

# ELECTRICITY AND MAGNETISM

UNIVERSITY PHYSICS SERIES

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BERNHARD KURRELMEYER

*Professor of Physics  
Brooklyn College of  
The City University of New York*

and

WALTER H. MAIS

*Professor of Physics  
Brooklyn College of  
The City University of New York*

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**ELECTRICITY  
AND  
MAGNETISM**

# UNIVERSITY PHYSICS SERIES

A Group of Textbooks for Intermediate and Advanced Courses

WALTER C. MICHELS, Ph.D.

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Dedicated to  
Lucy Hayner Kurrelmeyer  
and  
Mildred L. Mais

## PREFACE

This book is intended as a college text at the sophomore or junior level, for students who have had the equivalent of a year of calculus, a year of introductory physics, and a semester of intermediate mechanics. We believe that the selection of topics is appropriate for students intending to specialize in any branch of science or engineering. The emphasis is on the fundamental phenomena and on the principles which have been deduced from them. A certain amount of space has been devoted to the more important applications.

Most of the equations are written in such a way that they are valid in any system of units, provided that appropriate numerical values are assigned to the constant  $\epsilon_0$  appearing in Coulomb's law and to the constant  $\mu_0$  appearing in Ampère's law. We have used the rationalized MKSA system of units almost exclusively. A set of conversion factors to other units, and a table showing the dimensions of all the more commonly used electrical and magnetic quantities, are given in Appendix 5.

The first chapter contains a brief review of the basic experimental facts, and is followed by several chapters dealing with dc circuits. We have found this arrangement of material to be superior pedagogically to the historical approach beginning with electrostatics, for two reasons. First, students are in general more familiar with circuits and with the concepts involved, so that they acquire some much-needed confidence. Second, the laboratory work, which normally begins with dc measurements, proceeds much more smoothly with this introduction.

If it is desired to begin with electrostatics, the text assignments can begin with Chapter 5, where all the necessary definitions are given, leaving the first four chapters as reading assignments for laboratory work.

The course in electricity and magnetism is usually one of the most difficult in the undergraduate physics sequence, even for superior students. We have therefore made the presentation very detailed at the start, at the risk of repeating material covered in the introductory course. Students with good preparation can absorb this part of the book very rapidly. In later chapters the pace is gradually stepped up. Vector analysis is used sparingly at the start, and then increasingly as the

work progresses. All the necessary definitions and formulas of vector analysis are collected in Appendix 3.

It has been our experience that even the superior student has difficulty with the large number of new concepts appearing in this subject. We have therefore followed the rule of introducing a new concept only when it is really needed, that is, when the failure to introduce it would result in awkward formulas or extensive circumlocution. For example, as long as we are dealing with fields in vacuum, the vectors **E** and **B** are sufficient; the vectors **D** and **H** are not even mentioned until we come to phenomena in material media.

The book may be used for a course of either one or two semesters. At Brooklyn College, in a course of one semester with four class hours per week plus laboratory, we aim to cover the content of Chapters 1 through 14 and 17, either in the lectures or in the laboratory. Chapters 15, 16, and 18 provide reference material for later courses in electronics and advanced laboratory work in atomic physics.

Problems have been constructed with several objectives in view. Some are intended to promote familiarity with the most important results, and with the orders of magnitude of physical quantities occurring in typical processes and situations. These usually involve numerical substitution into formulas. Some involve derivations not given in detail in the text. Still others involve application of principles to new situations. They are intended to test comprehension of the principles and of the limitations under which they apply.

For the legion of students who still have difficulty with arithmetic, a special feature will be Appendix 2, "Mental Arithmetic, When and How."

We have given frequent references to the literature, especially where the content touches on recent developments. These can be used as the starting point for independent study projects.

We wish to express our gratitude to Professor Walter C. Michels, who as Editor of the University Series in Physics, offered initial encouragement and much helpful advice on scope and content during several revisions of the text. We would also like to acknowledge the helpful cooperation of the staff of D. Van Nostrand Co.

