Dynamic Modeling and Control of Engineering Systems

THIRD EDITION

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DYNAMIC MODELING AND CONTROL OF ENGINEERING SYSTEMS

THIRD EDITION

This textbook is ideal for a course in Engineering System Dynamics and Controls. The work is a comprehensive treatment of the analysis of lumped-parameter physical systems. Starting with a discussion of mathematical models in general, and ordinary differential equations, the book covers input–output and state-space models, computer simulation, and modeling methods and techniques in mechanical, electrical, thermal, and fluid domains. Frequency-domain methods, transfer functions, and frequency response are covered in detail. The book concludes with a treatment of stability, feedback control (PID, lag–lead, root locus), and an introduction to discrete-time systems. This new edition features many new and expanded sections on such topics as Solving Stiff Systems, Operational Amplifiers, Electrohydraulic Servovalves, Using MATLAB® with Transfer Functions, Using MATLAB with Frequency Response, MATLAB Tutorial, and an expanded Simulink® Tutorial. The work has 40 percent more end-of-chapter exercises and 30 percent more examples.

Bohdan T. Kulakowski, Ph.D. (1942–2006) was Professor of Mechanical Engineering at Pennsylvania State University. He was an internationally recognized expert in automatic control systems, computer simulations and control of industrial processes, systems dynamics, vehicle–road dynamic interaction, and transportation systems. His fuzzy-logic algorithm for avoiding skidding accidents was recognized in 2000 by *Discover* magazine as one of its top 10 technological innovations of the year.

John F. Gardner is Chair of the Mechanical and Biomedical Engineering Department at Boise State University, where he has been a faculty member since 2000. Before his appointment at Boise State, he was on the faculty of Pennsylvania State University in University Park, where his research in dynamic systems and controls led to publications in diverse fields from railroad freight car dynamics to adaptive control of artificial hearts. He pursues research in modeling and control of engineering and biological systems.

J. Lowen Shearer (1921–1992) received his Sc.D. from the Massachusetts Institute of Technology. At MIT, between 1950 and 1963, he served as the group leader in the Dynamic Analysis & Control Laboratory, and as a member of the mechanical engineering faculty. From 1963 until his retirement in 1985, he was on the faculty of Mechanical Engineering at Pennsylvania State University. Professor Shearer was a member of ASME's Dynamic Systems and Control Division and received that group's Rufus Oldenberger Award in 1983. In addition, he received the Donald P. Eckman Award (ISA, 1965), and the Richards Memorial Award (ASME, 1966).

Dedicated to the memories of Professor Bohdan T. Kulakowski (1942–2006), the victims of the April 16, 2007 shootings at Virginia Tech, and all who are touched by senseless violence. May we never forget and always strive to learn form history.

Preface

From its beginnings in the middle of the 20th century, the field of systems dynamics and feedback control has rapidly become both a core science for mathematicians and engineers and a remarkably mature field of study. As early as 20 years ago, textbooks (and professors) could be found that purported astoundingly different and widely varying approaches and tools for this field. From block diagrams to signal flow graphs and bond graphs, the diversity of approaches, and the passion with which they were defended (or attacked), made any meeting of systems and control professionals a lively event.

Although the various tools of the field still exist, there appears to be a consensus forming that the tools are secondary to the insight they provide. The field of system dynamics is nothing short of a unique, useful, and utterly different way of looking at natural and manmade systems. With this in mind, this text takes a rather neutral approach to the tools of the field, instead emphasizing insight into the underlying physics and the similarity of those physical effects across the various domains.

This book has its roots as lecture notes from Lowen Shearer's senior-level mechanical engineering course at Penn State in the 1970s with additions from Bohdan Kulakowski's and John Gardner's experiences since the 1980s. As such, it reveals those roots by beginning with lumped-parameter mechanical systems, engaging the student on familiar ground. The following chapters, dealing with types of models (Chapter 3) and analytical solutions (Chapter 4), have seen only minimal revisions from the original version of this text, with the exception of modest changes in order of presentation and clarification of notation. Chapters 5 and 6, dealing with numerical solutions (simulations), were extensively rewritten for the second edition and further updated for this edition. Although we made a decision to feature the industry-standard software package (MATLAB®) in this book (Appendices 3 and 4 are tutorials on MATLAB and Simulink®), the presentation was specifically designed to allow other software tools to be used.

Chapters 7, 8, and 9 are domain-specific presentations of electric, thermal, and fluid systems, respectively. For the third edition, these chapters have been extensively expanded, including operational amplifiers in Chapter 7, an example of lumped approximation of a cooling fin in Chapter 8, and an electrohydraulic servovalve in Chapter 9. Those using this text in a multidisciplinary setting, or for nonmechanical engineering students, may wish to delay the use of Chapter 2 (mechanical systems) to this point, thus presenting the four physical domains sequentially. Chapter 10 presents some important issues in dealing with multidomain systems and how they interact.

