geometric modeling for CAD applications

edited by m. j. wozny, h. w. mclaughlin and j. l. encarnaçao

IFIP

north-holland

GEOMETRIC MODELING FOR CAD APPLICATIONS

Selected and expanded papers from the IFIP WG 5.2 Working Conference
Rensselaerville, NY, USA, 12–16 May 1986

edited by

M. J. WOZNY
Rensselaer Polytechnic Institute
Troy, NY, USA

H. W. McLAUGHLIN
Rensselaer Polytechnic Institute
Troy, NY, USA

J. L. ENCARNAÇAO
Technische Hochschule Darmstadt
Darmstadt, BRD

N·H
P·C

1988

NORTH-HOLLAND
AMSTERDAM · NEW YORK · OXFORD · TOKYO

ISBN: 0 444 70416 7

Publishers:

ELSEVIER SCIENCE PUBLISHERS B.V.
P.O. Box 1991
1000 BZ Amsterdam
The Netherlands

Sole distributors for the U.S.A. and Canada:

ELSEVIER SCIENCE PUBLISHING COMPANY, INC.
52 Vanderbilt Avenue
New York, N.Y. 10017
U.S.A.

LIBRARY OF CONGRESS
Library of Congress Cataloging-in-Publication Data

IFIP WG 5.2 Working Conference on Geometric Modeling for CAD Applications (1986 : Rensselaerville, N.Y.)
Geometric modeling for CAD applications : selected and expanded papers from the IFIP WG 5.2 Working Conference, Rensselaerville, NY, USA, 12-14 May 1986 / edited by M.J. Wozny, H.W. McLaughlin, J.L. Encarnaçao.
p. cm.
Bibliography: p.
ISBN 0-444-70416-7
1. Geometry--Data processing--Congresses. 2. Computer graphics--Congresses. 3. Mathematical models--Congresses. 4. CAD/CAM systems--Congresses. I. Wozny, M. J. (Michael J.) II. McLaughlin, H. W. III. Encarnação, José Luis. IV. Title.
QA447.I35 1986
516'.0724--dc19

88-7175
CIP

PRINTED IN THE NETHERLANDS

IFIP WG 5.2 Working Conference on
Geometric Modeling for CAD Applications
Rensselaerville, NY, USA, 12–16 May 1986

NORTH-HOLLAND
AMSTERDAM · NEW YORK · OXFORD · TOKYO

PREFACE

This volume comprises the proceedings of the IFIP WG 5.2 Workshop on Geometric Modeling, 12-16 May 1986, Rensselaerville, New York, USA.

The five-day workshop was hosted by the Center for Interactive Computer Graphics of the Rensselaer Polytechnic Institute. The objective of the workshop was to classify the research sub-areas of geometry, and identify the open research issues. The papers in this volume give a good sampling of some of the important research ideas which are emerging in the field of geometric modelling as used in CAD/CAM.

Prof. Michael J. Wozny, Director, Center for Interactive Computer Graphics, opened the workshop by welcoming the 30 participants from 12 countries, and described the research activities in his Center. Peter R. Wilson, the first invited speaker, reviewed the research on solid modeling being done by his group at the General Electric Corporate Research Labs (GE CRD) in Schenectady, N.Y. Next, Professor Mark Shephard, Associate Director, Center for Interactive Computer Graphics, presented his invited talk on "Geometry Communication Operators Needed for Finite Element Modeling", emphasizing the interaction between geometry and finite element modeling.

For the "Open Sessions", the following six topic areas and session leaders were chosen:

1. Form Features in Solid Geometry (Representation, Extraction & Application) – Frank Tuijnman
2. Boundary Evaluation in Solids – John Dodsworth
3. Relationships between 2D & 3D Geometry, and User Interfaces – Kenneth Preiss
4. Topclogical Structures & Boundary Representation – Keven Weiler
5. Standardization & Data Bases Issues – Gunther Schrack
6. Free-Form Surfaces & Curves – Harry McLaughlin

The second day of the workshop was devoted to special invited presentations by Kevin Weiler of GE CRD and RPI/CICG (Topological Structures for Geometric Modeling), Josh Turner of IBM and RPI/CICG (Tolerances in Computer-Aided Geometric Design). The keynote address was given by Stuart Miller, Manager, Automation Systems Laboratory, GE CRD. Miller discussed CAD/CAM Technology Trends and Status.

