

Linear Algebra in Signals, Systems, and Control

siam

Linear Algebra in Signals, Systems, and Control

EDITED BY

Biswa Nath Datta
Northern Illinois University

Charles R. Johnson
College of William and Mary

Marinus A. Kaashoek
Vrije University

Robert J. Plemmons
North Carolina State University

Eduardo D. Sontag
Rutgers University

siam

Philadelphia 1988

All rights reserved. Printed in the United States of America. No part of this book may be reproduced, stored, or transmitted in any manner without the written permission of the publisher. For information, write the Society for Industrial and Applied Mathematics, 1400 Architects Building, 117 South 17th Street, Philadelphia, PA 19103-5052.

This work relates to Department of Navy grant N00014-86-G-0160 issued by the Office of Naval Research. The United States Government has a royalty free license throughout the world in all copyrightable material contained herein.

Library of Congress Catalog Card Number 88-60026
ISBN 0-89871-223-8

Copyright 1988 by the Society for Industrial and Applied Mathematics.

Linear Algebra in Signals, Systems, and Control

Proceedings of the Conference on Linear Algebra in Signals, Systems,
and Control, Boston, Massachusetts, August 12-14, 1986.

*This conference was sponsored by the SIAM Activity Group on Linear
Algebra and was supported in part by the National Science Foundation
under grant DMS8601261 and by the Office of Naval Research under
grant N00014-86-G-0160.*

Preface

In recent years, the scientific and engineering communities have looked for ways to bridge the gap dividing mathematicians, computer scientists, and engineers working in signals, systems, and control. The interdisciplinary SIAM conference on Linear Algebra in Signals, Systems, and Control, held in Boston on August 12-14, 1986, was organized to help bridge this gap. The most recent in a series of conferences in this highly active area of research, it follows the 1984 AMS-IMS-SIAM Summer Research Conference on Linear Algebra and Its Role in Systems Theory.

The present volume consists of papers selected from invited and contributed presentations at the conference. The papers can be divided into the following broad categories:

- 1) Core Linear Algebra;
- 2) Numerical Linear Algebra;
- 3) Algorithms for Signals, Systems, and Control;
- 4) Linear and Nonlinear Control and Systems Theory.

We hope that the conference, together with this volume, will help to promote further joint development in these fields.

We take this opportunity to thank the following research agencies for their generous support of the conference: the National Science Foundation, the Air Force Office of Scientific Research, the Army Research Office, and the Office of Naval Research. We also thank the SIAM Activity Group on Linear Algebra for sponsoring the conference and Professor David Carlson and Dr. Robert Ward, in particular, for their support and enthusiasm. Throughout the entire process of organizing the conference and putting together these final manuscripts, we received enthusiastic support from many colleagues, including those who served as referees. We express our sincere thanks to all of them,

especially to our referees, for their hard work. We also thank Professor Gregory Ammar of Northern Illinois University for his help in making the final program. Finally, we wish to thank Mrs. Sara Clayton and Mrs. Peggy Putzstuck of Northern Illinois University for their excellent secretarial assistance.

Biswa Nath Datta
Northern Illinois University

Charles R. Johnson
College of William and Mary

Marinus A. Kaashoek
Vrije University

Robert J. Plemmons
North Carolina State University

Eduardo Sontag
Rutgers University

Referees

Thomas Alexander
Gregory Ammar
A. C. Antoulas
Shankar Bhattacharyya
Sergio Bittanti
Rafael Bru
Richard Brualdi
James Bunch
Ralph Byers
Stephen Campbell
David Carlson
Eric Chu
Biswa Nath Datta
James Demmel
S. G. Ganeshan
B. Ghosh
Daniel Grubb

F. Hamano
Robert Hartwig
Ilse Ipsen
B. Jackubcyk
Charles Johnson
M. A. Kaashoek
Pramrod Khargonekar
S. Y. Kung
Bernard Levy
Michael Lindquist
Franklin Luk
Pradip Misra
Michael Neumann
Nancy Nichols
Jorge Nocedal
Peter Nylen
Chris Paige

C. T. Pan
Rajnikant Patel
R. Plemmons
V. Popov
Moshen Pourahmadi
Yucef Saad
J. M. Schumaker
Mark Shayman
Eduardo Sontag
Jeff Stuart
Tracy Summers
William Trench
Paul Van Dooren
George Verghese
Alan Wilsky
Stephen Wright

