$ and $Sc = 13$ exceeded the maximum amplitude found on an identical single cylinder (19). When the coupled cylinders were staggered relative to the free stream velocity the maximum amplitude decreased as seen in Fig. 5. It should be pointed out that the rigidly coupled cylinders were mounted on a cantilever with a single (transverse) degree of freedom.

High W . Excitation well beyond vortex shedding resonance has been discovered (12) in a row of cylinders arranged closely side-by-side. All the odd cylinders were fixed and all the even cylinders had only a streamwise degree of freedom. Large amplitude streamwise oscillations were excited for $W > 20$ even when $Sc > 40$. The cause of excitations was synchronization of the jet switch with the frequency of the oscillating cylinders, coupled with a hysteretic effect. The latter prolonged the narrow near-wake and a high drag force was found during the downstream stroke. A wide near-wake and low drag was found during the upstream stroke (12).

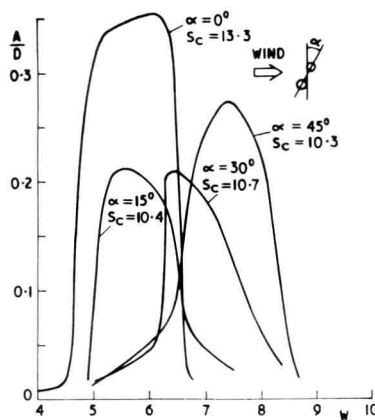


Fig. 5. Synchronization Ranges of Coupled Cylinders (19)

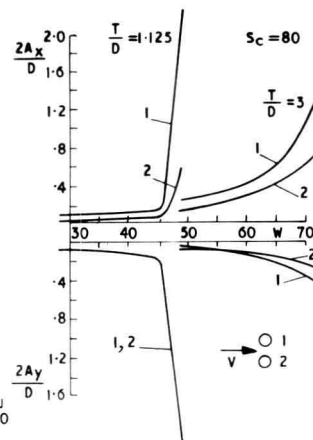


Fig. 6. Jet-switch Instability for Two Side-by-Side Arrangements for $Sc = 80$ (2)

A similar excitation of large amplitude oscillations was studied for two flexibly mounted cylinders having two degrees of freedom (20). The Scruton number was deliberately chosen to be so high ($Sc=80$) that all other excitations, at low and medium reduced velocities, were fully suppressed. The critical reduced velocity for self-excitation W_{cr} depended strongly on the spacing between the cylinders. Table 2 summarizes the observed W_{cr} and Fig. 6 shows the difference in the initial amplitude build up for the smallest spacing $T/D = 1.125$ and that near the end of an extended interference region, $T/D = 3$. The mode of oscillation was always out-of-phase and the interference region for the oscillating cylinders ($T/D > 3$) exceeded that found for the stationary cylinders ($T/D < 2.2$).

Table 2. Measured Critical Reduced Velocities for Side by Side Arrangements

T/D	1.125	1.25	1.5	2.0	3.0
W_{cr}	45	30	40	55	60-65

When the cylinders were sufficiently staggered ($L/D > 0.5$) the jet could not switch and oscillations were predominantly in the transverse direction (20). It may be noted that the rigid coupling of two cylinders, which was ineffective at medium W , should be fully effective for this type of excitation at high W .

The main difference between this type of self-excitation and all previous ones can be summarized as follows:

- 1) The critical velocity depends strongly on the Scruton number and instability can be delayed by increasing the Scruton number.
- 2) Once excited there is no upper limit of reduced velocity where self-excitation ceases. The synchronization range of jet-switching is open-ended and self-sustaining.

PROXIMITY-AND-WAKE INTERFERENCE

Low W . Forced symmetric vortex shedding is likely to occur when two tandem cylinders are in close proximity. Flow-visualizations in water, for $L/D = 2$ and $Sc = 0.75$, revealed (21) symmetric vortex shedding in the range $1.2 > W > 2.75$ behind the upstream cylinder only. The downstream cylinder was excited by the variation in pressure in the gap and it oscillated in the streamwise direction and out-of-phase relative to the upstream cylinder direction. When the spacing was increased beyond $L/D = 2.75$ only the upstream cylinder oscillated because the pressure variations behind the upstream cylinder could not reach and excite the downstream cylinder.

