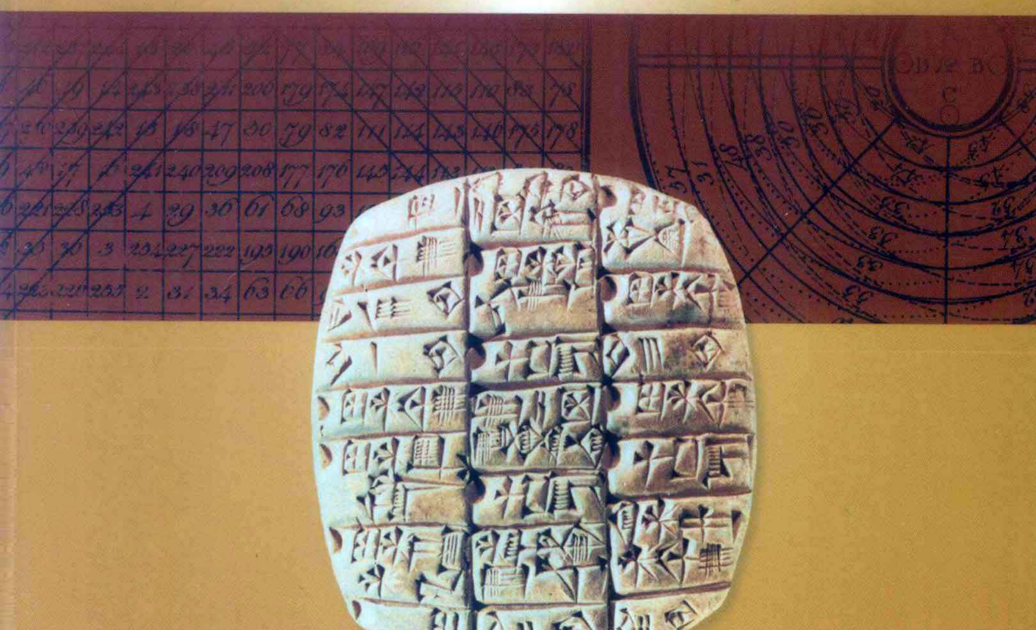
A HISTORY OF MATHEMATICS

THIRD EDITION



Uta C. Merzbach and Carl B. Boyer

Foreword by Isaac Asimov

A History of Mathematics

THIRD EDITION

Uta C. Merzbach and Carl B. Boyer

WILEY
John Wiley & Sons, Inc.

Published by John Wiley & Sons, Inc., Hoboken, New Jersey
Published simultaneously in Canada

Library of Congress Cataloging-in-Publication Data:

Boyer, Carl B. (Carl Benjamin), 1906–1976.
A history of mathematics / Carl B. Boyer and Uta Merzbach. — 3rd ed.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-470-52548-7 (pbk.); ISBN 978-0-470-63039-6 (ebk.);
ISBN 978-0-470-63054-9 (ebk.); ISBN 978-0-470-630563 (ebk.)
1. Mathematics—History. I. Merzbach, Uta C., 1933– II. Title.
QA21.B767 2010
510.9—dc22 2010003424

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

In memory of Carl B. Boyer
(1906–1976)
—U.C.M.

To the memory of my parents,
Howard Franklin Boyer and
Rebecca Catherine (Eisenhart) Boyer
—C.B.B.

Foreword to the Second Edition

By Isaac Asimov

Mathematics is a unique aspect of human thought, and its history differs in essence from all other histories.

As time goes on, nearly every field of human endeavor is marked by changes which can be considered as correction and/or extension. Thus, the changes in the evolving history of political and military events are always chaotic; there is no way to predict the rise of a Genghis Khan, for example, or the consequences of the short-lived Mongol Empire. Other changes are a matter of fashion and subjective opinion. The cave-paintings of 25,000 years ago are generally considered great art, and while art has continuously—even chaotically—changed in the subsequent millennia, there are elements of greatness in all the fashions. Similarly, each society considers its own ways natural and rational, and finds the ways of other societies to be odd, laughable, or repulsive.

But only among the sciences is there true progress; only there is the record one of continuous advance toward ever greater heights.

