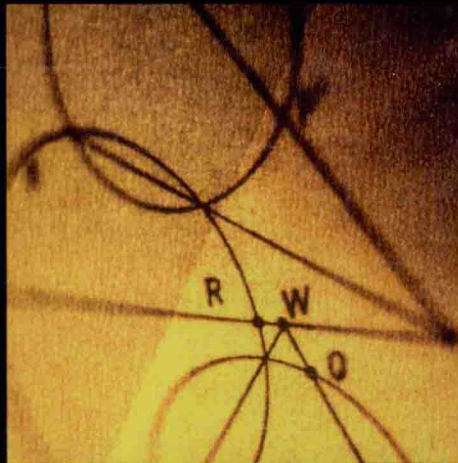


GEOMETRICAL and VISUAL OPTICS

a clinical introduction



STEVEN H. SCHWARTZ

GEOMETRICAL AND VISUAL OPTICS

A Clinical Introduction

Steven H. Schwartz, OD, PhD

Vice President and Dean for Academic Affairs
State University of New York
State College of Optometry
New York, New York

ion

New York Chicago San Francisco Lisbon London Madrid
Mexico City Milan New Delhi San Juan Seoul
Singapore Sydney Toronto

McGraw-Hill

A Division of The McGraw-Hill Companies



GEOMETRICAL AND VISUAL OPTICS: A Clinical Introduction

Copyright © 2002 by **The McGraw-Hill Companies, Inc.** All rights reserved. Printed in the United States of America. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a data base or retrieval system, without the prior written permission of the publisher.

1234567890 DOC/DOC 098765432

ISBN 0-07-137415-9

This book was set in Garamond by Hightstown Desktop Publishing.
The editors were Darlene Barela Cooke and Lester A. Sheinis.
The production manager was Phil Galea.
The illustration manager was Charissa Baker.
The cover designer was Aimée Nordin.
The indexer was Alexandra Nickerson.
R. R. Donnelley & Sons Company was printer and binder.

This book is printed on acid-free paper.

Library of Congress Cataloging-in-Publication Data

Schwartz, Steven H.

Geometrical and visual optics / Steven H. Schwatz.

p. ; cm.

Includes bibliographical references and index.

ISBN 0-07-137415-9

1. Physiological optics. 2. Geometrical optics. I. Title.

[DNLM: 1. Optics—Problems and Exercises. 2. Lenses—Problems and Exercises. 3. Refraction, Ocular—Problems and Exercises. 4. Vision—Problems and Exercises. WW 18.2 S399g 2002]

QP475 .S379 2002

617.7'5'076—dc21

2001044815

GEOMETRICAL
AND
VISUAL OPTICS

NOTICE

Medicine is an ever-changing science. As new research and clinical experience broaden our knowledge, changes in treatment and drug therapy are required. The author and the publisher of this work have checked with sources believed to be reliable in their efforts to provide information that is complete and generally in accord with the standards accepted at the time of publication. However, in view of the possibility of human error or changes in medical sciences, neither the author nor the publisher nor any other party who has been involved in the preparation or publication of this work warrants that the information contained herein is in every respect accurate or complete, and they disclaim all responsibility for any errors or omissions or for the results obtained from use of the information contained in this work. Readers are encouraged to confirm the information contained herein with other sources. For example and in particular, readers are advised to check the product information sheet included in the package of each drug they plan to administer to be certain that the information contained in this work is accurate and that changes have not been made in the recommended dose or in the contraindications for administration. This recommendation is of particular importance in connection with new or infrequently used drugs.

Preface

The goal of this book is to demystify geometrical and visual optics. It is intended to be a concise and learner-friendly resource for clinicians as they study optics for the first time and subsequently prepare for licensing and qualifying examinations. The emphasis is on those optical concepts and problem-solving skills that underlie contemporary clinical eye care and refraction.