Thirty presentations, divided into six sessions, were given on Wednesday and Thursday by the participants. All sessions were extremely exciting and resulted in extensive discussion. The Friday morning session was used to summarize the workshop presentations and discussions.

The attendees voted to make a permanent record of the deliberations of this workshop. Fifteen attendees agreed to rewrite their papers and extended abstract into full papers, subject to reviewing and editing, to be included in a book on geometric modeling.

TABLE OF CONTENTS

Preface v

Chapter 1: NON-MANIFOLD GEOMETRY

1.1 The Radial Edge Structure: A Topological Representation for Non-Manifold Geometric Boundary Modeling
K. Weiler 3

1.2 Boundary Graph Operators for Non-Manifold Geometric Modeling Topology Representation
K. Weiler 37

Chapter 2: SOLID GEOMETRY

2.1 Solid Modeling Research and Applications in the USA·
P.R. Wilson 69

2.2 Notes on the Synthesis of Surface and Volume Modelling
T. Varady 89

2.3 Multiple Representations of Solid Models
P.R. Wilson 99

2.4 The Role of Multidimensional Models in CAD Systems
T. Takala 115

Chapter 3: FORM FEATURES

3.1 A Taxonomy of Features for Solid Modeling
P.R. Wilson and M.J. Pratt 125

3.2 Form Feature Representations and Recognition in a Hierarchical Boundary Model
S. Ansaldi and B. Falcidieno 137

Chapter 4: TOLERANCES

4.1 A Mathematical Theory of Tolerances
J.U. Turner and M.J. Wozny 163

Chapter 5: SURFACE GEOMETRY AND SHAPE DESIGN

5.1 The Use of Triangular Patches in CAD
G. Farin 191

5.2 Blends in Geometric Modelling
T. Varady 195

5.3 Specification of Shape by Dimensions and other Geometric Constraints
G. Sunde 199

Chapter 6: STANDARDS

6.1 Data Transfer and Solid Modeling
P.R. Wilson 217

6.2 Open Issues and Trends on Graphics and Geometry Standards
J. Encarnaçao, J. Rix and J. Schönhut 255

Chapter 7: GEOMETRY APPLICATIONS

7.1 Geometric Modeling Needs of Finite Element Modeling
M.S. Shephard 271

7.2 Algorithms for Real-Time Tool Path Generation
G. Hermann 295

7.3 The Use of 3D CAD Models in Robot Vision
F. Tuijnman 307

Chapter 8: DATA MANAGEMENT

8.1 A Conceptual Framework for Data Management in Mechanical CAD
D.L. Spooner and M. Hardwick 317

NON-MANIFOLD GEOMETRY

Geometric Modeling for CAD Applications
M.J. Wozny, H.W. McLaughlin, J.L. Encarnaçao (eds.)
Elsevier Science Publishers B.V. (North-Holland)

The Radial Edge Structure: A Topological Representation for Non-Manifold Geometric Boundary Modeling

Kevin Weiler

General Electric
Corporate Research and Development
Schenectady, NY 12301

Abstract

Non-manifold geometric modeling using a boundary representation encompasses both the manifold and non-manifold object domain and allows the unified representation of wireframe, surface, solid, and cellular modeling forms simultaneously in the same modeling environment, while increasing the representable range of models beyond what is individually achievable in these other modeling forms.

The Radial Edge structure is the first complete non-manifold boundary modeling representation which explicitly represents topological adjacencies. Adjacencies between vertex, edge, face, and volume elements in a model are represented, providing a general topological framework which may be used with a variety of geometric curved surface representations in a geometric modeling system.

Non-manifold object modeling representations permit simpler and more uniform algorithms for object manipulations such as the Boolean shape operations. Also important, analysis such as finite element analysis may be carried out in the same representation as the original modeling representation, allowing the possibility of automated rather than current manual feedback in the industrial CAD design/analysis cycle.

This paper describes a computer implementation of the data structure and some of the issues behind its design.

