ABRAHAM ROBINSON

SELECTED PAPERS

VOLUME 1

MODEL THEORY
AND ALGEBRA

Selected Papers
of Abraham Robinson

Edited by
H. J. Keisler, S. Körner, W. A. J. Luxemburg, and A. D. Young

Volume 1   Model Th ₎ιy and Algebra
Edited and with an introduction by H. J. Keisler

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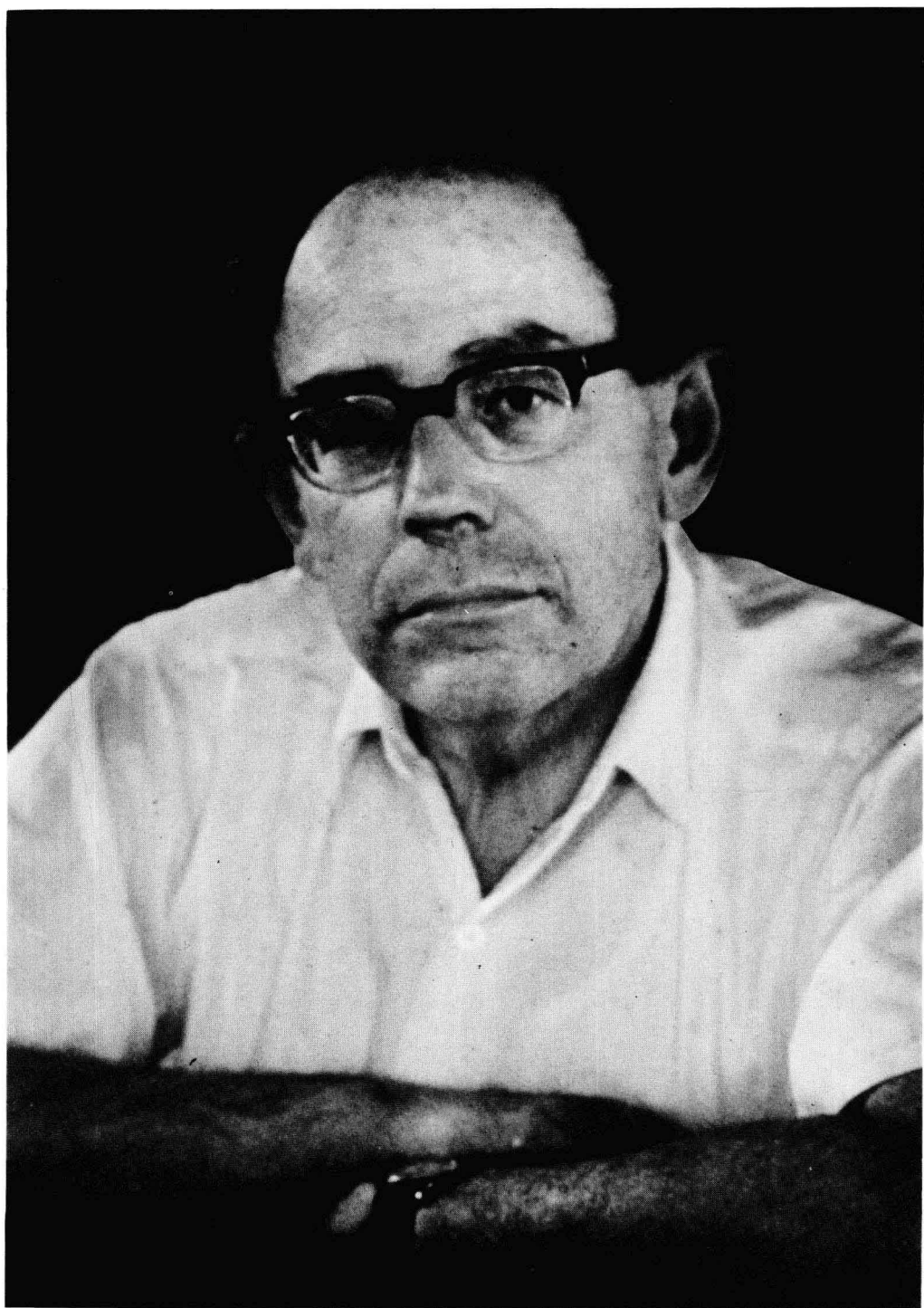
Abraham Robinson in Brasília, 1972.

