

INTRODUCTORY COLLEGE MATHEMATICS

HACKWORTH
and
HOWLAND

S AUNDERS
ERIES IN

M ODULAR
ATHEMATICS

Consumer Mathematics

INTRODUCTORY COLLEGE MATHEMATICS

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W. B. Saunders Company: West Washington Square
Philadelphia, PA 19105

12 Dyott Street
London, WC1A 1DB

833 Oxford Street
Toronto, Ontario M8Z 5T9, Canada

INTRODUCTORY COLLEGE MATHEMATICS
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ISBN 0-7216-4410-4

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Last digit is the print number: 9 8 7 6 5 4 3 2 1

PREFACE

Consumer Mathematics

This book is one of the sixteen content modules in the Saunders Series in Modular Mathematics. The modules can be divided into three levels, the first of which requires only a working knowledge of arithmetic. The second level needs some elementary skills of algebra and the third level, knowledge comparable to the first two levels. *Consumer Mathematics* is in level 1. The groupings according to difficulty are shown below.

Level 1

Tables and Graphs
Consumer Mathematics
Algebra 1
Sets and Logic
Geometry

Level 2

Numeration
Metric Measure
Probability
Statistics
Geometric Measures

Level 3

Real Number System
History of Real Numbers
Indirect Measurement
Algebra 2
Computers
Linear Programming

The modules have been class tested in a variety of situations: large and small discussion groups, lecture classes, and in individualized study programs. The emphasis of all modules is upon ideas and concepts.

Consumer Mathematics deals with some of the fundamental problems facing any consumer. Consequently, it is relevant for all students without regard to college majors. *Consumer Mathematics* is appropriate for freshmen and sophomore students.

Consumer Mathematics begins by presenting percent notation and developing skills with percent problems. Simple interest, compound interest, effective interest, and present value are presented with direct application to problems such as the mathematics of a mortgage closing statement. The module includes an introduction to the mathematics of insurance and indicates the type of factors effecting the premiums of various insurances.

In preparing each module we have been greatly aided by the valuable suggestions of the following excellent reviewers: William Andrews, Triton College, Ken Goldstein, Miami-Dade Community College, Don Hostetler, Mesa Community College, Karl Klee, Queensboro Community College, Pamela Matthews, Chabot College, Robert Nowlan, Southern Connecticut State College, Ken Seydel, Skyline College, Ara Sullenberger, Tarrant County Junior College, and Ruth Wing, Palm Beach Junior College. We thank them, and the staff at W. B. Saunders Company for their support.

Robert D. Hackworth
Joseph W. Howland

NOTE TO THE STUDENT

OBJECTIVES

Upon completion of this module the reader is expected to be able to demonstrate the following skills:

1. To be able to solve percent problems given any two of the three quantities; base, rate, or percentage.
2. To be able to use percent to solve increase, decrease, discount, commission, and fee problems.
3. To be able to find simple, effective, and compound interest.
4. To be able to read a mortgage closing statement correctly and solve associated interest problems.
5. To evidence an understanding of the nature of insurance and its rate structures.

Three types of problem sets with answers are used in this module. Progress Tests are always short. Questions asked on each Progress Test come from the section immediately preceding it.

Exercise Sets involve more problems and cover a greater number of concepts than the Progress Tests. Section I problems in the Exercise Sets are keyed directly to the module objectives. Section II problems of the Exercise Sets are Challenge Problems.

A Self-Test is found at the end of the module. The Self-Test problems are representative of the entire module.

In learning the material, the student is encouraged to try each problem set as it is encountered. Check answers and restudy those sections where difficulties are discovered. This procedure is guaranteed to be both efficient and effective.



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CONSUMER MATHEMATICS

INTRODUCTION

John Dewey, one of America's greatest philosophers, once wrote that a person is not really free until he/she is not threatened by society's economic system. For most individuals, those that are not born rich, their freedom will be dependent upon the skill they use in controlling their finances.

This module explains some of the skills for meeting the economic realities of real life situations. Because many of the problems of consumer mathematics involve percents and percentages, the module begins with those topics.