B. KURRELMEYER  
WALTER H. MAIS



# CONTENTS

## 1. BASIC FACTS, LAWS, AND DEFINITIONS

Introduction	1
1.1 Charging by Contact	1
1.2 Conductors and Insulators	2
1.3 Heating by Charges in Motion	3
1.4 Definitions of Quantity of Charge, Current, and Current Density	4
1.5 Definition of Potential Difference	5
1.6 Ohm's Law and Joule's Law: Definition of Resistance	9
1.7 Magnetic Effects of Charges in Motion	10
1.8 Electromagnetic Induction	10
1.9 Electrical Units	12
1.10 Dimensions of Electrical Quantities	14
References	15

## 2. SIMPLE ELECTRICAL CIRCUITS AND CIRCUIT ELEMENTS

Introduction	16
2.1 Steady Electric Currents	16
2.2 Series Circuit	17
2.3 Parallel Connection of Several Conductors	21
2.4 Resistivity	23
2.5 Temperature Variation of Resistance	24
2.6 The Potential Divider	26
2.7 The Current Divider or Shunt	27
References	28
Problems	28

## 3. KIRCHHOFF'S LAWS

Introduction	33
3.1 Kirchhoff's Laws	33
3.2 Method of Mesh Currents	38

3.3	The Unbalanced Wheatstone Bridge	40
	References	42
	Problems	42

#### 4. GENERAL NETWORK THEOREMS

	Introduction	44
4.1	Kirchhoff's Laws Applied to General Networks	44
4.2	The Superposition Theorem	47
4.3	The Reciprocity Theorem	48
4.4	The Substitution Theorem	48
4.5	The Compensation Theorem	48
4.6	Thévenin's Theorem	48
4.7	The Power Transfer Theorem	50
4.8	Corollaries to the Network Theorems	50
4.9	Example of the Use of the Network Theorems	51
	References	52
	Problems	52

#### 5. POINT CHARGES

	Introduction	57
5.1	Coulomb's Law	57
5.2	Principle of Superposition	58
5.3	Electric Intensity	58
5.4	Electric Intensity Due to Point Charges	59
5.5	Potential Due to Point Charges	64
5.6	Potential Difference Around a Closed Path	67
5.7	Gradient of the Potential	69
5.8	The Electric Dipole	71
5.9	Torque on a Dipole	73
5.10	Potential Energy of a Dipole	74
5.11	Force on a Dipole in an Inhomogeneous Field	75
5.12	Potential Due to an Assemblage of Charges	76
	References	77
	Problems	78

#### 6. EXTENDED CHARGE DISTRIBUTIONS

	Introduction	83
6.1	Conductors	83
6.2	Charging by Induction	84
6.3	Coordinate Systems	86
6.4	Solid Angle	88

6.5	Gauss' Theorem	90
6.6	Electric Intensity Due to a Solid or Hollow Charged Conducting Sphere	92
6.7	Concentric Spheres	94
6.8	Infinite Plane Sheet of Charge	94
6.9	Two Infinite Plane Sheets	97
6.10	Parallel Infinite Plane Conductors	97
6.11	Electric Intensity Due to an Infinite Wire	98
6.12	Electric Intensity Due to Coaxial Cylinders	99
6.13	Solution of Electrostatics Problems	100
6.14	Method of Electrical Images	105
6.15	Capacitance and Capacitors	108
6.16	Calculation of Capacitance	112
6.17	Capacitance of Parallel Cylinders	114
6.18	Capacitors in Parallel	117
6.19	Capacitors in Series	117
6.20	Energy of a Charged Capacitor	118
6.21	Force of Attraction Between Capacitor Plates	119
6.22	Force on a Charged Conducting Surface	120
6.23	Energy Density in the Electric Field	121
	References	123
	Problems	123

## 7. VOLUME DISTRIBUTIONS OF CHARGE; DIELECTRICS

	Introduction	130
7.1	Atoms and Molecules Subjected to an Electric Field	131
7.2	Macroscopic Theory of Dielectrics: The Dielectric Constant	134
7.3	Dielectric Constants of Gases, Liquids, and Solids	142
7.4	Boundary Conditions on $\mathbf{D}$ and $\mathbf{E}$	144
7.5	Application of the Boundary Conditions	148
7.6	$\mathbf{E}$ and $\mathbf{D}$ inside Dielectric Cavities	151
7.7	Local Field in a Dielectric	152
7.8	Energy Density in a Dielectric	154
	References	155
	Problems	156

## 8. THE MAGNETIC FORCES BETWEEN STEADY CURRENTS

	Introduction	159
8.1	The Experiments of Ampère	160
8.2	The Magnetic Induction Vector $\mathbf{B}$	165