EDITORS' NOTE

The tragic loss caused by the premature death of Abraham Robinson brought about a deeply felt desire by his close friends, colleagues, and pupils to commemorate his exceptional gifts and personality. An appropriate way to accomplish this appeared to be the selection and editing of his writings on applied mathematics, mathematics, mathematical logic, philosophy, and nonstandard analysis, and the publication of these papers in these volumes.

This edition presents only an incomplete picture of Robinson's work and influence on mathematics and mathematical logic. It is impossible to do full justice to his many lectures, his philosophical and expository articles, and his many contributions to the work of his pupils. We do feel, however, that the present selection, together with his many important books and monographs, will show the mark his work has left on the development of mathematics and mathematical logic in this century.

It is with great pleasure and with deep gratitude that we acknowledge the help and advice of his widow, Renée Robinson. We also would like to extend our sincere thanks to Mrs. Jane Isay of the staff of Yale University Press for her care and professionalism in the production of these volumes.

H. J. Keisler
S. Körner
W. A. J. Luxemburg
A. D. Young

On the Metamathematics of Algebraic Systems.

To be read to the Birkbeck College (University of London) Mathematics Seminar on Wednesday, 4th Feb, 1948

by

A. Robinson.

1. Introduction

Time and again in the development of mathematical Logic there arose a desire to apply the new science, not only to clarify and crystalize the concepts of mathematics proper, but as a tool for the discovery or demonstration of actual mathematical theorems. Such tendencies appeared in the very beginnings of Formal Logic, in Leibniz' writings. However, when Leibniz tried to demonstrate the applicability of his symbolic language to Huygens, the latter completely missed the point. A later instance which in some ways is parallel to the first, will be found in one of Poincaré's books. Poincaré quotes Peano as saying that symbolic Logic has provided mathematics with wings. He comments with some cynicism that Peano is a good mathematician, but that like every other mathematician he still has to walk ("marche à pied")

Conditions have changed during the last thirty years, and symbolic Logic now receives a good deal of attention even in purely mathematical circles. Nevertheless it remains true that the actual application of symbolic Logic is still mostly confined to the use of some of its symbols (e.g. the existential quantifier) as a mere abbreviation. And the results of symbolic Logic which are most widely quoted are those which, like the theorems of Skolem-Löwenheim or of Gödel, show up the incompleteness of certain calculi rather than their positive efficacity.

It would appear that this state of affairs is due

Robinson's first contribution to the metamathematics of algebra was his 1949 University of London Ph. D. thesis. Above is the first manuscript page of a report on this thesis to the Logic Group at Birkbeck College in 1948.

BIOGRAPHY OF ABRAHAM ROBINSON

George B. Seligman

Abraham Robinson was born in Waldenburg in Lower Silesia (then Germany) on October 6, 1918. He was the second of two children born to Abraham Robinsohn and Hedwig Lotte Robinsohn, born Bähr. His brother Saul was two years older. The apparent break with Jewish practice made in naming him after his father was justified on the grounds of the father's untimely death at the age of forty, shortly before young Abraham's birth.

The senior Abraham Robinsohn had studied literature and philosophy in Switzerland and had continued his study of philosophy in Oxford and London. He received a Ph.D. from the University of London for a dissertation on William James. He was a leader in Zionist movements from his student days in Switzerland, where he organized student groups for the study of modern Hebrew and the promotion of its use within Jewish society. For the rest of his life he served the Jewish communities of England and continental western Europe through his journalistic and organizational work, particularly in spreading the idea of a Jewish state among the younger people. He sought Jewish unity through language and drew up plans for organizing Jewish education around this linguistic core. His writing and editorial efforts were centered first in London and later in Berlin, where several periodicals owed to him much of their vitality, and perhaps even the very fact of their publication.