PERCENT NOTATION

Percent notation provides a convenient way to show fractions or ratios. For example, 1% means $\frac{1}{100}$ or .01. If Willie breaks 1 egg out of 100, he breaks 1% of the eggs. If Idellia misses 20 days of class out of 200, she missed 10% of the classes because $\frac{20}{200}$ is equivalent to $\frac{10}{100}$. Percent means one hundredths.

$$1\% = \frac{1}{100} = 0.01 = \text{one hundredth}$$

Because percent means hundredths, $10\% = \frac{10}{100} = 0.10 = \text{ten hundredths}$. The decimal equivalent to 5% can be found by writing the fraction $\frac{5}{100}$. Since the decimal equivalent of $\frac{5}{100}$ is 0.05, $5\% = \frac{5}{100} = 0.05$. Similarly, $68\% = \frac{68}{100} = 0.68$. $100\% = \frac{100}{100} = 1.00$.

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Following are more examples of equivalent percent forms. The fractions in the middle column are formed by placing the number of the percent over a denominator of 100. Since two decimal places represents hundredths, a percent may be changed to a decimal by moving its decimal point two places to the left.

PERCENT	FRACTION	DECIMAL
1%	$\frac{1}{100}$	0.01
10%	$\frac{10}{100}$	0.10
17%	$\frac{17}{100}$	0.17
50%	$\frac{50}{100}$	0.50
100%	$\frac{100}{100}$	1.00
125%	$\frac{125}{100}$	1.25

Two more examples of changing a percent to its decimal equivalent are shown below.

$$66\frac{2}{3}\% = \frac{66\frac{2}{3}}{100} = 66\frac{2}{3} \cdot \frac{1}{100} = \frac{200}{3} \cdot \frac{1}{100} = \frac{2}{3} = 0.667 \text{ rounded to the nearest one-thousandth.}$$

$$0.1\% = \frac{\frac{1}{10}}{100} = \frac{1}{10} \cdot \frac{1}{100} = \frac{1}{1000} = 0.001$$

$66\frac{2}{3}\% = 0.667$ and $0.1\% = 0.001$. Notice that changing 0.1% to a decimal fraction is equivalent to moving the decimal point two places to the left.

Three more examples of fractional percents are shown below.

$$\frac{3}{4}\% = 0.75\% = 0.0075$$

$$\frac{1}{2}\% = 0.50\% = 0.0050$$

$$\frac{1}{4}\% = 0.25\% = 0.0025$$

Progress Test

Give the decimal equivalents of the following percents:

1. 1%

4. $\frac{3}{5}\%$

2. 85%

5. 10%

3. 110%

6. 0.3%

WRITING FRACTIONS AND DECIMALS AS PERCENTS

An understanding of percents is useful for answering questions similar to the following. If Mr. Moula lost \$3,400 worth of his merchandise in a robbery last year, what percent of his \$17,000 income does the loss represent?

To solve Mr. Moula's problem the fraction of his loss compared to his total income is written:

$\frac{3,400}{17,000}$ of his income was lost. The fraction can be simplified as follows:

$$\frac{3400}{17000} = \frac{170 \cdot 20}{170 \cdot 100} = \frac{20}{100} = 20\%$$

20% of his income was lost.

The fraction $\frac{5}{8}$ can be changed to its percent equivalent by first finding its decimal equivalent. To make this change, the numerator, 5, can be divided by the denominator, 8. The numerator 5, is written as 5.00 so that hundredths will be found in the division answer.

$$8 \overline{) 5.00} \begin{array}{r} 0.62\frac{1}{2} \\ 48 \\ \hline 20 \\ 16 \\ \hline 4 \end{array} = 0.625$$

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$$0.625 = 62.5\%$$

$$\frac{5}{8} = 0.62\frac{1}{2} = 62\frac{1}{2}\% \text{ or } 62.5\%$$

To change $\frac{3}{17}$ to its percent equivalent, first use division to find its decimal equivalent in hundredths. Write 3 as 3.00 as shown in the example below.

$$\begin{array}{r} 0.1764 \\ 17 \overline{) 3.0000} \\ \underline{17} \\ 130 \\ \underline{119} \\ 110 \\ \underline{102} \\ 80 \\ \underline{68} \\ 12 \end{array}$$

$$\frac{3}{17} = 17.6\% \text{ rounded to the nearest one-tenth}$$

To change $\frac{5}{12}$ to percent it is necessary to find the number of hundredths equivalent to $\frac{5}{12}$. The number of hundredths are found by dividing 5.00 by 12. The zeros are necessary so that hundredths will be indicated in the division answer.

$$\begin{array}{r} 0.4166 \\ 12 \overline{) 5.00} \\ \underline{48} \\ 20 \\ \underline{12} \\ 80 \\ \underline{72} \\ 80 \end{array}$$

$$\frac{5}{12} = 41.7\% \text{ rounded to the nearest one-tenth}$$

Progress Test 2

Fill in the blanks to obtain a true statement.