List of Contributors

- W. N. Anderson, Jr., *Department of Mathematics and Computer Science, Fairleigh Dickinson University, Teaneck, New Jersey 07666*
- Gregory S. Ammar, *Department of Mathematical Sciences, Northern Illinois University, DeKalb, Illinois 60115*
- J. D. Aplevich, *Department of Electrical Engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1*
- Zoran Banjanin, *Cooperative Institute for Mesoscale Meteorological Studies, University of Oklahoma, Norman, Oklahoma 73109*
- Itzhack Y. Bar-Itzhack, *Department of Aeronautical Engineering, Technion-Israel Institute of Technology, Haifa 32000, Israel*
- T. Beelen, *Philips Research Laboratory, Advanced Systems Group, 5600 JA Eindhoven, the Netherlands*
- A. A. (Louis) Beex, *Department of Electrical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061*
- S. P. Bhattacharyya, *Electrical Engineering Department, Texas A&M University, College Station, Texas 77843*
- Sergio Bittanti, *Dipartimento di Elettronica, Politecnico di Milano, 20133 Milano, Italy*
- Anthony M. Bloch, *Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48109*
- Paola Bolzern, *Dipartimento di Elettronica, Politecnico di Milano, 20133 Milano, Italy*
- Adhemar Bultheel, *Department of Computer Science, Katholieke Universiteit Leuven, B-3030 Heverlee, Belgium*
- Stephen L. Campbell, *Department of Mathematics, North Carolina State University, Raleigh, North Carolina 27695*
- Daizhan Cheng, *Department of Systems Science and Mathematics, Washington University, St. Louis, Missouri 63130*
- Dipa Choudhury, *Mathematics Department, Loyola College, Baltimore, Maryland 21210*
- Patrizio Colaneri, *Dipartimento di Elettronica, Politecnico di Milano, 20133 Milano, Italy*
- Roberto Cristi, *Electrical and Computer Engineering Department, Naval Postgraduate School, Monterey, California 93943*
- J. R. Cruz, *School of Electrical Engineering and Computer Science, University of Oklahoma, Norman, Oklahoma 73109*
- Jose A. De Abreu-Garcia, *Department of Electrical Engineering, Queen's University, Kingston, Ontario, Canada K7L 3N6*
- Giuseppe de Nicolao, *Dipartimento di Elettronica, Politecnico di Milano, 20133 Milano, Italy*

- Victor DeBrunner, *Department of Electrical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061*
- James W. Demmel, *Computer Science Department, Courant Institute of Mathematical Sciences, New York University, New York, New York 10012*
- Mukund Desai, *The Charles Stark Draper Laboratory, Cambridge, Massachusetts 02139*
- Premal Desai, *Department of Electrical Engineering, Pennsylvania State University, University Park, Pennsylvania 16802*
- Frederick W. Fairman, *Queen's University, Kingston, Ontario, Canada K7L 3N6*
- K. Vince Fernando, *Numerical Algorithms Group, Ltd., Oxford OX2 7DE, United Kingdom*
- Daniel R. Fuhrmann, *Department of Electrical Engineering, Washington University, St. Louis, Missouri 63130*
- C. Ganesh, *Department of Electrical and Computer Engineering, Rice University, Houston, Texas 77251*
- Moshe Goldberg, *Department of Mathematics, Technion-Israel Institute of Technology, Haifa 32000, Israel*
- Ljubomir T. Gurjić, *Faculty of Mechanical Engineering, 1101 Belgrade, Yugoslavia*
- S. Gutman, *Department of Mechanical Engineering, Technion-Israel Institute of Technology, Haifa 32000, Israel*
- Fumio Hamano, *Department of Electrical and Computer Engineering, Florida Atlantic University, Boca Raton, Florida 33431*
- Sven J. Hammarling, *Numerical Algorithms Group, Ltd., Oxford OX2 7DE, United Kingdom*
- Jia-Yuan Han, *Department of Electrical Engineering, Southern Illinois University, Carbondale, Illinois 62901*
- Vicente Hernández, *Department of Mathematics, Polytechnical University of Valencia, Valencia, Spain*
- Ulf Holmberg, *Department of Automatic Control, Lund Institute of Technology, S-221 00 Lund, Sweden*
- Roger A. Horn, *Department of Mathematical Sciences, The Johns Hopkins University, Baltimore, Maryland 21218*
- Kang C. Jea, *Center for Numerical Analysis, University of Texas, Austin, Texas 78712*
- Bo Kagström, *Institute of Information Processing, University of Umeå, S-901 87 Umeå, Sweden*
- L. H. Keel, *Electrical Engineering Department, Texas A&M University, College Station, Texas 77843*
- Mats Lilja, *Department of Automatic Control, Lund Institute of Technology, S-221 00 Lund, Sweden*
- A. K. Mahalanabis, *Department of Electrical Engineering, Pennsylvania State University, University Park, Pennsylvania 16802*
- Tsun-Zee Mai, *Center for Numerical Analysis, University of Texas, Austin, Texas 78712*
- Bengt Mårtensson, *Department of Electrical Engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1*
- Volker Mehrmann, *Fakultät für Mathematik, Universität Bielefeld, 4800 Beilefeld, Federal Republic of Germany*
- Pradeep Misra, *Department of Electrical Engineering, Concordia University, Montreal, Quebec, Canada H3G 1M8*
- K. Morris, *University of Waterloo, Waterloo, Ontario, Canada N2L 3G1*
- Y. Oshman, *Department of Aeronautical Engineering, Technion-Israel Institute of Technology, Haifa 32000, Israel*
- Michael L. Overton, *Centre for Mathematical Analysis, Australian National University, Canberra, ACT 2601 Australia*
- Sydney R. Parker, *Department of Electrical and Computer Engineering, Naval Postgraduate School, Monterey, California 93943*
- R. V. Patel, *Department of Electrical Engineering, Concordia University, Montreal, Quebec, Canada H3G 1M8*
- Leonid I. Perlovsky, *Nichols Research Corporation, Edgewater Office Park, Wakefield, Massachusetts 01880*
- D. J. Pierce, *Boeing Computer Services, Seattle, Washington 98124*
- R. J. Plemmons, *Departments of Mathematics and Computer Science, North Carolina State University, Raleigh, North Carolina 27695*
- Charles M. Rader, *Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts 02173*
- M. Anisar Rahman, *Department of Electrical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061*