Leadership of the World Zionist Organization had passed, upon the death of Theodore Herzl in 1904, to David Wolffsohn, a Cologne businessman who had been a most enthusiastic supporter of the Organization and a personal friend of Herzl's. By 1912 Wolffsohn's health was failing and he was aware of his inability to do justice to all the responsibilities of his office. In particular, the extensive correspondence on behalf of the Organization, much of it necessarily in Hebrew, was a serious burden for a man who was primarily a fund-raiser. About the end of 1912, Wolffsohn engaged Abraham Robinsohn as his private secretary, with responsibility for this correspondence. Robinsohn also tutored Wolffsohn in Hebrew and served as his confidant and personal adviser. The latter capacity became more significant as Wolffsohn's health deteriorated and he shared with Robinsohn his visions and his concerns for the future of the Zionist movement.

David Wolffsohn died in the fall of 1914. His papers, and ultimately those of Herzl as well, became the responsibility of Robinsohn. The curator of these papers found himself in his later thirties, still unwed, but now with a hope for the financial security to undertake marriage. In early 1916 he married Hedwig Lotte Bähr, a teacher in Cologne who shared his dedication to Zionism. She was twenty-seven at the time. His duties with the Herzl and Wolffsohn archives took the couple to Berlin, where a son, Saul, was born in late 1916. They were only awaiting the end of the war to realize the dream of Jerusalem, where Robinsohn had been selected to head the Jewish national

library on Mt. Scopus, when he died, suddenly, in May of 1918. He left unpublished a long biography of Wolffsohn and other writings. A shorter version of the biography, in German, was published by the Jüdischer Verlag in Berlin in 1921. The full manuscript lies on Mt. Scopus awaiting publication, not the least of the reasons for the delay being the accounts it is said to contain of conflicts within the Zionist organization, involving participants who long outlived Wolffsohn. A short volume of fables in poetry and prose in Hebrew, possibly written with his children in mind, was published by Eschkol in Berlin in 1924.

The expecting widow and her small son found shelter in her father's home in Waldenburg. It was there that the new Abraham was born in October, and there that he spent most of the first seven years of his life. The home seems to have been one lively with intellectual and social activity. In addition to carrying on with several of her late husband's projects, Frau Robinsohn kept up the poetic efforts she had pursued from her youth and maintained a vigorous interest in artistic and Zionist affairs. Her father, Jacob Bähr, bore the informal title *Prediger* (preacher). Born in Königsberg in East Prussia, he was a man of considerable learning and of forceful character, much admired in the community. He was primarily a teacher, an early and vocal proponent of Zionism. A lecturer and writer on Jewish educational and cultural matters, he held positions as head of the organization of Jewish teachers in Silesia and as cultural leader of the Jewish community. He fought for better conditions of employment for Jewish teachers, particularly by seeking contributions from wealthier communities to help support teachers in poorer ones. His house boasted a fine library and those close to the family credit his books and his example, as well as his teaching, as decisive influences on his grandsons.

When Jacob Bähr retired in 1925, the family moved to Breslau. For the next three years, until his death in 1928, the boys spent much of their time in the care and company of their grandfather, while the mother worked as secretary to the Keren Hajessod, the organization for raising money to purchase land and otherwise to aid settlement of Jews in Palestine.

Frau Robinsohn had extensive contacts in Zionist and literary circles, and her family had a history of involvement in social movements dating back at least to the participation of her maternal grandfather, from Westphalia, in the German revolution of 1848. She had returned to Berlin for a time after Abraham's birth in connection with some of her husband's projects. Letters from Zionist leaders attest to the value of these services. It is also known from letters that she received Chaim Weizmann on a visit to Breslau in 1927, and we may assume that this was only the high point in a series of stimulating visits at the home of Saul and Abraham. The boys were encouraged to read widely in the classics and current literature, as well as in Zionist writings, and acquaintances of the time report on discussions of their readings that the mother initiated in the home. Their spirit is recalled as that of liberal German humanism, while the family consciously maintained an identification with Jewish nationalism.

