Algebra 1

An Incremental Development

THIRD EDITION

SAXON

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John H. Saxon, Jr.

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Preface

This text is the third edition of a widely used and now classic textbook. It is the first text in the Saxon secondary school mathematics series, which is comprised of three books: *Algebra 1*, *Algebra 2*, and *Advanced Mathematics*. Together, these three books form an integrated treatment of secondary school mathematics. Topics from secondary school mathematics, including a full year of plane geometry, have been integrated throughout the three-book series.

At the suggestion of classroom teachers, many changes have been incorporated into the third edition. One innovation is the addition of lesson reference numbers in the problem sets. Beneath each problem number in the problem sets is a number in parentheses; this number refers to the lesson where the concept or skill required to solve the problem is introduced. Should you have difficulty solving a particular problem, refer to the lesson where the concepts and skills required to solve that problem are discussed.

Unfortunately, John Saxon did not have the pleasure of seeing his work on this text come to fruition. He passed away in October 1996 in the middle of the revision process. Below is what I can best describe as his "exhortation" to students. In his unique and colorful style, John Saxon addresses students like yourself and explains his philosophy of teaching and learning. I urge you to read what Mr. Saxon says carefully. I hope that you also join the ranks of many thousands of students who have used the Saxon program and, as a result, have achieved success in mathematics.

philosophy

Algebra is not difficult. Algebra is just different. Time is required in order for things that are different to become things that are familiar. In this book we provide the necessary time by reviewing all the concepts in every problem set. Also, the parts of a particular concept are introduced in small units so that they may be practiced for a period of time before the next part of the same concept is introduced. Understanding the first part makes it easier to understand the second part. If you find that a particular problem is troublesome, get help at once because the problem won't go away. It will appear again and again in future problem sets.

The problem sets contain all the review that is necessary. **Your task is to work all the problems in every problem set.** The answers to odd-numbered problems are in the back of the book. You will have to check the answers to even-numbered problems with a classmate. Don't be discouraged if you continue to make mistakes. Everyone makes mistakes, often for a long period of time. A large part of learning algebra is devising defense mechanisms to protect you from yourself. If you work at it, you can find ways to prevent these mistakes. Your teacher is an expert because he or she has made the same mistakes many times and has finally found ways to prevent them. You must do the same. Each person must devise his or her own defense mechanisms.

The repetition in the problem sets in this book is necessary to permit all students to master all the concepts. Then, application of the concepts must be practiced for a long time to ensure retention. This practice has an element of drudgery to it, but it has been demonstrated that people who are not willing to practice fundamentals often find success elusive. Ask any athlete, musician, or artist about the necessity of practicing fundamental skills.

This book continues the study of the area, volume, and perimeter of geometric figures begun in *Algebra* $\frac{1}{2}$. The long-term practice of these problems will allow you to emblazon these fundamental concepts in your memory so that you will be able to use these concepts without effort for the rest of your life.

The book concentrates on teaching you the fundamental aspects of problem solving. Problem solving is simply the application of mathematical concepts in new situations. Problem solving is easier in many cases if a picture of the problem can be drawn. Thus we use diagrams when we work uniform motion problems, ratio problems, and percent problems. The percent diagrams are a little difficult at first, but after a while they are easy to draw.

This book will prove to you that mathematics is reasonable and that mathematics is not hard. If you do every problem in every problem set, you will be amazed at how easy it all becomes. We repeat ourselves by saying that algebra is not **difficult.** Algebra is just **different.** Things that are different become familiar things only after they have been practiced for a long time.

acknowledgments I thank editor Smith Richardson for completing the revision of this text in John Saxon's absence. I also thank Diana Stolfus for extensive help and for providing us the benefit of her wisdom and experience gained from many years in the classroom. I thank typesetter Lj Stephens for her very competent typesetting of the text. I thank editor Julie Webster for her very careful reading of the manuscript and her thoughtful suggestions. I thank artists John Chitwood, Chris Cope, Michael Lott, and Travis Southern for precisely and meticulously drawing the artwork required for the text. I thank our proofreaders for carefully reading the manuscript and working as well as checking the answers to the problems in the problem sets. I thank J. Travis Rose, pre-press manager, for overseeing the entire pre-press process, and Joan Coleman, production coordinator, for coordinating the manufacturing aspects of this revision process. Lastly, I thank the many, many Saxon teachers for their continued and abiding support of the Saxon program.