The book stresses a vergence approach to geometrical and visual optics. Schematic figures and clinical examples are used throughout the text to engage reader interest. Every effort is made to provide the reader with an intuitive and clinical sense of optics that will allow him or her to effectively care for patients.

To develop competence and facility in geometrical and visual optics, it is necessary to solve problems. Each chapter provides problems of varying complexity, with worked-out solutions given at the end of the book. The reader should make every attempt to solve the problems before resorting to the solutions.

Despite careful review and editing, mathematical errors are bound to occur in a text of this nature. Please send any corrections or comments to the author at <opticsbook@aol.com>.

This book grew out of my experiences as an educator and practitioner. Over the years, I have been afforded the opportunity to work with talented colleagues, teach motivated students, and provide care to a diverse spectrum of patients. My colleagues, students, and patients all, in their own ways, motivated this book and for this I am most thankful.

Drs. Raymond Applegate, Ian Bailey, Michael Barris, Lewis Reich, and Alan Riezman read and commented on portions of the manuscript. Their thoughtful input is very much appreciated.

Sally Barhydt, Charissa Baker, Darlene Cooke, and Lester Sheinis, all of McGraw-Hill, and Dr. Norman Haffner, President of the State University of New York, State College of Optometry, provided critical support at various stages of this project. The forbearance and encouragement of Lenge Hong throughout this past year is especially appreciated.

GEOMETRICAL
AND
VISUAL OPTICS

Contents

Preface	xi
1. Basic Terms and Concepts	1
• Objects, Light Rays, and Pencils	2
• Vergence	4
• Refraction	5
• Snell's Law	6
• Self-Assessment Problems	12
2. Refraction at Spherical Surfaces	13
• Converging and Diverging Surfaces	13
• More on Focal Points	16
• Refracting Power and Focal Lengths	18
• Another Way to Calculate Power	19
• Real Images	21
• Virtual Images	22
• Self-Assessment Problems	24
3. The Vergence Relationship	27
• Basic Concepts	27
• More on Vergence	28
• Sign Conventions	31
• Sample Problems	32
Converging Surface	32
Location of Focal Points	35
Diverging Surface	35
Locating the Object When Given the Image Location	37

Surface with No Power	38
• Self-Assessment Problems	41
4. Thin Lenses	43
• Focal Points	43
• Ray Tracing	47
• Paraxial Relationship	47
• Newton's Relation	50
• Self-Assessment Problems	52
5. Optical Systems with Multiple Surfaces	53
• Multiple Thin Lens Systems	53
• Virtual Objects	57
• Thick Lenses	59
• Self-Assessment Problems	62
6. Equivalent Lenses	63
• Definitions and Formulae	63
Equivalent Power	65
Front Vertex and Back Vertex Power	66
Principal Planes	67
• Sample Problem	67
• Locating an Image Using an Equivalent Lens	72
• Nodal Points	73
• Self-Assessment Problems	75
7. Schematic Eyes and Ametropia	77
• Gullstrand and Reduced Eye Models	77
• Emmetropia	79
• Myopia	80
• Hyperopia	82
• Far-Point in Emmetropia	85
• Far-Point Vergence Relationship	85
• Lens Effectivity	87
• Correction of Ametropia with Laser and Surgical Procedures	90
• Self-Assessment Problems	94
8. Accommodation	95
• Accommodation in the Emmetropic Eye	96
• Accommodation in Uncorrected Ametropia	98
• Near Point of Accommodation	101
• Accommodation in Corrected Ametropia	103
• Correction of Presbyopia	108
• Self-Assessment Problems	111