Abraham and Saul attended a private school in Breslau that was founded and headed by Rabbi Simonson. The school incorporated all levels from elementary school through high school, so the intellectual resources available to the boys stretched well

beyond what children their age would normally require. Without display but very clearly, they showed their ability to take full advantage of the opportunity. The death of their father had deprived them of his passionate encouragement in Hebrew at home, but they made rapid progress there and in the Talmud under the direction of Rabbi Simonson. The rabbi's widow reports on his recollections: ''The big boy was an extremely gifted child, but the little one was a genius.'' Eliezer Pinczover, a friend there and later in Jerusalem, remarks on Abby as a ''flower blossoming in hiding,'' in that he avoided all show of what he knew and could do. Both acquaintances were struck by the seriousness and maturity of his bearing. Indeed, the picture they paint is almost distressingly serious, and not easily reconciled with the zestful curiousity and enthusiasm for life that those who met him in mature years cherished in him.

There was still another member of the family whose influence in these early years was significant. He afforded them a direct contact with the world of science and with a richer and more cosmopolitan society than was theirs in Waldenburg or Breslau. Dr. Isaac Robinsohn, the only brother of the their father, was head of the Rothschild Hospital in Vienna and had an international reputation for techniques and medical discoveries made in the developing science of radiology. He took on the responsibility of guardianship for the boys after their father's death, and they were regular summer guests at his villa on the outskirts of Vienna. In addition to his research and administrative duties, he acted on his deeply felt responsibility on the part of the medical profession, and on the part of society generally, to enable those who could not otherwise afford it to share in the benefits of medical advances. He promoted better care for the poor both in his hospital and through more broadly based programs of outpatient services. It is probably impossible to assign a specific weight to his influence in developing the principles of generosity that were so much a part of Abby's character. However, it seems fair to give ample credit to the impression his example must have made.

Meanwhile the evil disease that had been incubating in Germany was taking full possession of its host. The national elections of March 5, 1933, saw the National Socialist German Workers' Party, with its leader Hitler already established as chancellor, with its unofficial apparatus of terror augmented by a highly developed propaganda machine, poll 44 percent of the vote. Detention of opposition members of the new Reichstag and physical bullying of some waverers produced a majority of those present for the Enabling Act of March 23, which granted Hitler dictatorial powers. Frau Robinsohn made hurried plans to leave for Palestine.

Matters worsened quickly. On March 29 a manifesto of the Nazi party announced a boycott of Jewish shops. On March 30 the Robinsohn household was shaken by the report that the passports of Jews were to be called in and stamped invalid for foreign travel. Hasty arrangements were made, and Frau Robinsohn and Saul took the night train to Berlin to settle affairs there before leaving the country. The next morning Abraham was to accompany a friend directly to Brno, in Czechoslovakia. He kept a diary for this ten-day period, beginning with his waiting at the Breslau station for the friend and the train to Brno, only to discover that an error in their understanding as to its time of departure had brought him to the station too late. He thereupon followed his

mother and brother to Berlin, where the family decided that their best chances lay by way of Munich, and from there through Italy.

His record of the trip is a sensitive chronicle of anxiety and hope, broken by episodes of eager sightseeing among the classical ruins of Rome and Athens, the natural beauty and teeming humanity of Naples, and even the grim hostility of Berlin, where the boycott was being proclaimed on every corner. (At this stage, Nazi control was inadequate to enforce the boycott and it was limited to one day.) Although twenty-four hours was more than the average stay at each of their stops, the boys and their mother managed to explore remarkably widely. This early manifestation of avid tourism in Abby will be not at all surprising to those who knew him later.