And yet, among most branches of science, the process of progress is one of both correction and extension. Aristotle, one of the greatest minds ever to contemplate physical laws, was quite wrong in his views on falling bodies and had to be corrected by Galileo in the 1590s. Galen, the greatest of ancient physicians, was not allowed to study human cadavers and was quite wrong in his anatomical and physiological conclusions. He had to be corrected by Vesalius in 1543 and Harvey in 1628. Even Newton, the greatest of all scientists, was wrong in his view of the nature of light, of the achromaticity of lenses, and missed the existence of spectral lines. His masterpiece, the laws of motion and the theory of universal gravitation, had to be modified by Einstein in 1916.

Now we can see what makes mathematics unique. Only in mathematics is there no significant correction—only extension. Once the Greeks had developed the deductive method, they were correct in what they did, correct for all time. Euclid was incomplete and his work has been extended enormously, but it has not had to be corrected. His theorems are, every one of them, valid to this day.

Ptolemy may have developed an erroneous picture of the planetary system, but the system of trigonometry he worked out to help him with his calculations remains correct forever.

Each great mathematician adds to what came previously, but nothing needs to be uprooted. Consequently, when we read a book like *A History of Mathematics*, we get the picture of a mounting structure, ever taller and broader and more beautiful and magnificent and with a foundation, moreover, that is as untainted and as functional now as it was when Thales worked out the first geometrical theorems nearly 26 centuries ago.

Nothing pertaining to humanity becomes us so well as mathematics. There, and only there, do we touch the human mind at its peak.

Preface to the Third Edition

During the two decades since the appearance of the second edition of this work, there have been substantial changes in the course of mathematics and the treatment of its history. Within mathematics, outstanding results were achieved by a merging of techniques and concepts from previously distinct areas of specialization. The history of mathematics continued to grow quantitatively, as noted in the preface to the second edition; but here, too, there were substantial studies that overcame the polemics of "internal" versus "external" history and combined a fresh approach to the mathematics of the original texts with the appropriate linguistic, sociological, and economic tools of the historian.

In this third edition I have striven again to adhere to Boyer's approach to the history of mathematics. Although the revision this time includes the entire work, changes have more to do with emphasis than original content, the obvious exception being the inclusion of new findings since the appearance of the first edition. For example, the reader will find greater stress placed on the fact that we deal with such a small number of sources from antiquity; this is one of the reasons for condensing three previous chapters dealing with the Hellenic period into one. On the other hand, the chapter dealing with China and India has been split, as content demands. There is greater emphasis on the recurring interplay between pure and applied mathematics as exemplified in chapter 14. Some reorganization is due to an attempt to underline the impact of institutional and personal transmission of ideas; this has affected most of the pre-nineteenth-century chapters. The chapters dealing with the nineteenth century have been altered the least, as I had made substantial changes for some of this material in the second edition. The twentieth-century material has been doubled, and a new final chapter deals with recent trends, including solutions of some longstanding problems and the effect of computers on the nature of proofs.

It is always pleasant to acknowledge those known to us for having had an impact on our work. I am most grateful to Shirley Surrette Duffy for responding judiciously to numerous requests for stylistic advice, even at times when there were more immediate priorities. Peggy Aldrich Kidwell replied with unfailing precision to my inquiry concerning certain photographs in the National Museum of American History. Jeanne LaDuke cheerfully and promptly answered my appeals for help, especially in confirming sources. Judy and Paul Green may not realize that a casual conversation last year led me to rethink some recent material. I have derived special pleasure and knowledge from several recent publications, among them *Klopfer* 2009 and, in a more leisurely fashion, *Szpiro* 2007. Great thanks are due to the editors and production team of John Wiley & Sons who worked with me to make this edition possible: Stephen Power, the senior editor, was unfailingly generous and diplomatic in his counsel; the editorial assistant, Ellen Wright, facilitated my progress through the major steps of manuscript creation; the senior production manager, Marcia Samuels, provided me with clear and concise instructions, warnings, and examples; senior production editors Kimberly Monroe-Hill and John Simko and the copyeditor, Patricia Waldygo, subjected the manuscript to painstakingly meticulous scrutiny. The professionalism of all concerned provides a special kind of encouragement in troubled times.

I should like to pay tribute to two scholars whose influence on others should not be forgotten. The Renaissance historian Marjorie N. Boyer (Mrs. Carl B. Boyer) graciously and knowledgeably complimented a young researcher at the beginning of her career on a talk presented at a Leibniz conference in 1966. The brief conversation with a total stranger did much to influence me in pondering the choice between mathematics and its history.

More recently, the late historian of mathematics Wilbur Knorr set a significant example to a generation of young scholars by refusing to accept the notion that ancient authors had been studied definitively by others. Setting aside the "*magister dixit*," he showed us the wealth of knowledge that emerges from seeking out the texts.