- Asok Ray, *Department of Mechanical Engineering, Pennsylvania State University, University Park, Pennsylvania 16802*
- Tzila Shamir, *Department of Applied Mathematics, Weizmann Institute of Science, Rehovot 76100, Israel*
- Jon Arne Sjogren, *NASA Langley Research Center, Hampton, Virginia 23665*
- Allan O. Steinhardt, *Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts 02173*
- Vassilios D. Tourassis, *Department of Electrical Engineering, University of Rochester, Rochester, New York 14627*
- George E. Trapp, *Department of Statistics and Computer Science, West Virginia University, Morgantown, West Virginia 26506*
- William F. Trench, *Department of Mathematics, Trinity University, San Antonio, Texas 78284*
- Srbijanka R. Turajlić, *Faculty of Electrical Engineering, University of Belgrade, 11001 Belgrade, Yugoslavia*
- Ana Urbano, *Department of Applied Mathematics, Agricultural Engineering, Polytechnical University of Valencia, Valencia, Spain*
- Marc Van Barel, *Department Computerwetenschappen, Katholieke Universiteit Leuven, B-3030 Heverlee, Belgium*
- P. Van Dooren, *Philips Research Laboratory, B-1170 Brussels, Belgium*
- Victor Vinnikov, *Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel*
- James R. Weaver, *Department of Mathematics and Statistics, University of West Florida, Pensacola, Florida 32514*
- Bostwick F. Wyman, *Department of Mathematics, Ohio State University, Columbus, Ohio 43210*
- David M. Young, Jr., *Center for Numerical Analysis, University of Texas, Austin, Texas 78712*
- Kai-Bor Yu, *Department of Electrical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061*

Table of Contents

Part I: Core Linear Algebra	1
Network Matrix Operations for Vectors and Quaternions <i>W. N. Anderson and G. E. Trapp</i>	3
Inertia Theorems for Lyapunov and Riccati Equations—An Updated View <i>Sergio Bittanti, Paolo Bolzern, and Patrizio Colaneri</i>	11
An Analogue of the Schur Triangular Factorization for Complex Orthogonal Similarity and Consimilarity <i>Dipa Choudhury and Roger A. Horn</i>	36
Mixed-Multiplicativity for l_p Norms of Matrices <i>Moshe Goldberg</i>	44
The Structure of Root Clustering Criteria <i>Shaul Gutman</i>	48
Extended Inertia Theorems for Discrete-Time Periodic Lyapunov and Riccati Equations <i>Vicente Hernández and Ana Urbano</i>	55
On the Discrete Relationship Between Matrix Continued Fractions and the Maximal (A,B)-Invariant Subspace in KerC <i>Tzila Shamir</i>	64
Determinantal Representations of Algebraic Curves <i>Victor Vinnikov</i>	73
Eigenvalues of Centrosymmetric Matrices <i>James R. Weaver</i>	100
Part II: Numerical Linear Algebra	105
Superfast Solution of Real Positive Definite Toeplitz Systems <i>Gregory S. Ammar and William B. Gragg</i>	107
Accurate Solutions of Ill-Posed Problems in Control Theory <i>James Demmel and Bo Kagström</i>	126