Frank Y. H. Wang, Ph.D.
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Norman, Oklahoma

Contents

LESSON 1 *Addition and Subtraction of Fractions • Lines and Segments*

1.A

addition and subtraction of fractions

To add or subtract fractions that have the same denominators, we add or subtract the numerators as indicated below, and the result is recorded over the same denominator.

$$\frac{5}{11} + \frac{2}{11} = \frac{7}{11} \qquad \frac{5}{11} - \frac{2}{11} = \frac{3}{11}$$

If the denominators are not the same, it is necessary to rewrite the fractions so that they have the same denominators.

	Problem	Rewritten with Equal Denominators	Answer
(a)	$\frac{1}{3} + \frac{2}{5}$	$\frac{5}{15} + \frac{6}{15}$	$\frac{11}{15}$
(b)	$\frac{2}{3} - \frac{1}{8}$	$\frac{16}{24} - \frac{3}{24}$	$\frac{13}{24}$

A **mixed number** is the sum of a whole number and a fraction. Thus the notation

$$13\frac{3}{5}$$

does not mean 13 multiplied by $\frac{3}{5}$ but instead 13 plus $\frac{3}{5}$.

$$13 + \frac{3}{5}$$

When we add and subtract mixed numbers, we handle the fractions and the whole numbers separately. In some subtraction problems it is necessary to borrow, as shown in (e).

	Problem	Rewritten with Equal Denominators	Answer
(c)	$13\frac{3}{5} + 2\frac{1}{8}$	$13\frac{24}{40} + 2\frac{5}{40}$	$15\frac{29}{40}$
(d)	$13\frac{3}{5} - 2\frac{1}{8}$	$13\frac{24}{40} - 2\frac{5}{40}$	$11\frac{19}{40}$
		Borrowing	
(e)	$13\frac{3}{5} - 2\frac{7}{8}$	$13\frac{24}{40} - 2\frac{35}{40} = 12\frac{64}{40} - 2\frac{35}{40}$	$10\frac{29}{40}$

1.B

lines and segments

It is impossible to draw a mathematical line because a mathematical line is a **straight line** that has **no width** and **no ends.** To show the location of a mathematical line, we draw a pencil line and put arrowheads on both ends to emphasize that the mathematical line goes on and on in both directions.

We can name a line by naming any two points on the line and using an overbar with two arrowheads. We can designate the line shown by writing $\overleftrightarrow{AX}$, $\overleftrightarrow{XA}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{CA}$, $\overleftrightarrow{XC}$, or $\overleftrightarrow{CX}$.

A part of a line is called a **line segment.** A line segment contains the endpoints and all points between the endpoints. To show the location of a line segment, we use a pencil line with no arrowheads. We name a segment by naming the endpoints of the segment.

This is segment MC or segment CM. We can indicate that two letters name a segment by using an overbar with no arrowheads. Thus $\overline{MC}$ means segment MC. If we use two letters without the overbar, we designate the length of the segment. Thus MC is the length of $\overline{MC}$.

example 1.1 Add: $\dfrac{10}{11} - \dfrac{5}{6} + \dfrac{1}{3}$

solution We begin by rewriting each fraction so that they have the same denominators. Then we add the fractions.