—Uta C. Merzbach
March 2010

Preface to the Second Edition

This edition brings to a new generation and a broader spectrum of readers a book that became a standard for its subject after its initial appearance in 1968. The years since then have been years of renewed interest and vigorous activity in the history of mathematics. This has been demonstrated by the appearance of numerous new publications dealing with topics in the field, by an increase in the number of courses on the history of mathematics, and by a steady growth over the years in the number of popular books devoted to the subject. Lately, growing interest in the history of mathematics has been reflected in other branches of the popular press and in the electronic media. Boyer's contribution to the history of mathematics has left its mark on all of these endeavors.

When one of the editors of John Wiley & Sons first approached me concerning a revision of Boyer's standard work, we quickly agreed that textual modifications should be kept to a minimum and that the changes and additions should be made to conform as much as possible to Boyer's original approach. Accordingly, the first twenty-two chapters have been left virtually unchanged. The chapters dealing with the nineteenth century have been revised; the last chapter has been expanded and split into two. Throughout, an attempt has been made to retain a consistent approach within the volume and to adhere to Boyer's stated aim of giving stronger emphasis on historical elements than is customary in similar works.

The references and general bibliography have been substantially revised. Since this work is aimed at English-speaking readers, many of whom are unable to utilize Boyer's foreign-language chapter references, these have been replaced by recent works in English. Readers are urged to consult the General Bibliography as well, however. Immediately following the chapter references at the end of the book, it contains additional works and further bibliographic references, with less regard to language. The introduction to that bibliography provides some overall guidance for further pleasurable reading and for solving problems.

The initial revision, which appeared two years ago, was designed for classroom use. The exercises found there, and in the original edition, have been dropped in this edition, which is aimed at readers outside the lecture room. Users of this book interested in supplementary exercises are referred to the suggestions in the General Bibliography.

I express my gratitude to Judith V. Grabiner and Albert Lewis for numerous helpful criticisms and suggestions. I am pleased to acknowledge the fine cooperation and assistance of several members of the Wiley editorial staff. I owe immeasurable thanks to Virginia Beets for lending her vision at a critical stage in the preparation of this manuscript. Finally, thanks are due to numerous colleagues and students who have shared their thoughts about the first edition with me. I hope they will find beneficial results in this revision.

—Uta C. Merzbach
Georgetown, Texas
March 1991

Preface to the First Edition

Numerous histories of mathematics have appeared during this century, many of them in the English language. Some are very recent, such as J. F. Scott's *A History of Mathematics*[1]; a new entry in the field, therefore, should have characteristics not already present in the available books. Actually, few of the histories at hand are textbooks, at least not in the American sense of the word, and Scott's *History* is not one of them. It appeared, therefore, that there was room for a new book—one that would meet more satisfactorily my own preferences and possibly those of others.

The two-volume *History of Mathematics* by David Eugene Smith[2] was indeed written "for the purpose of supplying teachers and students with a usable textbook on the history of elementary mathematics," but it covers too wide an area on too low a mathematical level for most modern college courses, and it is lacking in problems of varied types. Florian Cajori's *History of Mathematics*[3] still is a very helpful reference work; but it is not adapted to classroom use, nor is E. T. Bell's admirable *The Development of Mathematics*.[4] The most successful and appropriate textbook today appears to be Howard Eves, *An Introduction to the History of Mathematics*,[5] which I have used with considerable satisfaction in at least a dozen classes since it first appeared in 1953.

[1]London: Taylor and Francis, 1958.
[2]Boston: Ginn and Company, 1923–1925.
[3]New York: Macmillan, 1931, 2nd edition.
[4]New York: McGraw-Hill, 1945, 2nd edition.
[5]New York: Holt, Rinehart and Winston, 1964, revised edition.

I have occasionally departed from the arrangement of topics in the book in striving toward a heightened sense of historicalmindedness and have supplemented the material by further reference to the contributions of the eighteenth and nineteenth centuries especially by the use of D. J. Struik, *A Concise History of Mathematics.*[6]

The reader of this book, whether layman, student, or teacher of a course in the history of mathematics, will find that the level of mathematical background that is presupposed is approximately that of a college junior or senior, but the material can be perused profitably also by readers with either stronger or weaker mathematical preparation. Each chapter ends with a set of exercises that are graded roughly into three categories. Essay questions that are intended to indicate the reader's ability to organize and put into his own words the material discussed in the chapter are listed first. Then follow relatively easy exercises that require the proofs of some of the theorems mentioned in the chapter or their application to varied situations. Finally, there are a few starred exercises, which are either more difficult or require specialized methods that may not be familiar to all students or all readers. The exercises do not in any way form part of the general exposition and can be disregarded by the reader without loss of continuity.