9. Cylindrical Lenses and the Correction of Astigmatism	113
• Lens Crosses	114
• Lens Formulae/Prescriptions	116
• Image Formation: Point Sources	119
• Image Formation: Extended Sources	121
• Astigmatism: Definitions and Classifications	123
• Jackson Crossed-Cylinder Test	126
• Spherical Equivalency	127
• What Does the Astigmat See?	129
• Self-Assessment Problems	131
10. Prisms	133
• Angle of Deviation	133
• Prism Power	135
• Prismatic Effects of Lenses	137
• Prentice's Rule	137
• Clinical Applications	140
• Self-Assessment Problems	144
11. Depth of Field	145
• Blur Circles and Visual Acuity	145
• Depth of Field and Depth of Focus	150
• Hyperfocal Distance	154
• Self-Assessment Problems	157
12. Magnifying Devices	159
• Magnification by Plus Lenses	159
Lateral Magnification	159
Effective Magnification	160
Angular Magnification of a Plus Lens	162
The Problem with Magnification	163
• Prescribing Near-Plus Magnifiers	163
Magnifying Lens and Bifocal Add in Combination	166
Fixed-Focus Stand Magnifiers	167
Closed-Circuit Television	170
More on Near Magnification Devices	170
• Telescopes	170
Galilean Telescopes	171
Keplerian Telescopes	171
An Alternative Method of Determining a Telescope's Angular Magnification	173
Lens Caps	174
• Self-Assessment Problems	177

13. Retinal Image Size	179
• Linear Size of the Retinal Image in Uncorrected Ametropia	179
• Spectacle Magnification	181
• Angular Magnification in Corrected Ametropia	183
• Physical Image Size in Corrected Ametropia	184
• Summary	187
• Self-Assessment Problems	189
14. Reflection	191
• Ray Tracing: Concave, Convex, and Plane Mirrors	191
Concave Mirrors	191
Convex Mirrors	193
Plane Mirrors	195
• Power of Mirrors	196
• The Vergence (Paraxial) Relationship	198
• Reflections and Antireflective Coatings	203
• Purkinje Images	206
Location of Purkinje Image I	207
Location of Purkinje Image III	209
• Corneal Topography	213
• Keratometry and Contact Lenses	214
• Javal's Rule	219
• Self-Assessment Problems	223
15. Aberrations	225
• The Paraxial Assumption	225
• Seidel Aberrations	226
Spherical Aberration	227
Coma	229
Radial Astigmatism	234
Curvature of Field	235
Distortion	236
• Spherical Aberration of the Human Eye	237
• Wavefront Sensing and Adaptive Optics	237
Measurement of the Eye's Monochromatic Aberrations	238
Supernormal Vision	240
Imaging the Fundus	242
• Chromatic Aberrations	243
Dispersive Power and Constrigence	244
Achromatic Lenses	245
• Chromatic Aberrations of the Human Eye	247
• The Red-Green Refraction Technique	249
• Lateral (Transverse) Chromatic Aberration	249
• Self-Assessment Problems	252

Answers to Self-Assessment Problems	253
• Chapter 1 Basic Terms and Concepts	253
• Chapter 2 Refraction at Spherical Surfaces	254
• Chapter 3 The Vergence Relationship	256
• Chapter 4 Thin Lenses	260
• Chapter 5 Optical Systems with Multiple Surfaces	262
• Chapter 6 Equivalent Lenses	268
• Chapter 7 Schematic Eyes and Ametropia	275
• Chapter 8 Accommodation	278
• Chapter 9 Cylindrical Lenses and the Correction of Astigmatism ..	282
• Chapter 10 Prisms	287
• Chapter 11 Depth of Field	289
• Chapter 12 Magnifying Devices	291
• Chapter 13 Retinal Image Size	293
• Chapter 14 Reflection	295
• Chapter 15 Aberrations	302
 Index	 305

Basic Terms and Concepts

At any given moment, our eyes are inundated by an enormous quantity of electromagnetic radiation. This radiation ranges from short-wavelength gamma and x-ray radiation to longer-wavelength radar and radio waves. *Light* is the portion of the electromagnetic spectrum that is visible (Fig. 1-1).