Chapters 11 and 12 introduce the important concept of a transfer function and frequency-domain analysis. These two chapters are the most revised and (hopefully) improved parts of the text. In previous editions of this text, we derived the complex transfer function by using complex exponentials as input. For the third edition, we retain this approach, but have added a section showing how to achieve the same ends using the Laplace transform. It is hoped that this dual approach will enrich student understanding of this material. In approaching these, and other, revisions, we listened carefully to our colleagues throughout the world who helped us see where the presentation could be improved. We are particularly grateful to Sean Brennan (of Penn State) and Giorgio Rizzoni (of Ohio State) for their insightful comments.

This text, and the course that gave rise to it, is intended to be a prerequisite to a semester-long course in control systems. However, Chapters 13 and 14 present a very brief discussion of the fundamental concepts in feedback control, stability (and algebraic and numerical stability techniques), closed-loop performance, and PID and simple cascade controllers. Similarly, the preponderance of digitally implemented control schemes necessitates a discussion of discrete-time control and the dynamic effects inherent in sampling in the final chapters (15 and 16). It is hoped that these four chapters will be useful both for students who are continuing their studies in electives or graduate school and for those for which this is a terminal course of study.

Supplementary materials, including MATLAB and Simulink files for examples throughout the text, are available through the Cambridge University Press web site (http://www.cambridge.org/us/engineering) and readers are encouraged to check back often as updates and additional case studies are made available.

Outcomes assessment, at the program and course level, has now become a fixture of engineering programs. Although necessitated by accreditation criteria, many have discovered that an educational approach based on clearly stated learning objectives and well-designed assessment methods can lead to a better educational experience for both the student and the instructor. In the third edition, we open each chapter with the learning objectives that underlie each chapter. Also in this edition, the examples and end-of-chapter problems, many of which are based on real-world systems encountered by the authors, were expanded.

This preface closes on a sad note. In March of 2006, just as the final touches were being put on this edition, Bohdan Kulakowski was suddenly and tragically taken from us while riding his bicycle home from the Penn State campus, as was his daily habit. His family, friends, and the entire engineering community suffered a great loss, but Bohdan's legacy lives on in these pages, as does Lowen's. As the steward of this legacy, I find myself "standing on the shoulders of giants" and can take credit only for its shortcomings.

JFG
Boise, ID
May, 2007

DYNAMIC MODELING AND CONTROL OF ENGINEERING SYSTEMS

Contents

1 Introduction

LEARNING OBJECTIVES FOR THIS CHAPTER

1–1 To work comfortably with the engineering concept of a "system" and its interaction with the environment through inputs and outputs.

1–2 To distinguish among various types of mathematical models used to represent and predict the behavior of systems.

1–3 To recognize through (T-type) variables and across (A-type) variables when examining energy transfer within a system.

1–4 To recognize analogs between corresponding energy-storage and energy-dissipation elements in different types of dynamic systems.

1–5 To understand the key role of energy-storage processes in system dynamics.

1.1 SYSTEMS AND SYSTEM MODELS

The word "system" has become very popular in recent years. It is used not only in engineering but also in science, economics, sociology, and even in politics. In spite of its common use (or perhaps because of it), the exact meaning of the term is not always fully understood. A system is defined as a combination of components that act together to perform a certain objective. A little more philosophically, a system can be understood as a conceptually isolated part of the universe that is of interest to us. Other parts of the universe that interact with the system comprise the system environment, or neighboring systems.

All existing systems change with time, and when the rates of change are significant, the systems are referred to as dynamic systems. A car riding over a road can be considered as a dynamic system (especially on a crooked or bumpy road). The limits of the conceptual isolation determining a system are entirely arbitrary. Therefore any part of the car given as an example of a system – its engine, brakes, suspension, etc. – can also be considered a system (i.e., a subsystem). Similarly, two cars in a passing maneuver or even all vehicles within a specified area can be considered as a major traffic system.

The isolation of a system from the environment is purely conceptual. Every system interacts with its environment through two groups of variables. The variables in the first group originate outside the system and are not directly dependent on what happens in the system. These variables are called input variables, or simply inputs. The other group comprises variables generated by the system as it interacts with its

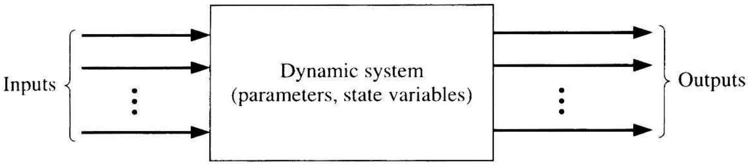

Figure 1.1. A dynamic system.

environment. Those dependent variables in this group that are of primary interest to us are called output variables, or simply outputs.