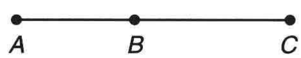
$$\frac{10}{11} - \frac{5}{6} + \frac{1}{3} = \frac{60}{66} - \frac{55}{66} + \frac{22}{66} \qquad \text{common denominators}$$

$$= \frac{5}{66} + \frac{22}{66} \qquad \text{added}$$

$$= \frac{27}{66} \qquad \text{added}$$

$$= \mathbf{\frac{9}{22}} \qquad \text{simplified}$$

example 1.2 Segment AC measures $10\frac{1}{4}$ units. Segment AB measures $4\frac{3}{7}$ units. Find BC.



solution We need to know the length of segment BC. We know AC and AB. We subtract to find BC.

$$BC = AC - AB$$

$$= 10\frac{1}{4} - 4\frac{3}{7} \qquad \text{substituted}$$

$$= 10\frac{7}{28} - 4\frac{12}{28} \qquad \text{common denominators}$$

$$= 9\frac{35}{28} - 4\frac{12}{28} \qquad \text{borrowed}$$

$$= \mathbf{5\frac{23}{28}\ units} \qquad \text{subtracted}$$

problem set 1

Add or subtract as indicated. Write answers as proper fractions reduced to lowest terms or as mixed numbers.

1. $\frac{1}{5} + \frac{2}{5}$ **2.** $\frac{3}{8} - \frac{2}{8}$ **3.** $\frac{4}{3} - \frac{1}{3} + \frac{2}{3}$

Different denominators:

4. $\frac{1}{3} + \frac{1}{5}$ **5.** $\frac{3}{8} - \frac{1}{5}$ **6.** $\frac{2}{3} - \frac{1}{8}$

7. $\frac{1}{13} + \frac{1}{5}$ **8.** $\frac{14}{15} - \frac{2}{3}$ **9.** $\frac{5}{9} + \frac{2}{5}$

10. $\frac{14}{17} - \frac{6}{34}$ **11.** $\frac{5}{13} + \frac{1}{26}$ **12.** $\frac{4}{7} - \frac{2}{5}$

13. $\frac{4}{7} + \frac{1}{8} + \frac{1}{2}$ **14.** $\frac{3}{5} + \frac{1}{8} + \frac{1}{8}$ **15.** $\frac{5}{11} - \frac{1}{6} + \frac{2}{3}$

Addition of mixed numbers:

16. $2\frac{1}{2} + 3\frac{1}{5}$ **17.** $7\frac{3}{8} + 6\frac{1}{3}$ **18.** $1\frac{1}{8} + 7\frac{2}{5}$

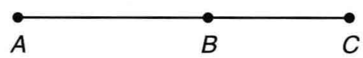
Subtraction with borrowing:

19. $15\frac{1}{3} - 7\frac{4}{5}$ **20.** $42\frac{3}{8} - 21\frac{3}{4}$ **21.** $22\frac{2}{5} - 13\frac{7}{15}$

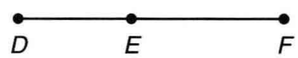
22. $42\frac{1}{11} - 18\frac{2}{3}$ **23.** $78\frac{2}{5} - 14\frac{7}{10}$ **24.** $43\frac{1}{13} - 6\frac{5}{8}$

25. $21\frac{1}{5} - 15\frac{7}{13}$ **26.** $21\frac{2}{19} - 7\frac{7}{10}$ **27.** $43\frac{3}{17} - 21\frac{9}{10}$

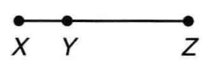
28. The length of $\overline{AB}$ is $7\frac{1}{8}$ units. The length of $\overline{BC}$ is $5\frac{2}{7}$ units. Find AC.



29. DF is $42\frac{1}{7}$ units. EF is $24\frac{2}{11}$ units. Find DE.



30. XZ is $12\frac{11}{16}$ units. XY is $3\frac{5}{8}$ units. Find YZ.



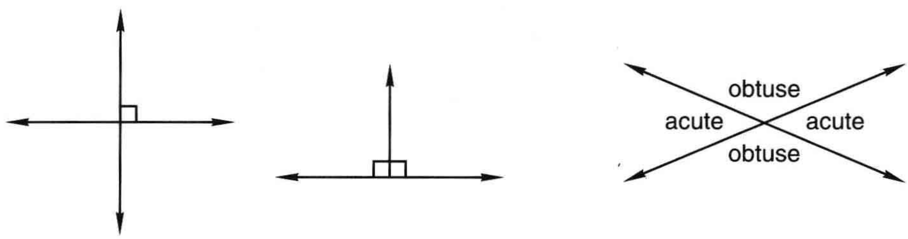
LESSON 2 Angles • Polygons • Triangles • Quadrilaterals

2.A angles

If two lines cross, we say that the lines **intersect.** The place where the lines cross is called the **point of intersection.** Two lines in the same plane either intersect or do not intersect. If two lines in the same plane do not intersect, we say that the lines are **parallel lines.** The perpendicular distance between two parallel lines is everywhere the same.