Synchronized alternate vortex shedding was found for $Sc < 1$ in the range $2.5 > W > 4$ with in-phase oscillations of the cylinders in the streamwise direction (12,21). In this case, the downstream cylinder always had greater maximum amplitude than the upstream one and in excess of those found for a single cylinder (21). Clashing of oscillating cylinders for $L/D = 1.5$ was observed in water (16) in the range $2.8 > W > 4.8$. The measured amplitude was less than half of the spacing and the out-of-phase mode, excited by symmetric vortex shedding, continued unabated throughout the clashing range. When the spacing was increased to $L/D = 1.75$, there was a change to the in-phase mode and clashing did not occur. Also there was no second maximum of amplitude for the vortex shedding type of excitation (16).

When two elastic cantilevered cylinders were joined rigidly at their free-ends both types of excitation were observed (21). Fig. 7 shows that the coupled cylinders oscillated vigorously for the first excitation. This

could be explained by symmetric vortex shedding being forced behind both cylinders because they oscillated in-phase. For $L/D = 2.25$ a considerable displacement of the first excitation is seen in Fig. 7 and for $L/D < 2$ the excitation was suppressed because presumably the fixed gap was too small for the formation of symmetric vortices.

The slightly inclined staggered arrangements have not yet attracted research. It might be expected that the asymmetric interference would inhibit the formation of symmetric vortices and the first excitation might not occur. The second type of excitation might develop only on the downstream cylinder. These speculations require experimental verification.

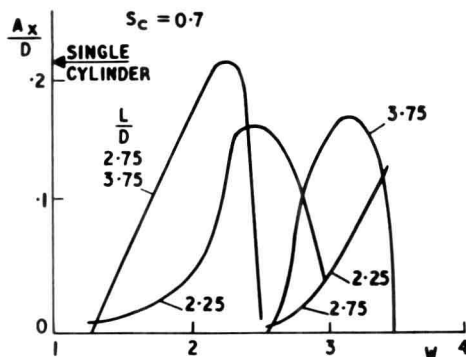


Fig. 7. Streamwise Excitations of Coupled Tandem Cylinders in Water for $Sc = 0.7$ (21)

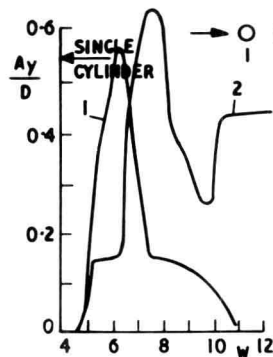


Fig. 8. Synchronization ranges for Tandem Cylinders $L/D = 2.5$ (21)

Medium W. Proximity-and-wake interference prevents vortex shedding behind the upstream cylinder when two stationary cylinders are in a tandem arrangement. When the cylinders are elastic the transverse displacement of the cylinders in an out-of-phase mode could trigger and synchronize vortex shedding behind the upstream cylinder. Fig. 8 shows a typical excitation of two elastic cantilevered cylinders in water arranged at $L/D = 2.5$ (21). The excitation starts with both cylinders oscillating out-of-phase (16). Synchronized vortex shedding behind the upstream cylinder produces a maximum amplitude at $W = 6.3$. Further increase in reduced velocity changes the phase angle between the excitation and oscillations and results in a rapid decrease in amplitude of the upstream cylinder. The synchronized vortex street behind the downstream cylinder produces a maximum amplitude at $W = 7.7$ which exceeds the previous maximum amplitude produced by the upstream cylinder. The appearance of yet another maximum amplitude for $W = 11$ will be discussed in the next section.

Similar responses were found for slightly inclined staggered arrangements at $Sc = 1.7$ (22) and $Sc = 23$ (20). Systematic tests have been carried out when a downstream cylinder having a transverse degree of freedom was placed in the nearwake of a fixed upstream cylinder (23, 24) and vice versa (23, 25).

Two coupled tandem cylinders were tested in water (21) and air (19). Fig. 9 shows two distinct peaks. The coupled cylinders can only oscillate in an in-phase mode and the initial excitation is produced by the vortex shedding behind the downstream cylinder, the first peak in Fig. 9. The second and smaller peak was probably excited by the downstream cylinder but the mechanism is not known. The excitation was reduced considerably when the cylinders were in staggered arrangement (19).