Keywords:

geometric modeling, data structures, non-manifold, wireframe modeling, surface modeling, solid modeling, topology

Contents:

1. Introduction
2. Geometric Modeling
3. Topology as a Framework
4. Domain
5. The Non-Manifold Topological Elements
6. Adjacency Relationships in a Non-Manifold Model
7. Sufficiency
8. Design Issues in Non-Manifold Representations
    - 8.1 Direct Representation of Adjacency Uses
    - 8.2 Non-Manifold Conditions Along an Edge
    - 8.3 Non-Manifold Conditions at a Vertex
    - 8.4 Non-Manifold Wireframe Representation
    - 8.5 Separation Surfaces
9. The Radial Edge Data Structure
    - 9.1 Design Decisions
    - 9.2 Data Structures
    - 9.3 Geometry and Other Attributes
    - 9.4 Variations in Data Structures
10. Detecting Volume Closure by Face Additions
11. Application Areas for Expanded Modeling Capabilities
12. Conclusion

Acknowledgements

Bibliography

1. Introduction

Geometric modeling representation techniques, of which solid modeling is a more recent example, have found increasingly wider use in applications ranging from industrial mechanical part design to motion picture production.

One kind of geometric modeling representation that has found especially wide application is the *boundary representation* technique, where the surfaces, edges, and corner vertices of objects are represented explicitly. Wireframe and surface as well as some varieties of solid model representations are examples of boundary modeling techniques. Many of the more sophisticated boundary representations explicitly store topological information about the positional relationships among surfaces, edges, and vertices.

Until recently, boundary based solid modeling techniques which explicitly stored topological adjacency relationships were restricted to representing only manifold domains. This disallows such conditions as two surfaces touching at a single point, two surfaces touching along an open or closed curve, two distinct enclosed volumes sharing a face as a common boundary, and a wire edge emanating from a point on a surface. Yet common modeling operations, such as the Boolean set operations (both standard and regularized versions), glueing operations, and others, can produce results with these conditions, even with strictly manifold input.

These conditions have come to be known in the geometric modeling field as *non-manifold* conditions. The *non-manifold geometric modeling* domain, also referred to here as the non-manifold domain, encompasses both manifold and non-manifold conditions and is therefore quite general.

There are many advantages to a true non-manifold geometric modeling representation. Non-manifold representations provide a single unified representation for any combination of wireframe, surface, and solid modeling forms. Composite cellular objects and interior structures in objects can be modeled directly. Analysis applications such as finite element analysis can be directly supported in the modeling representation environment, allowing direct access by both modeling and analysis functions. Closed form Boolean operations are possible in a non-manifold representation.

While some non-boundary based representations do allow non-manifold conditions, they do not allow unrestricted representation of wire edge and surface elements; non-manifold boundary representations therefore also have a representational advantage over other representation forms. A few existing boundary modeling techniques have attempted to represent some specific non-manifold conditions, but have not been able to address the non-manifold domain in general.

This paper describes recently developed boundary modeling techniques which can represent all non-manifold conditions and which explicitly represent topological adjacencies. In particular, a new data structure, the Radial Edge structure, which provides access to topological adjacencies in a general non-manifold boundary representation, is discussed.

The main focus in this paper is in describing the actual data structures for the non-manifold topology representation and discussing some of the issues and resolutions involved in their original design. First a general discussion of geometric modeling forms is given to set a context for non-manifold representations. Second, a discussion on the use of topological representations as a framework for geometric modeling implementations is given. Third, a description of the non-manifold domain addressed is given. Fourth, the topological elements, their adjacency relationships, and topological sufficiency are discussed. Fifth, some design issues for non-manifold representations are briefly discussed. Sixth, the actual data structures of the Radial Edge structure are described. Seventh, an example of how the adjacencies in the Radial Edge representation assist in maintaining volume closure information is given. Lastly, improvements to some existing and new application areas through the use of non-manifold modeling techniques are briefly identified.

A companion paper, "Boundary Graph Operators for Non-Manifold Geometric Modeling Topology Representation" [Weiler 86b] describes a general set of non-manifold topology manipulation operators which is independent of a specific data structure and is useful for insulating higher levels of geometric modeling functionality from the specifics and complexities of underlying data structures.