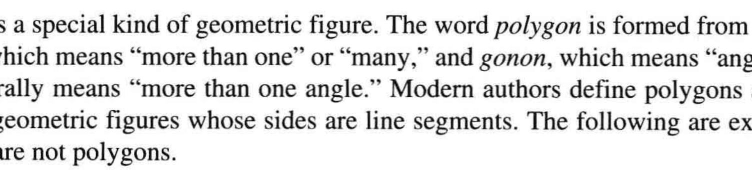
Intersecting lines — Parallel lines

If two lines make square corners at the point of intersection, we say that the lines are **perpendicular.** The angles made by perpendicular lines are called **right angles.** We can draw a little square at the point of intersection to indicate that all four angles formed are right angles. Two right angles form a **straight angle.** An angle smaller than a right angle is called an **acute angle.** An angle greater than a right angle but less than a straight angle is called an **obtuse angle.**



4 right angles — Straight angle — 2 acute angles and 2 obtuse angles

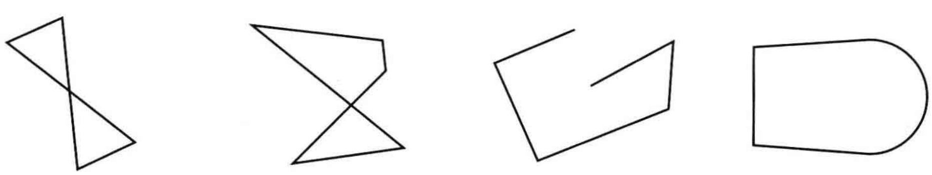
If a right angle is divided into 90 parts, we say that each part has a measure of 1 degree. Thus, a right angle has a measure of 90 degrees, which can also be written as 90°. Two right angles form a straight angle. Thus, a straight angle has a measure of 180 degrees, which can also be written as 180°. Four right angles have a measure of 360 degrees, which can also be written as 360°. Thus, the measure of a circle is 360°.



90° in a right angle — 180° in a straight angle — 360° in a circle

2.B polygons

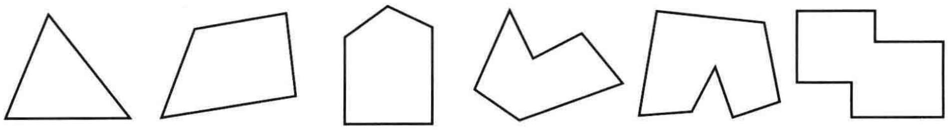
A **polygon** is a special kind of geometric figure. The word *polygon* is formed from the Greek roots *poly*, which means "more than one" or "many," and *gonon*, which means "angle." Thus, polygon literally means "more than one angle." Modern authors define polygons as simple, closed, flat geometric figures whose sides are line segments. The following are examples of figures that are not polygons.



(a) Not a polygon — (b) Not a polygon — (c) Not a polygon — (d) Not a polygon

Figures (a) and (b) are not polygons because the line segments cross and therefore the figures are not simple figures. Figure (c) is not a polygon because it is not closed. Figure (d) is not a polygon because one of the "sides" is curved.