Electromagnetic radiation is typically specified by its wavelength or frequency. Wavelength and frequency are inversely proportional and related to each other as shown by the following equation:

$$\nu = c/\lambda$$

where

ν = the frequency of light

c = the speed of light

λ = the wavelength of light

The wavelength of light ranges from about 380 to 700 nanometers (nm).¹ It is emitted in discrete packages of energy referred to as *photons* or *quanta*. The amount of energy in a photon is given by the following relationship:

$$E = h\nu$$

where

E = the amount of energy per photon

h = Planck's constant

¹One nanometer (nm) is equal to 10⁻⁹ meters (m).

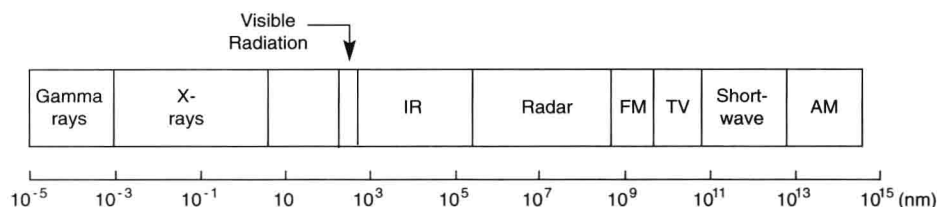


FIGURE 1-1. Light (visible radiation) is a small portion of the electromagnetic spectrum. (From Schwartz SH. Visual Perception: A Clinical Orientation. Copyright 1999. Reprinted by permission of McGraw-Hill, Inc.)

By substitution, we have:

$$E = \frac{hc}{\lambda}$$

As the wavelength decreases, the amount of energy per photon increases. For this reason, the absorption of short-wavelength radiation by body tissues is typically more damaging than the absorption of longer-wavelength radiation. The development of cataracts and basal cell carcinoma is promoted by exposure to short-wavelength, high-energy ultraviolet radiation.

OBJECTS, LIGHT RAYS, AND PENCILS

We see *objects* because they emit or reflect light, and this light is focused on our retina. A *point source* of light, such as a star, emits waves of light in much the same way that a pebble dropped into a quiet pond of water generates waves of water (Fig. 1-2). Light *rays* are perpendicular to light wavefronts and are represented by arrows.

A bundle of rays is called a *pencil* (Fig. 1-3). The light rays that form a pencil can be diverging, converging, or parallel. A *diverging pencil* is produced by a point source of light, such as a star. When light rays are focused at a point, they create a *converging pencil*. A converging optical system (e.g., a magnifying lens) is required to create converging light. An object located infinitely far away forms a *parallel pencil*.²

An *extended object*, such as an arrow, is composed of an infinite number of point sources (Fig. 1-4). Diverging light rays emerge from the point sources.

²Consider the waves that are created when a pebble is dropped into a quiet pond of water (Fig. 1-2). The wavefronts closest to the source (the pebble) are more curved than the wavefronts further from the source. At very far distances, the wavefronts are flat. Since rays are perpendicular to wavefronts, the rays are parallel to each other.

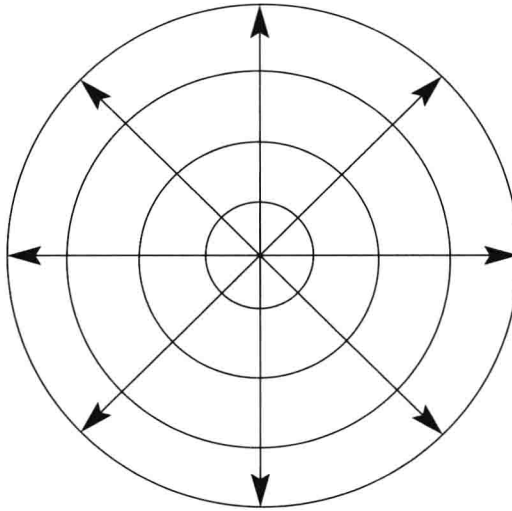


FIGURE 1-2. A point source of light emits concentric waves of light in much the same way a pebble dropped into a quiet pond of water produces waves of water. Light rays, represented by arrows, are orthogonal to the wavefronts.