8.3	Magnetic Forces Acting on Charges Moving in Vacuum	167
8.4	Magnetic Induction Due to a Circular Loop of Current in Vacuum	170
8.5	Magnetic Induction Due to a Uniform Helix or Solenoid	175
8.6	Magnetic Induction Due to a Long Straight Wire	177
8.7	Torque on a Current Loop Due to an External Uniform $\mathbf{B}$	179
	References	182
	Problems	183

## 9. CONSEQUENCES OF AMPÈRE'S LAW

	Introduction	186
9.1	The Divergence of $\mathbf{B}$	186
9.2	Ampère's Circuital Law	190
9.3	The Magnetic Scalar Potential	197
9.4	The Concept of Magnetic Flux	202
9.5	Energy of a Current Loop in an External Magnetic Field	204
	References	207
	Problems	207

## 10. INDUCED ELECTROMOTIVE FORCES

	Introduction	209
10.1	The Experiments of Faraday and of Henry	210
10.2	Emf Induced in a Moving Wire	212
10.3	Emf Induced in a Rotating Rectangular Coil	215
10.4	Electromagnetic Damping of a Moving-Coil Galvanometer	217
10.5	Self-Inductance of a Circuit	223
10.6	Mutual Inductance of Two Circuits	227
10.7	Energy of Currents in Inductive Circuits	229
10.8	Absolute Determination of the Ampere	234
10.9	Absolute Determination of the Ohm	235
10.10	Measurement of Magnetic Induction, Flux, and Inductance	236
	References	238
	Problems	239

## 11. MAGNETIC MATERIALS

	Introduction	242
11.1	The Electronic Origins of Magnetic Properties	243
11.2	Macroscopic Theory of Magnetic Materials	244

11.3	Magnetic Susceptibility	249
11.4	Boundary Conditions in Magnetic Media	250
11.5	Energy Density in a Magnetic Medium	251
11.6	Forces on Magnetized Materials	254
11.7	Diamagnetism	256
11.8	Paramagnetism	259
11.9	Magnetic Resonance	263
	References	265
	Problems	265

## 12. FERROMAGNETISM

	Introduction	268
12.1	Saturation Magnetization	269
12.2	The Magnetization Curve and the Hysteresis Curve	273
12.3	The Nature of the Atomic Moments	279
12.4	Ferromagnetic Domains	279
12.5	High-Permeability Materials	285
12.6	Permanent Magnet Materials	287
12.7	Magnetic Circuits	290
12.8	Ferrimagnetism and Antiferromagnetism: Ferrites	292
	References	294
	Problems	295

## 13. TRANSIENT CURRENTS

	Introduction	298
13.1	Linear Differential Equations	298
13.2	Transients in a $C, R$ Circuit	300
13.3	Transients in an $L, R$ Circuit	303
13.4	Transients in an $L, C$ Circuit	304
13.5	Transients in an $L, C, R$ Circuit	305
13.6	Transients in Two $L, R$ Circuits with Mutual Inductance	310
13.7	Finite Sawtooth Voltage Pulse	313
	References	315
	Problems	316

## 14. ALTERNATING CURRENTS

	Introduction	319
14.1	Series Circuit of Inductance, Capacitance, and Resistance	319
14.2	Special Cases of Series $L, C, R$ Circuits	321

14.3	Voltage Relations in the Series $L, C, R$ Circuit: The Vector Diagram	323
14.4	Power and Power Factor	325
14.5	Series Resonance	327
14.6	Transients in the Series $L, C, R$ Circuit	331
14.7	Complex Quantities	332
14.8	The Series $L, C, R$ Circuit in Complex Notation	334
14.9	Two Parallel Impedances	337
14.10	Parallel Resonance	338
14.11	Non-Sinusoidal Voltage and Current	342
	References	344
	Problems	345
15. INDUCTIVELY COUPLED CIRCUITS AND TRANSFORMERS		
	Introduction	351
15.1	Air-Core Transformer	352
15.2	Identical Tuned Coupled Circuits: Frequency Variation	357
15.3	The Iron-Core Inductance	362
15.4	The Ideal Transformer	367
15.5	The Iron-Core Transformer	368
15.6	Equivalent Circuit for Simple Iron-Core Transformer	373
15.7	Power Transfer Theorem	373
	References	375
	Problems	376
16. TRANSMISSION LINES		
	Introduction	380
16.1	Transmission Line with Lumped Constants	380
16.2	Uniform Transmission Line	382
16.3	Voltage and Current Distribution on a Long Uniform Line	387
16.4	Finite Line Terminated in its Characteristic Impedance	389
	References	390
	Problems	390
17. MAXWELL'S EQUATIONS; ELECTROMAGNETIC WAVES		
	Introduction	392
17.1	Generalization of Ampère's Circuital Law: Displacement Current	393
17.2	Maxwell's Equations	395