The following are examples of figures that are polygons:



Triangle 3 sides 3 vertices
Quadrilateral 4 sides 4 vertices
Pentagon 5 sides 5 vertices
Hexagon 6 sides 6 vertices
Heptagon 7 sides 7 vertices
Octagon 8 sides 8 vertices

Each segment of a polygon is called a **side.** Each endpoint of a side is called a **vertex** of the polygon. The plural of vertex is **vertices. For each polygon, the number of sides is always equal to the number of vertices.**

Polygons are named according to the number of sides they have:

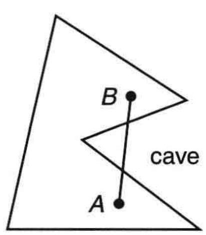
- The polygon with the fewest number of sides is the triangle.
- A polygon with 4 sides is called a *quadrilateral.*
- A polygon with 5 sides is called a *pentagon.*
- A polygon with 6 sides is called a *hexagon.*
- A polygon with 7 sides is called a *heptagon.*
- A polygon with 8 sides is called an *octagon.*
- A polygon with 9 sides is called a *nonagon.*
- A polygon with 10 sides is called a *decagon.*
- A polygon with 11 sides is called an *undecagon.*
- A polygon with 12 sides is called a *dodecagon.*

Some polygons of more than 12 sides have special names, but these names are not used often. Instead, we use the word *polygon* and tell the number of sides or use the number of sides with the suffix *-gon*. Thus, if a polygon has 42 sides, we would call it a polygon with 42 sides or a 42-gon. For easy reference, we list the names of special polygons in the following table.

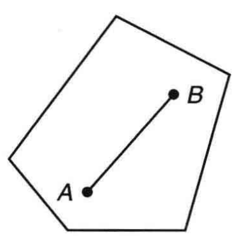
Number of Sides	Name of Polygon	Number of Sides	Name of Polygon
3	Triangle	9	Nonagon
4	Quadrilateral	10	Decagon
5	Pentagon	11	Undecagon
6	Hexagon	12	Dodecagon
7	Heptagon	n	n-gon
8	Octagon		

concave and convex polygons

If a polygon has an indentation (a cave), the polygon is called a **concave polygon.** Any polygon that does not have an indentation is called a **convex polygon.** Any two points in the interior of a convex polygon can be connected by a line segment that does not cut a side of the polygon. This statement is not true for concave polygons. Most of the polygons that you will study will be convex polygons. In this book, when *polygon* is used it will mean *convex polygon* unless stated otherwise.

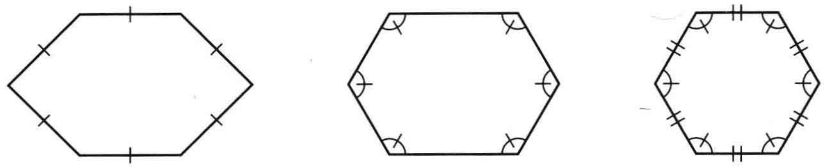


Concave polygon



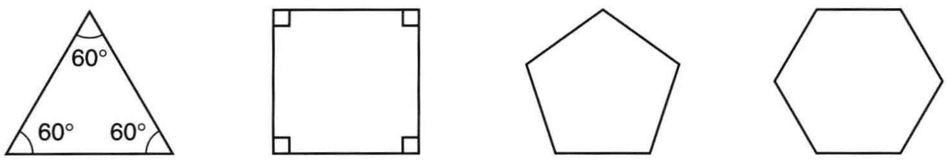
Convex polygon

regular polygons If all the sides of a polygon have the same length, the polygon is called an **equilateral polygon.** If all the angles of a polygon have the same measure, the polygon is called an **equiangular polygon.** Polygons in which all sides have the same length and all angles have the same measure are called **regular polygons.** In some of the following figures we use **tick marks** to denote sides whose lengths are equal and angles whose measures are equal.



Equilateral polygon Equiangular polygon Regular polygon

The following are examples of polygons that are regular polygons.



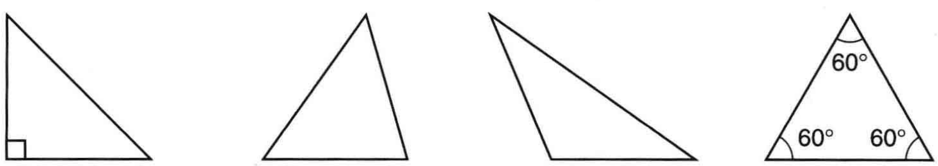
Regular triangle (Equilateral triangle) Regular quadrilateral (Square) Regular pentagon Regular hexagon

A regular triangle is called an equilateral triangle. All the angles in an equilateral triangle are 60° angles. We remember that a quadrilateral is a polygon with four sides. A regular quadrilateral is a square.