- | | FRACTION | = | DECIMAL | = | PERCENT |
|----|-----------------|---|---------|---|---------|
| 1. | $\frac{1}{100}$ | = | _____ | = | 1% |
| 2. | _____ | = | 0.10 | = | 10% |
| 3. | $\frac{3}{5}$ | = | _____ | = | _____ |
| 4. | _____ | = | 0.55 | = | _____ |
| 5. | _____ | = | _____ | = | 150% |
6. If a \$4,000 compact car depreciates \$1,000 in a year, what percent of the cost is the depreciation?
-

FINDING PERCENTS OF NUMBERS

To solve a problem such as "Find 3% of \$300", it is necessary to understand the meaning of the word "of". The word "of" when used in statements like $\frac{5}{6}$ of 16 = ? means to multiply $\frac{5}{6}$ by 16.

$$\frac{5}{6} \cdot \frac{16}{1} = \frac{80}{6} = \frac{40}{3} = 13\frac{1}{3}$$

Similarly, in "3% of \$200" the word "of" means to multiply 3% by \$200. Before multiplication is attempted, 3% should be written as its decimal equivalent.

$$3\% \text{ of } \$200 = \underline{\hspace{2cm}}$$

$$0.03 \cdot 200 = \$6.00$$

The following steps are used to find 25% of 60.

$$25\% \text{ of } 60 = \underline{\hspace{2cm}}$$

$$0.25 \cdot 60 = 15$$

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In " $\frac{1}{4}\%$ of 50" the word "of" means to multiply:

$$\frac{1}{4}\% \text{ of } 50 = .0025 \cdot 50 = .125$$

Following are three more examples showing how to solve a "percent of" problem:

$$1\% \text{ of } 40 = 0.01 \cdot 40 = 0.4$$

$$12\% \text{ of } 52 = 0.12 \cdot 52 = 6.24$$

$$0.5\% \text{ of } 20 = 0.005 \cdot 20 = 0.1$$

Progress Test 3

1. 6% of 40 = ?
 2. Find 10% of 58.
 3. What is 13% of \$500.
 4. Find 250% of 3,000.
 5. Ms. Jurski borrowed \$800 for a year. The bank charged Ms. Jurski 2% of the amount borrowed as a service charge on the loan. How much was the service charge?
 6. Mr. Roget can buy a \$550 lawn mower for 12% off if he pays for it with cash. How much discount would he get for cash?
-

USING PERCENT TO FIND COMMISSIONS AND FEES

In many vocations and professions, people are not paid by the hour for their services but receive commissions or fees as their compensation. The commissions or fees are usually based on a percent of the money involved in the transaction. An architect, for example, may charge a 10% fee for the planning and supervision of a construction job. If the cost of the building is \$130,000 the fee would be 10% of \$130,000 or \$13,000.

$$10\% \text{ of } \$130,000 = ?$$

$$0.10 \cdot \$130,000 = \$13,000$$

Lawyers commonly compute fees on a percentage basis. The fee may be a percent of the judge's award in a case. If an attorney wins an award of \$25,000 for his client in an automobile damage suit his fee may be 25% of the award.

$$\begin{aligned}25\% \text{ of } \$25,000 &= 0.25 \cdot \$25,000 \\ &= \$6,250\end{aligned}$$

The fee would be \$6,250.

Commissions are used as an incentive for salespeople. A salesperson who sells \$375,000 worth of pollution control equipment and makes a commission of 2% of sales will receive 2% of \$375,000.

$$\begin{aligned}2\% \text{ of } \$375,000 &= 0.02 \cdot \$375,000 \\ &= \$7,500\end{aligned}$$

The next example concerns a person who receives a salary plus commission on sales.

Margot Yaguchi is offered a job where she would receive \$100 a week plus 5% of her sales. She believes she can sell about \$400 worth of merchandise a week. How much will she make a week? To answer the previous question she first needs to find 5% of \$400.

$$5\% \text{ of } \$400 = 0.05 \cdot \$400 = \$20$$

She made \$20 on the sales. Therefore, she will make \$100 + \$20 or \$120 a week when she sells \$400 worth of goods.

Progress Test 4

1. Mr. Trot, an attorney, agrees to take a damage case provided he will receive 45% of the award in the case. Find the amount of his fee if his client receives \$350,000.
2. An urban planning company will be paid 1% of the cost of a \$3,500,000 project as its consulting fee. Find the amount of the fee.
3. Herman Goldman, a shoe salesman, makes \$150 a week plus 3% of his sales. How much will he make in a week when he sells \$700 worth of shoes?