A Product Induced Singular Value Decomposition (IISVD) for Two Matrices and Balanced Realization	128
<i>K. Vince Fernando and Sven Hammarling</i>	
An Algorithm for Subspace Computation, with Applications in Signal Processing	141
<i>Daniel R. Fuhrmann</i>	
On Minimizing the Maximum Eigenvalue of a Symmetric Matrix	150
<i>Michael L. Overton</i>	
A Two-Level Preconditioned Conjugate Gradient Scheme	170
<i>D. J. Pierce and R. J. Plemmons</i>	
Hyperbolic Householder Transforms	186
<i>Charles M. Rader and Allan O. Steinhardt</i>	
Total Least Squares Approach for Solving the Linear Prediction Equation	209
<i>M. A. Rahman and Kai-Bor Yu</i>	
Iterative Methods in the Solution of Dependability Models	220
<i>J. A. Sjogren</i>	
Numerical Solution of the Eigenvalue Problem for Symmetric Rationally Generated Toeplitz Matrices	244
<i>W. F. Trench</i>	
Preconditioned Conjugate Gradient Algorithms and Software for Solving Large Sparse Linear Systems	260
<i>David M. Young, Kang C. Jea, and Tsun-Zee Mai</i>	
Part III: Algorithms for Signals, Systems, and Control	285
Controller Parameterization and Recursive Design Using Implicit Systems	287
<i>J. D. Aplevich and K. Morris</i>	
A Pencil Approach for Embedding a Polynomial Matrix into a Unimodular Matrix	300
<i>T. Beelen and P. Van Dooren</i>	
Two Techniques for the Solution of the Discrete-Time Periodic Riccati Equation	315
<i>Sergio Bittanti, Patrizio Colaneri, and Giuseppe De Nicolao</i>	
Parallel Processing in the Adaptive Control of Linear Systems	332
<i>Roberto Cristi</i>	
Applications of the Quotient-Difference Algorithm to Modern Spectral Estimation	343
<i>J. R. Cruz and Zoran Banjanin</i>	
Sensitivity Analysis of Digital Filter Structures	355
<i>Victor E. DeBrunner and A. A. (Louis) Beex</i>	
Algorithms for the Interpolation with Outer Functions	375
<i>C. Ganesh</i>	
Integrating Different Symbolic and Numeric Tools for Linear Algebra and Linear Systems Analysis	384
<i>Ulf Holmberg, Mats Lilja, and Bengt Mårtensson</i>	
A Symplectic Orthogonal Method for Single Input or Single Output Discrete Time Optimal Control Problems	401
<i>Volker Mehrmann</i>	
A Determinant Identity and Its Applications in Evaluating Frequency Response Matrices	437
<i>P. Misra and R. V. Patel</i>	
Square Root $\sqrt{\lambda}$ -Filtering Using Normalized State Estimates	446
<i>Yaakov Oshman and Itzhack Y. Bar-Itzhack</i>	

A Minimal Partial Realization Algorithm for Data with Relative Errors <i>Marc Van Barel and Adhemar Bultheel</i>	459
Part IV: Linear and Nonlinear Control and Systems Theory	479
A Geometric Approach to Errors-In-Variables Models <i>Anthony M. Bloch</i>	481
Bilinear Nonlinear Descriptor Control Systems <i>Stephen L. Campbell</i>	493
Noninteracting Decomposition of Linear Systems <i>Daizhan Cheng</i>	512
Balanced Realization via Permutation Symmetric Jordan Realizations <i>Jose A. De Abreu-Garcia and Frederick W. Fairman</i>	522
Algebraic Conditions for Absolute Tracking Control of Continuous-Time Lurie Systems <i>Ljubomir T. Grujić</i>	535
Necessary and Sufficient Conditions for Subspace Reachability and Controllability of Discrete-Time, Time-Invariant Linear Dynamical Systems <i>Fumio Hamano</i>	556
Reachability of Linear Systems with Subspace Open Constraints <i>Jia-Yuan Han and Bostwick F. Wyman</i>	564
Robust Stability and Stabilization in Parameter Space <i>L. H. Keel and S. P. Bhattacharyya</i>	576
A Comparative Study of Two Minimum Variance Regulators for Linear Stochastic Systems <i>A. K. Mahalanabis and Premal Desai</i>	594
Dynamic High-Gain Stabilization of Multivariate Linear Systems, with Application to Adaptive Control <i>Bengt Mårtensson</i>	610
Frequency Estimates for Simple Oscillating Systems Under Random Forcing <i>L. I. Perlovsky</i>	619
Fault Detection Using a Linear Algebraic Approach <i>Asok Ray and Mukund Desai</i>	627
A Unifying Framework for the Design and Implementation of Robot Controllers <i>Vassilios D. Tourassis</i>	642
A State Space Approach to the Design of Orthogonal Models <i>Srbijanka R. Turajlić and Sydney R. Parker</i>	651