Here and there in the text are references to footnotes, generally bibliographical, and following each chapter there is a list of suggested readings. Included are some references to the vast periodical literature in the field, for it is not too early for students at this level to be introduced to the wealth of material available in good libraries. Smaller college libraries may not be able to provide all of these sources, but it is well for a student to be aware of the larger realms of scholarship beyond the confines of his own campus. There are references also to works in foreign languages, despite the fact that some students, hopefully not many, may be unable to read any of these. Besides providing important additional sources for those who have a reading knowledge of a foreign language, the inclusion of references in other languages may help to break down the linguistic provincialism which, ostrichlike, takes refuge in the mistaken impression that everything worthwhile appeared in, or has been translated into, the English language.

The present work differs from the most successful presently available textbook in a stricter adherence to the chronological arrangement and a stronger emphasis on historical elements. There is always the temptation in a class in history of mathematics to assume that the fundamental purpose of the course is to teach mathematics. A departure from mathematical standards is then a mortal sin, whereas an error in history is venial. I have striven to avoid such an attitude, and the purpose of the

[6]New York: Dover Publications, 1967, 3rd edition.

book is to present the history of mathematics with fidelity, not only to mathematical structure and exactitude, but also to historical perspective and detail. It would be folly, in a book of this scope, to expect that every date, as well as every decimal point, is correct. It is hoped, however, that such inadvertencies as may survive beyond the stage of page proof will not do violence to the sense of history, broadly understood, or to a sound view of mathematical concepts. It cannot be too strongly emphasized that this single volume in no way purports to present the history of mathematics in its entirety. Such an enterprise would call for the concerted effort of a team, similar to that which produced the fourth volume of Cantor's *Vorlesungen über Geschichte der Mathematik* in 1908 and brought the story down to 1799. In a work of modest scope the author must exercise judgment in the selection of the materials to be included, reluctantly restraining the temptation to cite the work of every productive mathematician; it will be an exceptional reader who will not note here what he regards as unconscionable omissions. In particular, the last chapter attempts merely to point out a few of the salient characteristics of the twentieth century. In the field of the history of mathematics perhaps nothing is more to be desired than that there should appear a latter-day Felix Klein who would complete for our century the type of project Klein essayed for the nineteenth century, but did not live to finish.

A published work is to some extent like an iceberg, for what is visible constitutes only a small fraction of the whole. No book appears until the author has lavished time on it unstintingly and unless he has received encouragement and support from others too numerous to be named individually. Indebtedness in my case begins with the many eager students to whom I have taught the history of mathematics, primarily at Brooklyn College, but also at Yeshiva University, the University of Michigan, the University of California (Berkeley), and the University of Kansas. At the University of Michigan, chiefly through the encouragement of Professor Phillip S. Jones, and at Brooklyn College through the assistance of Dean Walter H. Mais and Professors Samuel Borofsky and James Singer, I have on occasion enjoyed a reduction in teaching load in order to work on the manuscript of this book. Friends and colleagues in the field of the history of mathematics, including Professor Dirk J. Struik of the Massachusetts Institute of Technology, Professor Kenneth O. May at the University of Toronto, Professor Howard Eves of the University of Maine, and Professor Morris Kline at New York University, have made many helpful suggestions in the preparation of the book, and these have been greatly appreciated. Materials in the books and articles of others have been expropriated freely, with little acknowledgment beyond a cold bibliographical reference, and I take this opportunity to express to these authors my warmest gratitude. Libraries and publishers have been very helpful in providing information and

illustrations needed in the text; in particular it has been a pleasure to have worked with the staff of John Wiley & Sons. The typing of the final copy, as well as of much of the difficult preliminary manuscript, was done cheerfully and with painstaking care by Mrs. Hazel Stanley of Lawrence, Kansas. Finally, I must express deep gratitude to a very understanding wife. Dr. Marjorie N. Boyer, for her patience in tolerating disruptions occasioned by the development of yet another book within the family.

—Carl B. Boyer
Brooklyn, New York
January 1968

Contents