The work described in these papers is reported in greater detail in the PhD. thesis by the author, "Topological Structures for Geometric Modeling", RPI, August 1986 [Weiler 86a], which also contains other work concerning the use of topology in geometric modeling.

2. Geometric Modeling

Geometric modeling techniques have evolved considerably over the last twenty years, embodying more and more information about the physical shape of the objects they model.

Wireframe modeling, one of the earliest geometric modeling techniques, represents objects by edge curves and points on the surface of the object.

Surface modeling techniques go one step further by also providing mathematical descriptions of the surface shapes of the object. Surface modeling techniques allow graphic display and numerical control machining of carefully constructed models, but usually offer few integrity checking features.

Solid modeling, a more recently developed technique, contains even more information about the closure and connectivity of shapes and is becoming an increasingly important part of the process of computer aided modeling of solid physical objects for design, analysis, manufacturing, simulation, and other applications. Solid modeling offers a number of advantages over previous surface modeling techniques, including an ability to calculate mass properties such as weight, volume and center of gravity, as well as a greater ability to guarantee the integrity and physical realizability of the model.

A wide variety of representational forms have been developed for solid models, each with its own strengths and weaknesses in the context of different applications. A taxonomy was developed in which solid modeling representations can be differentiated on at least three independent criteria: whether they are boundary based or volume based, object based or spatially based, and evaluated or unevaluated [Weiler 84]. A representation is boundary based if the solid volume is specified by its surface boundary; if the solid is specified directly by its volume it is volume based. A representation is object based if it is fundamentally organized according to the characteristics of the actual solid shape itself; it is spatially based when the representation is organized around the characteristics of the spatial coordinate system it uses. The evaluated/unevaluated characterization is roughly a measure of the amount of work necessary to obtain information about the objects being represented with respect to a stated goal; here, for simplicity, we assume the goal is wireframe display of an object.

Non-manifold geometric modeling is a modeling form which embodies all of the capabilities of the previous three modeling forms in a unified representation and extends the domain beyond that of the previous modeling forms.

In a manifold (two-manifold) representation, every point on a surface is two-dimensional; that is, every point has a neighborhood which is homeomorphic to a two-dimensional disk. In other words, even though the surface exists in three-dimensional space it is topologically "flat" when the surface is examined closely enough in a small area around any given point. Historically, boundary based solid modeling systems have used manifold representations ([Baumgart 72], [Baumgart 75], [Braid, Hillyard, & Stroud 78], and [Eastman & Weiler 79]).

A non-manifold representation is essentially a mesh embedded in space with optional bounded surface pieces which may or may not enclose volumes. A given point on a surface need not necessarily be "flat" in that the neighborhood of the point need not be a two-dimensional disk (see Figure 1). In this case the point is essentially at the intersection of two or more topologically two-dimensional surfaces, or a two-dimensional surface and a one-dimensional curve.

A set of common solid modeling operations, the Boolean set operations, are not closed under manifold representations. A modification of the Boolean operations, called the *regularized set operators* [Requicha 77], is designed to permit only volume filling results from the Boolean operations. The regularized set operations therefore avoid a subset of the non-manifold results which can result from applying the Boolean operations on manifold inputs. However, with some manifold inputs the results of Boolean

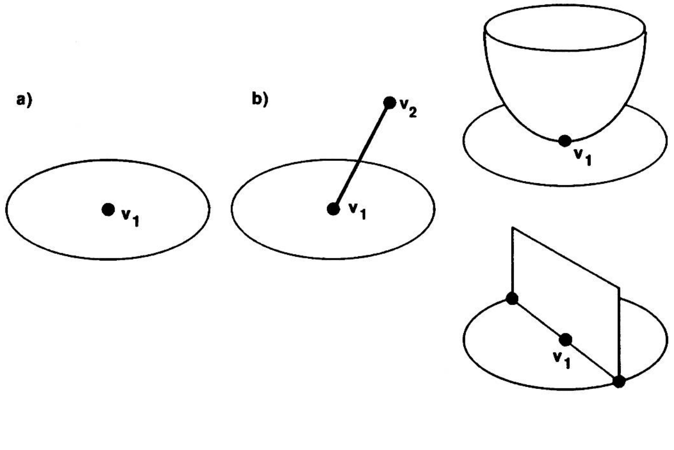

Figure 1. The 2-dimensional disk around points on a surface

operations, regularized or not, are non-manifold and therefore not representable under manifold representations. For example, an appendage reaching out from the main volume of an object and then touching back on the surface of the same object at a single point is not directly representable with manifolds, and creation of such an object even with regularized set operations cannot yield a valid manifold result (see Figure 2).