In describing the system itself, one needs a complete set of variables, called state variables. The state variables constitute the minimum set of system variables necessary to describe completely the state of the system at any given instant of time; and they are of great importance in the modeling and analysis of dynamic systems. Provided the initial state and the input variables have all been specified, the state variables then describe from instant to instant the behavior, or response, of the system. The concept of the state of a dynamic system is discussed in more detail in Chap. 3. In most cases, the state-variable equations used in this text represent only simplified models of the systems, and their use leads to only approximate predictions of system behavior.

Figure 1.1 shows a graphical presentation of a dynamic system. In addition to the state variables, parameters also characterize the system. In the example of the moving car, the input variables would include throttle position, position of the steering wheel, and road conditions such as slope and roughness. In the simplest model, the state variables would be the position and velocity of the vehicle as it travels along a straight path. The choice of the output variables is arbitrary, determined by the objectives of the analysis. The position, velocity, or acceleration of the car, or perhaps the average fuel flow rate or the engine temperature, can be selected as the output(s). Some of the system parameters would be the mass of the vehicle and the size of its engine. Note that the system parameters may change with time. For instance, the mass of the car will change as the amount of fuel in its tank increases or decreases or when passengers embark or disembark. Changes in mass may or may not be negligible for the performance of a car but would certainly be of critical importance in the analysis of the dynamics of a ballistic missile.

The main objective of system analysis is to predict the manner in which a system will respond to various inputs and how that response changes with different system parameter values. In the absence of the tools introduced in this book, engineers are often forced to build prototype systems to test them. Whereas the data obtained from the testing of physical prototypes are very valuable, the costs, in time and money, of obtaining these data can be prohibitive. Moreover, mathematical models are inherently more flexible than physical prototypes and allow for rapid refinement of system designs to optimize various performance measures. Therefore one of the early major tasks in system analysis is to establish an adequate mathematical model that can be used to gain the equivalent information that would come from several different physical prototypes. In this way, even if a final prototype is built to verify the mathematical model, the modeler has still saved significant time and expense.

A mathematical model is a set of equations that completely describes the relationships among the system variables. It is used as a tool in developing designs or control algorithms, and the major task for which it is to be used has basic implications for the choice of a particular form of the system model.

In other words, if a model can be considered a tool, it is a specialized tool, developed specifically for a particular application. Constructing universal mathematical models, even for systems of moderate complexity, is impractical and uneconomical. Let us use the moving automobile as an example once again. The task of developing a model general enough to allow for studies of ride quality, fuel economy, traction characteristics, passenger safety, and forces exerted on the road pavement (to name just a few problems typical for transportation systems) could be compared to the task of designing one vehicle to be used as a truck, for daily commuting to work in New York City, and as a racing car to compete in the Indianapolis 500. Moreover, even if such a supermodel were developed and made available to researchers (free), it is very likely that the cost of using it for most applications would be prohibitive.

Thus, system models should be as simple as possible, and each model should be developed with a specific application in mind. Of course, this approach may lead to different models being built for different uses of the same system. In the case of mathematical models, different types of equations may be used in describing the system in various applications.

Mathematical models can be grouped according to several different criteria. Table 1.1 classifies system models according to the four most common criteria: applicability of the principle of superposition, dependence on spatial coordinates as well

Table 1.1. Classification of system models

Type of model	Classification criterion	Type of model equation
Nonlinear	Principle of superposition does not apply	Nonlinear differential equations
Linear	Principle of superposition applies	Linear differential equations
Distributed	Dependent variables are functions of spatial coordinates and time	Partial differential equations
Lumped	Dependent variables are independent of spatial coordinates	Ordinary differential equations
Time-varying	Model parameters vary in time	Differential equations with time-varying coefficients
Stationary	Model parameters are constant in time	Differential equations with constant coefficients
Continuous	Dependent variables defined over continuous range of independent variables	Differential equations
Discrete	Dependent variables defined only for distinct values of independent variables	Time-difference equations

as on time, variability of parameters in time, and continuity of independent variables. Based on these criteria, models of dynamic systems are classified as linear or nonlinear, lumped or distributed, stationary time invariant or time varying, continuous or discrete, respectively. Each class of models is also characterized by the type of mathematical equations employed in describing the system. All types of system models listed in Table 1.1 are discussed in this book, although distributed models are given only limited attention.

1.2 SYSTEM ELEMENTS, THEIR CHARACTERISTICS, AND THE ROLE OF INTEGRATION

The modeling techniques developed in this text focus initially on the use of a set of simple ideal system elements found in four main types of systems: mechanical, electrical, fluid, and thermal. Transducers, which enable the coupling of these types of system to create mixed-system models, will be introduced later.

