Linear Algebra in Signals, Systems, and Control

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Preface

In recent years, the scientific and engineering communities have looked for ways to bridge the gap dividing mathematicians, computer scientists, and engineers working in signals, systems, and control. The interdisciplinary SIAM conference on Linear Algebra in Signals, Systems, and Control, held in Boston on August 12-14, 1986, was organized to help bridge this gap. The most recent in a series of conferences in this highly active area of research, it follows the 1984 AMS-IMS-SIAM Summer Research Conference on Linear Algebra and Its Role in Systems Theory.

The present volume consists of papers selected from invited and contributed presentations at the conference. The papers can be divided into the following broad categories:

1) Core Linear Algebra;
2) Numerical Linear Algebra;
3) Algorithms for Signals, Systems, and Control;
4) Linear and Nonlinear Control and Systems Theory.

We hope that the conference, together with this volume, will help to promote further joint development in these fields.

We take this opportunity to thank the following research agencies for their generous support of the conference: the National Science Foundation, the Air Force Office of Scientific Research, the Army Research Office, and the Office of Naval Research. We also thank the SIAM Activity Group on Linear Algebra for sponsoring the conference and Professor David Carlson and Dr. Robert Ward, in particular, for their support and enthusiasm. Throughout the entire process of organizing the conference and putting together these final manuscripts, we received enthusiastic support from many colleagues, including those who served as referees. We express our sincere thanks to all of them, especially to our referees, for their hard work. We also thank Professor Gregory Ammar of Northern Illinois University for his help in making the final program. Finally, we wish to thank Mrs. Sara Clayton and Mrs. Peggy Putzstuck of Northern Illinois University for their excellent secretarial assistance.

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Part I:
Core Linear Algebra

Network Matrix Operations for Vectors and Quaternions

W. N. ANDERSON* AND G. E. TRAPP†

ABSTRACT. Based on the concepts of the series and parallel sum of vectors, the shorted operator and hybrid addition are defined and examined. Matrix means are also generalized to the case of vectors since the series and parallel sums yield the arithmetic and harmonic means. An iterative definition of the geometric mean is reviewed and the Gaussian mean is then considered. It is shown that many properties of the network matrix operations and matrix means hold for vectors. Two special cases, quaternions and Cayley numbers, are also discussed.

INTRODUCTION. We are interested in a real n-dimensional Euclidian space. We will denote vectors by a, b, ... The Euclidian inner product will be written $\langle a,b \rangle$, and the norm induced by the inner product is $||a|| = \mathrm{sqrt}(\langle a,a \rangle)$. Given two vectors a and b with $a+b = 0$, we define their parallel sum, denoted a:b, as follows:

$$a:b = (a||b||^2 + b||a||^2)/(||a+b||^2).$$

In [5], we show that this is the natural vector generalization of the matrix parallel sum. The other fundamental network operation - series - corresponds to the normal sum of two vectors a + b. Using these two operations as building blocks, we will define other vector operations that generalize commonly used matrix operations and matrix means. In the next section we will define the shorted operator, the hydrid sum, and the contraharmonic mean of

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vectors. In section 3.0, we will review the iteration procedure used to define the geometric mean, and consider other related iteration procedures. In particular we will show that the Gaussian mean of two vectors may be defined. The last section of the paper considers two special vector cases: n=4 (quaternions) and n=8 (Cayley numbers).

We conclude this section by summarizing the basic properties of the parallel sum. The reader should consult [5] for additional information and proofs.

Theorem 1: If a and b are vectors such that $a + b \neq 0$ then the following hold:

i. $a:b = b:a$
ii. $(ka):(kb) = k(a:b)$
iii. $||a:b|| = ||a|| ||b|| / ||a+b||$
iv. $a:(b:c) = (a:b):c$ whenever all of the parallel sums are defined.

<u>NETWORK OPERATIONS DEFINED FOR VECTORS</u>. The series and parallel sums are motivated by the series and parallel connections of networks. Since other network connections are commonly used, we will define the corresponding network operations for vectors. The mixture of the series and parallel connections is the hybrid connection. In terms of networks, some of the connections are made in series and the remaining are made in parallel. We model this behavior by adding some of the components of the vectors in series and others in parallel. Results concerning the hybrid sum of matrices may be found in [10] or [12].

Let the n x n matrix P be the projection onto a subspace S of V; and let Q be the matrix I-P (here I is the identity matrix). Given vectors a and b such that $Qa + Qb \neq 0$, define the hydrid sum, denoted a*b, as follows:

$$a*b = Pa + Pb + (Qa):(Qb)$$

The following lemma concerning the projections P and Q and the parallel sum is required in the proof of Theorem 3.

Lemma 2: With P and Q as above and Qa and Qb parallel summable vectors, the following hold:

i. $PPa = Pa$ and $QQa = Qa$
ii. $PQa = QPa = 0$
iii. $P(Qa:Qb) = 0$ and $Q(Qa:Qb) = Qa:Qb$

Proof: Only item iii. requires a comment. The parallel sum of two vectors is a linear combination of the vectors, therefore since Qa and Qb are in the subpace $S^{\perp}$, their parallel sum is also in $S^{\perp}$. Thus P applied to that parallel sum is the zero vector and Q applied to the parallel sum is invariant.

QED

Theorem 3: Given vectors a, b and c with all hybrid sums defined, the following hold:

i. $a*b = b*a$
ii. $ka*kb = k(a*b)$
iii. $a*(b*c) = (a*b)*c$