Non-manifold representations avoid such singularities by representing such non-manifold situations directly instead of restricting the domain of the output.

Overall, non-manifold representations have superior flexibility, can represent a larger variety of objects, and can support a wider variety of applications than manifold representations, at a cost of a larger size data structure. Boolean operation implementations operating on either manifold or non-manifold representations must detect and deal with non-manifold results in some fashion; however, in a non-manifold representation such results are uniformly and cleanly represented and manipulated. Non-manifold representations are still required if accurate closed form Boolean operations with faithful representation of non-manifold results are desired. Non-manifold representations are also required if one is interested in the interior volume structures in an object and the relationships between them, such as in composite or cellular objects and finite element meshes.

Perhaps most importantly, generalized non-manifold representations can represent wireframe, surface, and solid modeling representations simultaneously in a single uniform format. This uniformity offers significant advantages to the staging and delivery of geometric modeling systems as well as providing enhanced functionality and simplicity.

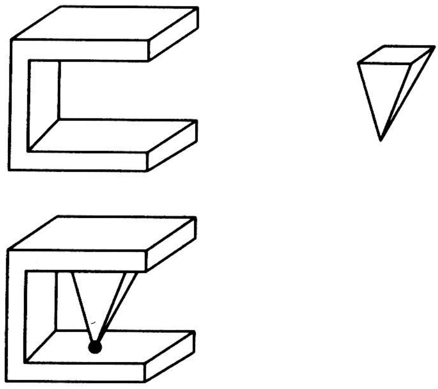

Figure 2. The Boolean union of two-manifold objects yielding a non-manifold result

3. Topology as a Framework

Geometry is considered here to represent essentially all information about the geometric shape of an object including where it lies in space and the precise geometric location of all aspects of its various elements.

Topology, by definition, is an abstraction, a coherent subset, of the information available from the geometry of a shape. More formally, it is a set of properties invariant under a restricted set of geometric transformations. Invariance under transformation implies that all information is not present in topology; topology is incomplete information which can theoretically be derived from the complete geometric specification. In the context of geometric modeling, when we think of topology we most often think of the adjacencies between topological elements such as vertices, edges, and faces.

Use of topological properties can simplify modeling algorithms and greatly improve their efficiency. However, for several reasons, topology can be even more useful when it serves as a framework around which the geometric modeling representation can be built [Weiler 84]. First, once the topological and geometric domain which the representation is intended to cover has been defined, and the corresponding topological representation has been selected, the topological portion of the *implementation* generally is less subject to change than the geometric portion of the implementation. Second, separation of topological and geometric information in a geometric modeling representation provides a more systematic approach to implementation, providing for simpler creation, verification, and analysis of the model.

4. Domain

The *domain* of a representation is the complete set of possibilities for which the representation is valid. The domain addressed by any representation should be carefully specified; it is the only measure of success of the representation and is the starting point for any formal proof of correctness.

This paper concentrates on the topological framework data structures for non-manifold boundary based object based evaluated representations. A wide variety of geometric surface representation formats can be added to this framework to provide a complete non-manifold modeling system. Related constraints on the geometric surface representations used are addressed where relevant.

A series of specifications on the geometric and topological domain for the non-manifold data structure representation to be described, which goes beyond the non-manifold boundary based object based evaluated characterization, now follows.

1. *Non-manifold Surfaces* - The representation is a non-manifold topological representation which allows the uniform representation of wireframe, surface, and solid modeling representations, allows Boolean operations in a closed form, and provides an extended domain which includes representation of the interior features of objects.

   The representation contains topological information in a graph structure embedded in three-dimensional Euclidean space. This embedded topological boundary graph structure provides a framework for the remaining geometric model information. The entire non-manifold structure is finite in extent. Any surfaces in it are orientable in the sense that the identity of the volume on each side of the surface is known.