This set of ideal linear elements is shown in Table 1.2, which also provides their elemental equations and, in the case of energy-storing elements, their energy-storage equations in simplified form. The variables, such as force F and velocity v used in mechanical systems, current i and voltage e in electrical systems, fluid flow rate Q_f and pressure P in fluid systems, and heat flow rate Q_h and temperature T in thermal systems, have also been classified as either T-type (through) variables, which act through the elements, or A-type (across) variables, which act across the elements. Thus force, current, fluid flow rate, and heat flow rate are called T variables, and velocity, voltage, pressure, and temperature are called A variables. Note that these designations also correspond to the manner in which each variable is measured in a physical system. An instrument measuring a T variable is used in series to measure what goes *through* the element. On the other hand, an instrument measuring an A variable is connected in parallel to measure the difference *across* the element. Furthermore, the energy-storing elements are also classified as T-type or A-type elements, designated by the nature of their respective energy-storage equations: for example, mass stores kinetic energy, which is a function of its velocity, an A variable; hence mass is an A-type element. Note that although T and A variables have been identified for each type of system in Table 1.2, both T-type and A-type energy-storing elements are identified in mechanical, electrical, and fluid systems only. In thermal systems, the A-type element is the thermal capacitor but there is no T-type element that would be capable of storing energy by virtue of a heat flow through the element.

In developing mathematical models of dynamic systems, it is very important not only to identify all energy-storing elements in the system but also to determine how many energy-storing elements are independent or, in other words, in how many elements the process of energy storage is independent. The energy storage in an element is considered to be independent if it can be given any arbitrary value without changing any previously established energy storage in other system elements. To put it simply, two energy-storing elements are not independent if the amount of energy stored in one element completely determines the amount of energy stored in the other element. Examples of energy-storing elements that are not independent are rack-and-pinion gears, and series and parallel combinations of springs, capacitors, inductors,

Table 1.2. Ideal system elements (linear)

System type	Mechanical translational	Mechancial rotational	Electrical	Fluid	Thermal
A-type variable	Velocity, v	Velocity, Ω	Voltage, e	Pressure, P	Temperature, T
A-type element	Mass, m	Mass moment of inertia, J	Capacitor, C	Fluid Capacitor, C_f	Thermal capacitor, C_h
Elemental equations	$F = m\frac{dv}{dt}$	$T = J\frac{d\Omega}{dt}$	$i = C\frac{de}{dt}$	$Q_f = C_f\frac{dP}{dt}$	$Q_h = C_h\frac{dT}{dt}$
Energy stored	Kinetic	Kinetic	Electric field	Potential	Thermal
Energy equations	$\mathscr{E}_k = \frac{1}{2}mv^2$	$\mathscr{E}_k = \frac{1}{2}J\Omega^2$	$\mathscr{E}_e = \frac{1}{2}Ce^2$	$\mathscr{E}_p = \frac{1}{2}C_f P^2$	$\mathscr{E}_t = \frac{1}{2}C_h T^2$
T-type variable	Force, F	Torque, T	Current, i	Fluid flow rate, Q_f	Heat flow rate, Q_h
T-type element	Compliance, $1/k$	Compliance, $1/K$	Inductor, L	Inertor, I	None
Elemental equations	$v = \frac{1}{k}\frac{dF}{dt}$	$\Omega = \frac{1}{K}\frac{dT}{dt}$	$e = L\frac{di}{dt}$	$P = I\frac{dQ_f}{dt}$	
Energy stored	Potential	Potential	Magnetic field	Kinetic	
Energy equations	$\mathscr{E}_P = \frac{1}{2k}F^2$	$\mathscr{E}_P = \frac{1}{2K}T^2$	$\mathscr{E}_m = \frac{1}{2}Li^2$	$\mathscr{E}_k = \frac{1}{2}IQ_f^2$	
D-type element	Damper, b	Rotational damper, B	Resistor, R	Fluid resistor, R_f	Thermal resistor, R_h
Elemental equations	$F = bv$	$T = B\Omega$	$i = \frac{1}{R}e$	$Q_f = \frac{1}{R_f}P$	$Q_h = \frac{1}{R_h}T$
Rate of energy dissipated	$\frac{dE_D}{dt} = Fv = \frac{1}{b}F^2 = bv^2$	$\frac{dE_D}{dt} = T\Omega = \frac{1}{B}T^2 = B\Omega^2$	$\frac{dE_D}{dt} = ie = Ri^2 = \frac{1}{R}e^2$	$\frac{dE_D}{dt} = Q_f P = R_f Q_f^2 = \frac{1}{R_f}P^2$	$\frac{dE_D}{dt} = Q_h$

Note: A-type variable represents a spatial difference across the element.