The Robinsohns' train left Berlin on the evening of April 1, crossed into Austria at noon the next day, and brought them through the Brenner Pass and into Italy by nightfall. By early morning on April 3 they were in Rome, leaving for Naples on the afternoon of April 4. On the evening of April 5 they boarded ship at Naples for Haifa via Athens, where the daylight hours of April 7 were spent on shore. Early in the morning of April 9 they arrived at Haifa.

The new home must have been spiritually exhilarating, but it was also necessary to make a living. Frau Robinsohn opened a *pension* on Rothschild Boulevard in Tel Aviv, where the boys, then fourteen and sixteen, entered the high school. Evidence of Abby's special interest in mathematics is available from this time, in the form of a carefully written set of notes in German on properties of conics and a somewhat later set, dated 1936 and in Hebrew, on the geometrical optics of lenses with surfaces generated by conics.

Within two years Saul had left Tel Aviv for the University in Jerusalem and the strain of running the *pension* had overcome the strength of Frau Robinsohn. The family decided to move to Jerusalem, taking an apartment in the house of friends. Abby spent his last school years there.

The time was one of great turmoil. Arab opposition to the influx of Jews and their acquisitions of property, financed largely from abroad, had hardened and was becoming more violent. Along with most of his companions, Abby was involved in Jewish defense activities. From at least as early as 1937, he was on duty in the principal defense organization, the Haganah. However, he does not seem to have been nearly so distracted from his studies by this excitement as were many of his classmates. One of them, Miriam Hermann of Jerusalem, recalls that the atmosphere was one of rebellion against the "irrelevant" academic requirements. It seems that the students were able to get hold of examination questions in advance and to set up a cooperative to produce answers for all. As one who knew him might expect, Abby shocked his classmates by quite pleasantly but firmly insisting that he would have nothing to do with the scheme. The lesson in integrity from a newcomer, probably somewhat younger than the norm for his class, must have been hard to take, but Mrs. Hermann reports that Abby's quiet commitment to principle under pressure had its effect on her character and on that of the others as well.

Abby took up his university studies in 1936. In addition to Haganah duty, which was unpaid, both he and Saul did tutoring in order to support themselves and their mother.

Although he was very well read in the humanities, mathematics had won his heart. The lectures of Abraham Fraenkel introduced him to research in logic and set theory. Professor Ernst Straus of UCLA, who was a younger student at the Hebrew University in those days, tells how Abby enjoyed the reputation among the other students of mathematics as being "the one to ask if you wanted to understand something." Straus adds that Abby fully lived up to that reputation and cites a lecture by Abby in the mathematics club as the start of his own interest in the Riemann zeta-function. By 1938 Fraenkel was proclaiming that there was nothing more he could teach Abby. Meanwhile Abby was taking on the most strenuous duties in the Haganah, with an uncomplaining acceptance and a quiet determination to do what had to be done. Companions in this service recall being impressed by his vast knowledge, particularly of German philosophy.

His first research papers date from these years, when he was not older than twenty. It is appropriate to his later career that his first paper was in logic, communicated by Fraenkel to Bernays for publication in the *Journal of Symbolic Logic*. His other teachers included Fekete and Levitzki, who was working on questions of nilpotency for rings all of whose elements are nilpotent. In 1938 Hopkins and Levitzki had independently published affirmative results for rings with minimum condition. Their proofs were somewhat labored, and even in the early 1950s this was counted as one of the more troublesome theorems in the theory of artinian rings. It was surprising to discover among Abby's files the galley proofs for a one-page proof, dated 1939 but never published, that yields nothing in the way of simplicity or incisiveness to the best proofs known today. The note is reproduced in these *Papers*.

In 1939 he won a scholarship to the Sorbonne and arrived in France at about the time the war started. Besides his studies, he managed to subscribe to a course of lessons in ballroom dancing and to learn something about skiing during the French winter.