17.3	Plane Wave in Empty Space	398
17.4	Poynting's Vector: Transfer of Energy	400
17.5	Plane Waves in Isotropic Dielectrics	404
17.6	Plane Waves in Conducting Media	405
17.7	Reflection and Refraction at Normal Incidence	407
17.8	Oblique Incidence at a Dielectric Surface	410
17.9	Oblique Incidence on a Perfect Conductor	416
17.10	Wave Guides	423
	References	426
	Problems	427

## 18. MOTION OF CHARGED PARTICLES IN ELECTRIC AND MAGNETIC FIELDS

	Introduction	431
18.1	Motion in Static Electric Fields	432
18.2	Alternating Electric Fields	438
18.3	Uniform Magnetic Field	439
18.4	Non-Uniform Magnetic Field	442
18.5	Larmor's Theorem	445
18.6	Combined Electric and Magnetic Fields	446
	References	449
	Problems	450

## APPENDIX 1. MATHEMATICAL FORMULAS

1.1	Trigonometric and Related Formulas	454
1.2	Series	454
1.3	Determinants	454
1.4	Solid-Angle Relations	457

## APPENDIX 2. MENTAL ARITHMETIC: WHEN AND HOW

2.1	What Accuracy is Required?	459
2.2	What Calculation Techniques are Available?	459
2.3	Equipment Needed for Approximate Mental Calculation	460
2.4	Special Techniques in Multiplication and Division	460
2.5	Estimation of Errors in Mental Arithmetic	461

## APPENDIX 3. VECTOR ANALYSIS

	Introduction	463
3.1	Coordinate Systems and Unit Vectors	463

3.2	Scalar Product of Two Vectors	465
3.3	Vector Product of Two Vectors	466
3.4	Differentiation and Integration of Vectors	468
3.5	Gradient of a Scalar	469
3.6	Divergence of a Vector	471
3.7	The Curl of a Vector	475
3.8	Energy Density in Electrostatic and Magnetostatic Fields	478
3.9	Summary of Principal Formulas	482
	References	486
	Problems	486

#### APPENDIX 4. SELECTED CONSTANTS AND VALUES

4.1	Selected Physical Constants	488
4.2	Electrical Resistivity of Some Elements and Alloys	489

#### APPENDIX 5. UNITS AND DIMENSIONS

 490

INDEX	495
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# CHAPTER I

## BASIC FACTS, LAWS, AND DEFINITIONS

### Introduction

In this chapter we give a brief review of the basic experimental facts of electricity and magnetism. The definitions of electrical quantities and units are given in terms of these facts. The definitions are the link between operations of measurement and the operations of mathematical character which form the foundations of electromagnetic theory.

### 1.1 Charging by Contact

Phenomena that are typically electrical are observable through mechanical, thermal, chemical, or optical effects which appear when matter is brought into certain definite conditions by special procedures. One very simple electrical phenomenon, charging by contact, is detected by mechanical means. Two bodies A and B, of different materials, when brought into close contact and then separated, will be found to exert forces on each other. The forces are attractions, are many times larger than the force of gravitation between the two bodies, and act over considerable distances.

Let us prepare two identical specimens of material A, labeled  $A_1$  and  $A_2$ , and also two specimens  $B_1$  and  $B_2$ . Suppose  $A_1$  is first brought into contact with  $B_1$ , while  $A_2$  is brought into contact with  $B_2$ . Then the two A's will be found to repel each other, and the two B's will also repel each other, while either A will attract either B. We can now try specimens of the material A of different shapes and sizes, and we can try other pairs of materials. The results of all these experiments can be described by a single statement: The materials that attract the original A will repel the original B, and vice versa.

These and other similar experiments are interpreted to mean that any kind of matter contains particles of two kinds, which are normally present in such numbers that they counteract or *neutralize* each other as far as externally observable effects are concerned. All these particles are said to carry electrical charges. Our picture of charging by contact is that a certain number of particles of the one kind move from A to B, leaving behind an equivalent number of the other kind in excess on A. In agreement with this view is the fact that if A and B, after being