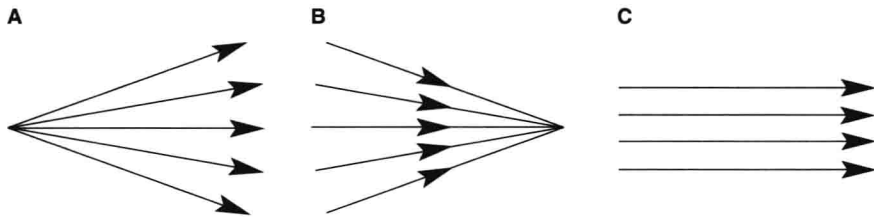


FIGURE 1-3A. A diverging pencil of light rays emerges from a point source. **B.** A converging pencil of light rays is focused at a point. **C.** An object located at infinity produces a parallel pencil of light rays.

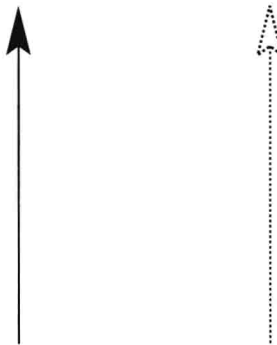


FIGURE 1-4. An extended object, such as an arrow, may be considered to consist of an infinite number of point sources. Each point emits diverging light rays.

VERGENCE

For solving clinical optical problems, it is useful to quantify the convergence or divergence of light. **The amount of convergence or divergence of light rays (i.e., the *vergence* of the light) is (1) the reciprocal of the distance to a point source or (2) the reciprocal of the distance to a point of focus. To arrive at the correct units for vergence—*diopters (D)*—the distance must be in meters. By convention, diverging light is always labeled with a negative sign and converging light with a positive sign.**

Consider Figure 1-5, which shows diverging light rays. At a distance of 10.00 cm from the point source, the vergence is -10.00 diopters, or -10.00 D.³ At distances of 20.00 and 50.00 cm, the vergence is -5.00 and -2.00 D, respectively. The further the distance from the point source, the less the (absolute) magnitude of the divergence.⁴

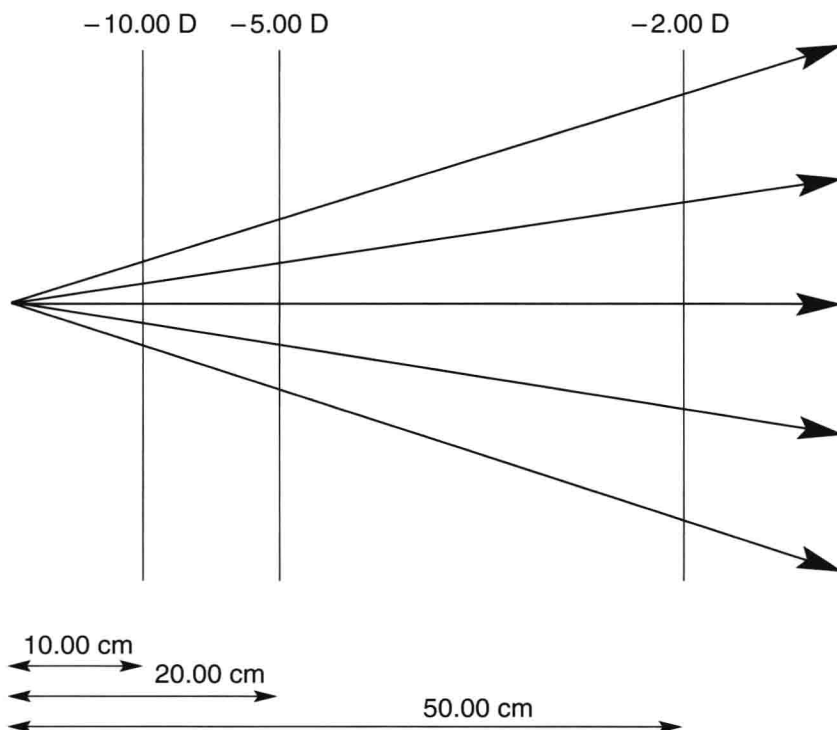


FIGURE 1-5. Diverging light rays have negative vergence. The absolute magnitude of the divergence *decreases* as the distance from the object *increases*.