The worsening condition of France left him little opportunity to develop his intersts. By June 14, 1940 the German army had entered Paris. With a fellow student from Jerusalem (now Professor Jacob Talmon of the Hebrew University), Abby joined the stream of refugees from the capital. Once again the flight was recorded in his notebooks, this time in Hebrew. At times they were in earshot of artillery, but they managed to reach the darkened station at Orleans in a freight car. From there they finally caught a train to Bordeaux, where he notes that the station was fully illuminated. The city was full of refugees; even the French government made Bordeaux its last station of withdrawal. There Paul Reynaud had to give up his desperate struggle to keep France alive. On June 16 he turned over the government to those committed to peace with Germany, and Marshal Pétain asked for an armistice the following day.

That was Monday, June 17. A notice was posted on the door of the British consulate instructing those British and British-protected citizens wishing to travel to England to come to the consulate with provisions. Abby and Talmon joined the line in the morning. In the afternoon they were brought by special train to the port of le Verdon and by launch to a ship standing offshore. Later in the day, bombs were dropped at le Verdon and ship's guns were engaged. The ship did not leave until the following day, when numerous officials and journalists joined the flight. Meanwhile Abby watched

air battles overhead. Once under way that evening, they found craft of every sort sailing from ports in North Africa and western France along their course to Britain.

They were interned for three weeks at Crystal Palace, on the southern outskirts of London. Through the intercession of a friend from the days in the Haganah (now Professor Schimon Abramsky of Oxford and London), they were released. They were billeted, at government expense, with a working-class family in Brixton, in what Talmon describes as "a quarter of ill repute." The Jewish Agency for Palestine helped them with pocket money.

Intensive bombing of London had begun. Talmon recalls night after night, in those months as the nights lengthened, in air-raid shelters in the subways. The song and hilarity brought forth there to make the best of matters contrasted sharply with the depressing views of damaged London offered by the cold, drizzly days. One morning they returned from the shelter to find that their home had received a direct hit. For more than a week they wandered the streets homeless, lounging in railway and subway stations and spending their nights in the shelters.

Their friends and families in Jerusalem had been making desperate inquiries after them. Somehow the search had extended from the Hebrew University to Professor Norman Bentwich, a well-connected British Jew associated with the University of London. He managed to locate them and referred them to the International Student Service. Talmon, who had received an M.A. and had begun work on a doctoral dissertation, was given a scholarship at the London School of Economics and Political Science, which later evacuated to Cambridge. Abby had no degree whatever and would have had to be taken on as a beginning student at a British university with no means of support. They remained in close contact with each other and with the Abramskys even after Talmon had left London for Cambridge.

Although Abby tried to join the British armed forces, they were not yet ready to accept so new an arrival, especially a born German. In November 1940 he finally found a way to contribute to the war effort by joining one of the regiments being organized by the Free French under de Gaulle, who had flown to England from Bordeaux on the day Abby sailed. It is not clear why Abby was assigned to the air force, whether by choice or by chance. In either case, there could hardly have been much forethought. The affiliation shaped his career for many years to come.

We do not have a record of his activities with the Free French, except to know that he visited the Abramskys in London from time to time and that he held the rank of sergeant at the end of 1941. At one time, he volunteered for service in Africa. He underwent the full range of immunizations customary for military personnel heading for that sector, but the assignment failed to develop.

It was during this period that he submitted to the *Proceedings of the Royal Society of Edinburgh* his note on the distributive law in fields, published there in 1941. His scholarship and his talents gained recognition; among his souvenirs are several signed photographs from Free French officers, saluting him as "friend, collaborator, professor."

The occasion for the presentation of photographs was his parting from service with the Free French in January 1942 to join the British forces, to whom he had been on loan

since December 1941. At that time he was appointed Scientific Officer in the Ministry of Aircraft Production, with assignment to the Royal Aircraft Establishment in Farnborough.