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4. A treasure diver recovers \$85,000 in rare gold coins. The state allows him to keep 50% of the value of sunken treasure he finds. How much of the \$85,000 can he keep?
 5. Alice Thesaures makes \$125 a week plus 5% of her sales. If she sells \$450 worth of merchandise in a week how much is her compensation for the week?
 6. Wylie and Maureen ask a home designer to plan their proposed \$35,000 home. The designer charges 1% of the house value for the plans. How much is the fee?
-

Exercise Set 1

- I. 1. Give decimal equivalents of the following percents:

- | | | |
|--------|---------|--------------------|
| a. 1% | e. 50% | i. 125% |
| b. 3% | f. 60% | j. $\frac{1}{5}\%$ |
| c. 10% | g. 89% | k. $\frac{3}{4}\%$ |
| d. 27% | h. 100% | l. $\frac{1}{2}\%$ |

2. Fill in the blanks to give a true statement:

Fraction		Decimal		Percent
a. _____	=	0.01	=	_____
b. _____	=	_____	=	10%
c. $\frac{3}{4}$	=	_____	=	_____
d. _____	=	0.65	=	_____
e. _____	=	_____	=	100%
f. $\frac{4}{5}$	=	_____	=	_____

3. A grocery store announced that the price of steak would be reduced by $\frac{1}{5}$ on Saturday. By what percent is the price reduced on steak?

4. If the price of a box of oranges increased by $\frac{1}{10}$ by what percent did it increase?
5. An auto insurance company announced that the prices of all of its policies would be one-eighth higher next year. By what percent will the cost increase?
6. An auto repair shop found that it could not collect 6% of its charges of \$20,000. How much of its money is uncollectable?
7.
 - a. 7% of 80 = ?
 - b. 10% of \$1,000 = ?
 - c. $\frac{1}{2}\%$ of \$50 = ?
 - d. $\frac{3}{4}\%$ of \$20,000 = ?
 - e. 2% of \$15,000 = ?
 - f. $1\frac{1}{2}\%$ of \$10,000 = ?
8. If a farmer grew 800 more bushels of beans this year than last year when he grew 16,000 bushels, what is the percent of increase of his crop?
9. A fire destroyed \$2,000 worth of an antique store's inventory of \$100,000. What is its percent of loss?
10. Mr. Boyd paid \$600 in taxes this year. He will be charged 10% more next year. How much more will he pay?
11. A sky diving club found that main chutes failed to open 2% of the time. If 3,600 jumps are planned next year, how many main chute failures are likely?
12. An architect received 8% for planning and supervising building projects. How much is the fee for a \$38,000 project?
13. If an attorney charges $33\frac{1}{3}\%$ for obtaining a \$150,000 judgment in a damage suit, how much is his fee?

14. Milly takes a sales position which pays \$500 a month plus 2% of her sales. If she has sales of \$20,000 in a month, how much will she make?
15. A pharmaceutical representative makes 1% of his sales plus \$1,000 a month. How much does he make if he has sales of \$55,000 in a month?
16. Ms. Awaar found a brand of canned vegetables that had the same food value as a more expensive brand but was priced $\frac{1}{10}$ or _____ percent lower.
17. By using cheaper cuts of meat Mrs. Notice lowered her grocery bill by $\frac{1}{9}$ or _____ percent.
18. Because of 26 years of service, a retired worker will receive 0.65 of his average pay. What percent of the pay will be received?
19. Jerry's solar water heater lowered his light bill by $\frac{1}{5}$. By what percent was his bill lowered?
20. After three years, the money accumulated by an investment made at 8% compounded annually is $\frac{67}{50}$ as great as the original investment. The accumulated money is what percent of the original investment?
21. Jill, in a burst of compulsive shopping, spent \$22 more than her \$50 clothing allowance in her budget. Her overspending is what percent of her clothing budget?
22. Herman obtained 5,000 more miles of wear from his tires than the usual 18,000 miles by inflating the tires to 30 pounds pressure. $\frac{5000}{18000}$ is equivalent to what percent?
23. Marsha increased the mileage on her car by 4 miles per gallon by driving 55 miles per hour instead of 65 miles per hour. Formerly she got 13 miles per gallon. What percent is equivalent to $\frac{4}{13}$?