³The reciprocal of 0.10 is 10.00.

⁴Returning to Figure 1-2, the wavefronts that are closer to the source are more curved than those further from the source. You can think of the curvature of a wavefront as a measure of vergence—the more curved the wavefront, the greater the vergence. In the extreme case—at an infinite distance from the source—the wavefront is flat (the rays are parallel), and the vergence is zero.

In Figure 1-6, converging light rays are focused at a point. With respect to this point of focus, the vergence at 50.00 cm is +2.00 D. Likewise, at distances of 20.00 and 10.00 cm, the vergence is +5.00 and +10.00 D, respectively. As the distance from the point of focus increases, the magnitude of the convergence decreases.

What is the vergence of parallel light rays? These rays originate from an object at optical infinity; the reciprocal of infinity is zero.⁵ Or think of it this way: since the rays are neither diverging nor converging, their vergence is zero.

REFRACTION

When light travels from one transparent material (e.g., air) to a more optically dense transparent material (e.g., water), its velocity decreases. This decrease in velocity can cause light rays to deviate from their original direction, a phenomenon referred to as *refraction*. In Figure 1-7A, a light ray traveling through air

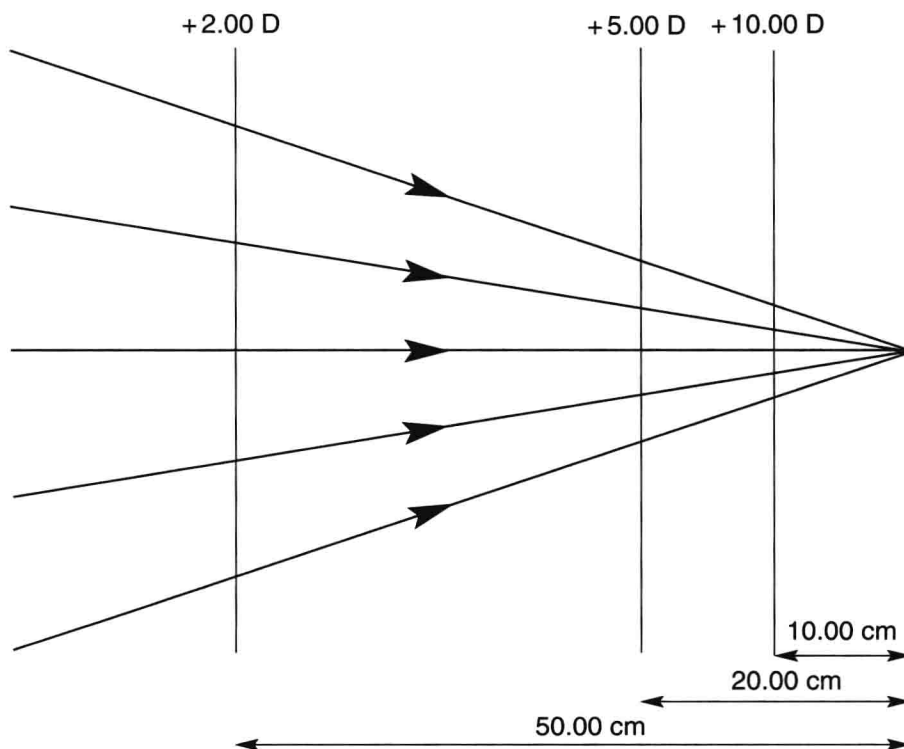


FIGURE 1-6. Converging light rays have positive vergence. As the distance from the point of focus increases, the absolute magnitude of the convergence decreases.

⁵In clinical practice, optical infinity is typically a distance of 20 ft (or 6 m).