His first duties were in the department of structures, where he threw himself into the work with characteristic dedication and soon produced technical reports that caught the attention of workers in the department of aerodynamics. Later he was transferred to that section. Meanwhile he was familiarizing himself with the theoretical and practical tools of the aeronautical engineer. His companion at work, Hermann Jahn (now Professor Emeritus of Applied Mathematics at Southampton), recalls his reading aeronautics by flashlight in bed after the landlord had turned off the electricity at night. He carried his program of study to the stage of official qualification, by examination, as an aeronautical engineer. According to Jahn, Abby enjoyed a reputation for infallibility among his colleagues at Farnborough. Alec Young, who worked with him there and later, reports in an interview:

> It soon became apparent that Abraham Robinson was an exceptionally good applied mathematician and, more than that, he had an outstanding ability for understanding in physics and in the problems he dealt with, so that he could apply mathematics in a way that was extraordinarily stimulating as well as elegant.
>
> In a very short time he had mastered the subject of wing theory and in a number of extraordinarily brilliant papers expanded its frontiers, particularly in the field of supersonic aerodynamics. . . . By the end of the war I was aware of Abraham Robinson as one of the country's experts in the field of wing theory.

Much of his work was of course classified, so his mastery of aerodynamics became evident to scientists at large only with his postwar publications. The problems dealt with often had their origins in actual failures or other difficulties encountered in flight. Later on, as supersonic flight became an obvious development for the future, fundamental questions of design and performance were considered. His fellow workers report that Abby was quick to recognize oversimplifications that had led to error. In those cases he was most able in resolving the more complex problem, frequently seeing in it features of an important much more general class of problems and casting his solution in such a form as to contribute significantly to these. Some of the valuable unclassified work he did during this time, and later at Cranfield, was subsequently published by the Ministry of Supply through the Reports and Memoranda of the Aeronautics Research Council. They are listed individually in our bibliography.

He was not content with his scientific contributions to the war effort, but volunteered for service with the Home Guard, in anticipation of a possible invasion of Britain. As with the Haganah, he spent much of his off-duty time in maneuvers and hard training. Fortunately the threat of actual combat came to naught this time.

His spare time still allowed him to visit London regularly, often by bicycle, where he indulged his passion for sightseeing and took in galleries, concerts and theatrical performances. On one of these visits, on January 30, 1943, he met at the Abramskys' an artist, a young woman from Vienna, Renée Kopel, who was working in London as a fashion designer and acting with a group of exiled German actors who had estab-

lished a German theater in London. Through the spring and summer months they spent more time together, discovering common pleasures not only in the cultural world of the metropolis but also in long quiet walks in the English countryside. As fall came on, he called for her one night after the theater with a present. She recalls that it was a calendar, and he urged her to examine it closely. She did not have to get past Januaary, where she found January 30, 1944, the anniversary of their meeting, marked as the date of their wedding.

Colleagues who have sat on committees with Abby will recall that he was patient and tolerant in listening to all sides of a question, and showed tact in presenting his own point of view. Still he made his point so well that he usually prevailed. His courtship went likewise, and the marriage took place as appointed. After a honeymoon in Cornwall, they found an apartment in a cottage in West Byfleet, between London and Farnborough, where there was a housekeeper who, according to Renée, was able to prepare delicious meals despite wartime shortages. Both continued to work daily in their usual jobs, although Renée had given up acting. The location offered splendid opportunities for walking, always one of their favorite recreations, and for cycling through the beautiful countryside.

After a year the cottage was sold and they took an apartment with a similar housekeeping arrangement (but with a cuisine of a decidedly lower order) in a house in Surbiton, closer to London but farther from Farnborough. It was clear by then that the end of the war was not far off. Abby worked hard at Farnborough, but yearned to carry on his work free of the obligation to serve military objectives above all others.

