

New Developments in Mathematical Physics

**Edited by
Heinrich Mitter and Ludwig Pittner**

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New Developments in Mathematical Physics

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Heinrich Mitter and Ludwig Pittner, Graz

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Starting from quantum field theory in its “algebraic form” the past decade of research in mathematical physics has developed formulations which have unified to a large degree the structures of field theory and of classical equilibrium statistical mechanics. At the same time it became clear that the mathematical theory of (generalized) stochastic processes was destined not only to play a central role in these developments but that it would also receive important new impulses for its own further development.

In the present volume contributions by leading researchers in these fields are collected to reflect this unification. Topics treated comprise: the mathematical theory of stochastic processes and its application to quantum theory, path integrals, the “semi-classical limit”, the renormalization group and dynamical systems, normed algebras and group representations, W^* -categories and cohomology of observable-nets, applications of probabilistic concepts to field theory and statistical mechanics, as well as the investigation of specific models of interaction.

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FOREWORD

The papers contained in this volume are lectures and seminars presented at the 20th "Universitätswochen für Kernphysik" in Schladming in February 1981. The goal of this school was to review some rapidly developing branches in mathematical physics. Thanks to the generous support provided by the Austrian Federal Ministry of Science and Research, the Styrian Government and other sponsors, it has been possible to keep up with the - by now already traditional - standards of this school. The lecture notes have been reexamined by the authors after the school and are now published in their final form, so that a larger number of physicists may profit from them. Because of necessary limitations in space all details connected with the meeting have been omitted and only brief outlines of the seminars were included. It is a pleasure to thank all the lecturers for their efforts, which made it possible to speed up the publication. Thanks are also due to Mrs. Krenn for the careful typing of the notes.

H.Mitter

L.Pittner

CLASSICAL SCATTERING THEORY⁺

by

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1. INTRODUCTION

It was first recognized by Hunziker [1] that the notions of scattering theory play an important role in classical mechanics. It turned out [2] that it leads to non-trivial information for the global properties of the solutions of the classical trajectories. For instance it shows that in the three body problem there are large regions in phase space with $2n - 1 = 17$ constants of motion and all trajectories in this region are homotopic to straight lines. Furthermore Wigner's [3] time delay has a simple geometrical meaning [4] for the trajectories. Recently Bollé and Osborn [5] succeeded in deriving even a classical analogue to Levinson's theorem. In these lectures I shall, following [6], show in detail how the phase shift corresponds to the generator of the S-transformation. For this purpose we define in the next section canonical coordinates for a one-, two- and three-dimensional configuration space such that this statement assumes a

⁺Lectures given at the XX.Internationale Universitätswochen für Kernphysik, Schladming, Austria, February 17-26, 1981.

simple form. This sheds some light on how trajectories with large time delays or loopings generate resonances of the quasiclassical phase shift. In the following section we give an alternative proof of the classical form of Levinson's theorem and illustrate its subtle feature by some examples. Finally we give a simple derivation of how a Dollard's [7] change in the free motion leads to the Coulomb phase shift as generator of the classical S-transformation for a $1/r$ -potential.

We shall employ the following

Notations:

$$\theta(x) \equiv \begin{matrix} 1 & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{matrix} \quad (\text{step function}).$$

$$f \circ g(x) = f(g(x)) \quad (\text{composition of maps}).$$

$$\sup_x f(x) = \text{least upper bound of } f.$$

$$\vec{a} \wedge \vec{b} = \text{vector product}.$$

2. THE S-TRANSFORMATION

Scattering theory investigates the asymptotic behaviour of the trajectories in phase space. Only some observables, typically functions of the momentum, will converge for $t \rightarrow \pm \infty$. This suffices to define the scattering angle

$$\theta = \angle(p_+, p_-), \quad p_{\pm} = \lim_{t \rightarrow \pm \infty} p(t),$$

but if the potentials decrease faster than $1/r$ for $r \rightarrow \infty$ additional information becomes available. Then time

evolution $\phi_t: (x(0), p(0)) \rightarrow (x(t), p(t))$ though not tending to a limit for $t \rightarrow \pm \infty$ approaches the free time evolution ϕ_t^0 such that $\phi_{-t} \circ \phi_t^0$ tends to a limit in some regions D_{\pm} :

$$\Omega_{\pm} = \lim_{t \rightarrow \pm \infty} \phi_{-t} \circ \phi_t^0.$$

This implies the convergence of $\vec{x}(t) - \vec{p}(t)$. t and allows the definition of the time delay compared with the free motion. Since ϕ and ϕ^0 are canonical transformations the "Möller-transformations" Ω_{\pm} will in general be a local canonical transformation mapping the domains D_{\pm} into ranges R_{\pm} . Closed orbits will be excluded from R_{\pm} but in reasonable [8] cases their union with R_+ or R_- will fill almost all of phase space. The scattering transformation

$$S = \Omega_+^{-1} \circ \Omega_- = \lim_{t \rightarrow \infty} \phi_{-t/2}^0 \circ \phi_t \circ \phi_{-t/2} \quad (1)$$

transforms D_- into D_+ . If ϕ_t^0 is the free time evolution having straight lines as trajectories, S has a simple geometrical meaning. For negative t , $\phi_{-t} \circ \phi_t^0$ means that you follow the straight trajectory back for a time $-|t|$ and then continue with the actual time evolution for the same length of time. If the forces have a finite range and ϕ and ϕ^0 coincide outside a certain region then $\phi_{-t} \circ \phi_t^0$ will become independent of t as soon as t leads you outside this region (Fig. 1). Then the limit is attained, Ω_- maps the straight line onto this trajectory of ϕ_t which is asymptotically tangent to it. Similar arguments for Ω_+ show that S maps (Fig. 1) the straight lines tangent for $t \rightarrow -\infty$ onto the ones tangent for $t \rightarrow +\infty$. It follows from its definition that S commutes with the free time evolution: $S \circ \phi_t^0 = \phi_t^0 \circ S$. As a canonical transformation one should be able to exhibit its generator which actually is possible. We first study the special cases.