New Developments in Mathematical Physics

Edited by
Heinrich Mitter and Ludwig Pittner

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SEMINARS

FOREWORD

The papers contained in this volume are lectures and seminars presented at the 20th "Universitätswochen für Kernphysik" in Schladming in February 1981. The goal of this school was to review some rapidly developing branches in mathematical physics. Thanks to the generous support provided by the Austrian Federal Ministry of Science and Research, the Styrian Government and other sponsors, it has been possible to keep up with the - by now already traditional - standards of this school. The lecture notes have been reexamined by the authors after the school and are now published in their final form, so that a larger number of physicists may profit from them. Because of necessary limitations in space all details connected with the meeting have been omitted and only brief outlines of the seminars were included. It is a pleasure to thank all the lecturers for their efforts, which made it possible to speed up the publication. Thanks are also due to Mrs. Krenn for the careful typing of the notes.

H.Mitter L.Pittner

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CLASSICAL SCATTERING THEORY+

by

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1. INTRODUCTION

It was first recognized by Hunziker [1] that the notions of scattering theory play an important role in classical mechanics. It turned out [2] that it leads to non-trivial information for the global properties of the solutions of the classical trajectories. For instance it shows that in the three body problem there are large regions in phase space with 2n - 1 = 17 constants of motion and all trajectories in this region are homotopic to straight lines. Furthermore Wigner's [3] time delay has a simple geometrical meaning [4] for the trajectories. Recently Bollé and Osborn [5] succeeded in deriving even a classical analogue to Levinson's theorem. In these lectures I shall, following [6], show in detail how the phase shift corresponds to the generator of the S-transformation. For this purpose we define in the next section canonical coordinates for a one-, two- and three-dimensional configuration space such that this statement assumes a

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simple form. This sheds some light on how trajectories with large time delays or loopings generate resonances of the quasiclassical phase shift. In the following section we give an alternative proof of the classical form of Levinson's theorem and illustrate its subtle feature by some examples. Finally we give a simple derivation of how a Dollard's [7] change in the free motion leads to the Coulomb phase shift as generator of the classical S-transformation for a 1/r-potential.

We shall employ the following

Notations:

$$\Theta(x) \equiv \begin{cases} 1 & \text{for} \quad x > 0 \\ 0 & \text{for} \quad x < 0 \end{cases} \qquad \text{(step function).}$$

$$f \circ g(x) = f(g(x)) \qquad \text{(composition of maps).}$$

$$\sup_x f(x) = \text{least upper bound of } f.$$

$$\vec{a} \wedge \vec{b} = \text{vector product.}$$

2. THE S-TRANSFORMATION

Scattering theory investigates the asymptotic behaviour of the trajectories in phase space. Only some observables, typically functions of the momentum, will converge for $t \to \pm\infty$. This suffices to define the scattering angle

$$\Theta = \measuredangle (p_+, p_-) , \qquad p_\pm = \lim_{t\to\pm\infty} p(t) ,$$

but if the potentials decrease faster than 1/r for $r \to \infty$ additional information becomes available. Then time evolution Φ_t: $(x(0),p(0)) \to (x(t),p(t))$ though not tending to a limit for $t \to \pm\infty$ approaches the free time evolution Φ_t^o such that $\Phi_{-t} \circ \Phi_t^o$ tends to a limit in some regions $D_\pm$:

$$\Omega_\pm = \lim_{t\to\pm\infty} \Phi_{-t} \circ \Phi_t^o .$$

This implies the convergence of $\vec{x}(t) - \vec{p}(t) . t$ and allows the definition of the time delay compared with the free motion. Since Φ and Φ^o are canonical transformations the "Möller-transformations" $\Omega_\pm$ will in general be a local canonical transformation mapping the domains $D_\pm$ into ranges $R_\pm$. Closed orbits will be excluded from $R_\pm$ but in reasonable [8] cases their union with R_+ or R_- will fill almost all of phase space. The scattering transformation

$$S = \Omega_+^{-1} \circ \Omega_- = \lim_{t\to\infty} \Phi^o_{-t/2} \circ \Phi_t \circ \Phi_{-t/2} \qquad (1)$$

transforms D_- into D_+. If Φ_t^o is the free time evolution having straight lines as trajectories, S has a simple geometrical meaning. For negative $t, \Phi_{-t} \circ \Phi_t$ means that you follow the straight trajectory back for a time $-|t|$ and then continue with the actual time evolution for the same length of time. If the forces have a finite range and Φ and Φ^o coincide outside a certain region then $\Phi_{-t} \circ \Phi_t^o$ will become independent of t as soon as t leads you outside this region (Fig. 1). Then the limit is attained, Ω_- maps the straight line onto this trajectory of Φ_t which is asymptotically tangent to it. Similar arguments for Ω_+ show that S maps (Fig. 1) the straight lines tangent for $t \to -\infty$ onto the ones tangent for $t \to +\infty$. It follows from its definition that S commutes with the free time evolution: $S \circ \Phi_t^o = \Phi_t^o \circ S$. As a canonical transformation one should be able to exhibit its generator which actually is possible. We first study the special cases.