At the end of the war he joined Hermann Jahn and two other scientists on a mission to Germany to study the progress of German aeronautical research. Jahn recalls Abby's insisting on a side trip to Breselenz, where Riemann was born and, having arrived, questioning villagers (with little success) about the house of his birth. Their trip carried them to Frankfurt, to Hannover, and into upper Bavaria. Despite a cold and fever, Abby was eager to see everything.

When the College of Aeronautics at Cranfield was established after the war, Abby was offered a position in its faculty of mathematics. The appointment was somewhat irregular since his studies had been interrupted without his receiving a degree. However, it was agreed that his work fully qualified him to receive an M.Sc. degree from the Hebrew University, and he was sent to Jerusalem for the conferring of the degree. At the same time the Hebrew University offered him a faculty position. He declined on the grounds of his obligations in England, but made it clear that he hoped one day to return.

The Robinsons moved into a house belonging to the college, bought furniture, and established a household of their own. Renée reduced her schedule of work somewhat, staying overnight in London during midweek because of the difficulty of commuting such a distance. Meanwhile Abby was continuing his research on subsonic and supersonic wing theory. While Renée was off in London, he quietly took up a course of lessons in flying in order to be able to add another dimension of his understanding of flight. Besides his aerodynamical works of this period, he published papers on algebraic curves (with Motzkin), on nonassociative systems, on hyperbolic differen-

tial equations, and on functional equations and summability, all based on research done in or before 1948.

He had made contact with Professor Dienes, at Birkbeck College of the University of London, and visited London regularly in midweek to see Renée and to participate in seminars in logic, in which he was resuming activity. The immediate result of this activity was a thesis, "The Metamathematics of Algebraic Systems," for which he received the Ph.D. in London in 1949. An abstract of a portion of this work was sent to the organizers of the section on logic and foundations at the International Congress of Mathematicians in Cambridge (Mass.) in 1950, and it so impressed them that he was invited to address the congress. From this time on, logic dominates his research; publications in aeronautics continue, with diminishing frequency. He had been appointed Deputy Head of his department at Cranfield in 1950, and the project of writing a book on wing theory with his former student J. A. Laurmann was begun about then. When he was offered a position in 1951 as Associate Professor of Applied Mathematics at the University of Toronto, the chance to move to a first-class university with experts in all fields was an irresistible lure.

One of the drawbacks of moving to the New World was that it became harder to pursue in the Old his ardor for traveling and sight-seeing. Photographs show him and Renée in Italy, Switzerland, Scandinavia, Ireland, and the Lake District of England. These tours, together with his trips to Jerusalem in 1946 and to America in 1950, show that he made major excursions in every year from 1946 through 1951, more than one in some years. Of course he followed closely the establishment of the state of Israel and its battle against Arab efforts to strangle it at birth.

Some recollections of his early days in Toronto come from Professor J. A. Steketee, now at Delft Institute of Technology, who had come to Toronto the year before. He remembers teaching Abby to drive a small and temperamental English Ford that Abby had bought. By the end of the year the Ford had carried Abby and Renée to the Rockies and back. In a subsequent summer it made the long journey to Mexico, where Abby was so taken with the country, its relics from Aztec times and earlier, and its modern art and architecture that it took over first place on his tourist's list.

The Department of Applied Mathematics occupied an old private home on St. George Street in Toronto. Abby kept regular hours (about 8:30 to 5:30), often bringing sandwiches and eating at his desk. His self-discipline included an insistence on the completion of a daily quota of work on the book on wing theory. His teaching involved a graduate course on wing theory and less advanced courses on fluid dynamics and partial differential equations, as well as a share in the introductory mathematics courses taught as a service to engineering students.

Professor G. F. D. Duff of Toronto recalls with gratitude Abby's encouragement in his work on mixed initial boundary value problems for hyperbolic partial differential equations, and with admiration Abby's quickness in understanding and analyzing problems. Abby himself published a joint paper on the subject with his student L. L. Campbell in 1955. Duff emphasizes Abby's gift for going to fundamentals and for putting any problem in perspective against the broad background that his knowledge afforded.