2.C triangles

We remember that the polygon with the fewest number of sides is the triangle. Triangles have three sides and three angles. **The sum of the measures of the three angles in any triangle is 180°.** Triangles can be classified according to the measures of their angles or according to the lengths of their sides.

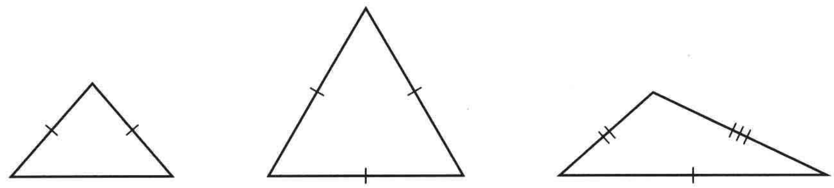
If a triangle has a right angle, the triangle is a **right triangle.** If all angles have a measure less than 90°, the triangle is an **acute triangle.** If one angle has a measure greater than 90°, the triangle is an **obtuse triangle.** The Latin prefix *equi* - means "equal" and the Latin word *angulus* means "angle." We can put them together to form *equiangular*, which means "equal angles." An **equiangular triangle** is a triangle in which the measures of all angles are equal. Each angle in an equiangular triangle must have a measure of 60° because $3 \times 60°$ equals 180°.



Right triangle Acute triangle Obtuse triangle Equiangular triangle

Triangles are also classified according to the relative lengths of their sides. The Greek prefix *iso* - means "equal" and the Greek word *skelos* means "leg." We can put them together to form *isosceles*, which means "equal legs." An **isosceles triangle** is a triangle that has at least two sides of equal length. Since the Latin prefix *equi*- means "equal" and the Latin word *latus* means "side," we can put them together to form *equilateral*, which means "equal sides." An

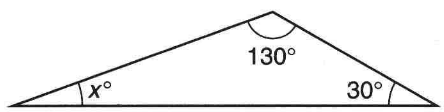
equilateral triangle is a triangle in which the lengths of all sides are equal. If all the sides of a triangle have different lengths, the triangle is called a **scalene triangle.**



Isosceles triangle   Equilateral triangle   Scalene triangle

The lengths of the sides of a triangle and the measures of the angles opposite these sides are related. In any triangle, the angles opposite sides of equal lengths have equal measures. Also, the sides opposite angles of equal measures have equal lengths. We remember an isosceles triangle is a triangle that has at least two sides of equal length. The angles opposite these sides have equal measures. All three sides in an equilateral triangle have the same length. Since the angles opposite these sides have equal measures, an equilateral triangle is also an equiangular triangle. All three angles in an equiangular triangle have equal measures. Since the sides opposite these angles have equal lengths, an equiangular triangle is also an equilateral triangle. The scalene triangle has no equal sides, so no two angles have equal measures.

example 2.1 Find x.

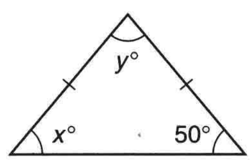


solution **The sum of the measures of the three angles in any triangle is 180°.** The two given angles have measures of 30° and 130°. The sum of the measures of these angles is 160°. Therefore, angle x must have a measure of 20° because

$$20 + 30 + 130 = 180$$

Therefore, x = **20.**

example 2.2 Find x and y.



solution The identical tick marks on two sides of the triangle tell us that these two sides have equal lengths. **In any triangle, the angles opposite sides of equal lengths have equal measures.** Therefore, angle x must have a measure of 50°. So, x = **50.** The sum of the measures of the three angles must be 180°. So angle y must have a measure of 80° because

$$50 + 50 + 80 = 180$$

Therefore, y = **80.**

example 2.3 Find x and y.