Part I:

Core Linear Algebra

Network Matrix Operations for Vectors and Quaternions

W. N. ANDERSON* AND G. E. TRAPP†

ABSTRACT. Based on the concepts of the series and parallel sum of vectors, the shorted operator and hybrid addition are defined and examined. Matrix means are also generalized to the case of vectors since the series and parallel sums yield the arithmetic and harmonic means. An iterative definition of the geometric mean is reviewed and the Gaussian mean is then considered. It is shown that many properties of the network matrix operations and matrix means hold for vectors. Two special cases, quaternions and Cayley numbers, are also discussed.

INTRODUCTION. We are interested in a real n -dimensional Euclidian space. We will denote vectors by a, b, \dots . The Euclidian inner product will be written $\langle a, b \rangle$, and the norm induced by the inner product is $\|a\| = \sqrt{\langle a, a \rangle}$. Given two vectors a and b with $a+b \neq 0$, we define their parallel sum, denoted $a:b$, as follows:

$$a:b = (\|a\|^2 \|b\|^2 + \|a+b\|^2) / (\|a+b\|^2).$$

In [5], we show that this is the natural vector generalization of the matrix parallel sum. The other fundamental network operation - series - corresponds to the normal sum of two vectors $a + b$. Using these two operations as building blocks, we will define other vector operations that generalize commonly used matrix operations and matrix means. In the next section we will define the shorted operator, the hybrid sum, and the contraharmonic mean of

*Department of Mathematics and Computer Science,
Fairleigh Dickinson University Teaneck, NJ 07666.

†Departments of Mathematics, Statistics and Computer
Science, West Virginia University, Morgantown, WV, 26506.

vectors. In section 3.0, we will review the iteration procedure used to define the geometric mean, and consider other related iteration procedures. In particular we will show that the Gaussian mean of two vectors may be defined. The last section of the paper considers two special vector cases: $n=4$ (quaternions) and $n=8$ (Cayley numbers).

We conclude this section by summarizing the basic properties of the parallel sum. The reader should consult [5] for additional information and proofs.

Theorem 1: If a and b are vectors such that $a + b \neq 0$ then the following hold:

- i. $a:b = b:a$
- ii. $(ka):(kb) = k(a:b)$
- iii. $||a:b|| = ||a|| ||b|| / ||a+b||$
- iv. $a:(b:c) = (a:b):c$ whenever all of the parallel sums are defined.

NETWORK OPERATIONS DEFINED FOR VECTORS. The series and parallel sums are motivated by the series and parallel connections of networks. Since other network connections are commonly used, we will define the corresponding network operations for vectors. The mixture of the series and parallel connections is the hybrid connection. In terms of networks, some of the connections are made in series and the remaining are made in parallel. We model this behavior by adding some of the components of the vectors in series and others in parallel. Results concerning the hybrid sum of matrices may be found in [10] or [12].

Let the $n \times n$ matrix P be the projection onto a subspace S of V ; and let Q be the matrix $I-P$ (here I is the identity matrix). Given vectors a and b such that $Qa + Qb \neq 0$, define the hydrid sum, denoted $a*b$, as follows:

$$a*b = Pa + Pb + (Qa):(Qb)$$

The following lemma concerning the projections P and Q and the parallel sum is required in the proof of Theorem 3.

Lemma 2: With P and Q as above and Qa and Qb parallel summable vectors, the following hold:

- i. $PPa = Pa$ and $QQa = Qa$
- ii. $PQa = QPa = 0$
- iii. $P(Qa:Qb) = 0$ and $Q(Qa:Qb) = Qa:Qb$

Proof: Only item iii. requires a comment. The parallel sum of two vectors is a linear combination of the vectors, therefore since Qa and Qb are in the subspace S^\perp , their parallel sum is also in S^\perp . Thus P applied to that parallel sum is the zero vector and Q applied to the parallel sum is invariant.

QED

Theorem 3: Given vectors a , b and c with all hybrid sums defined, the following hold:

- i. $a*b = b*a$
- ii. $ka*kb = k(a*b)$
- iii. $a*(b*c) = (a*b)*c$