LINEAR ALGEBRA
IDEAS AND APPLICATIONS

THIRD EDITION

RICHARD C. PENNEY

WILEY

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LINEAR ALGEBRA

Preface

I wrote this book because I have a deep conviction that mathematics is about ideas, not just formulas and algorithms, and not just theorems and proofs. The text covers the material usually found in a one or two semester linear algebra class. It is written, however, from the point of view that knowing *why* is just as important as knowing *how*.

To ensure that the readers see not only why a given fact is true, but also why it is important, I have included a number of the beautiful applications of linear algebra.

Most of my students seem to like this emphasis. For many, mathematics has always been a body of facts to be blindly accepted and used. The notion that they personally can decide mathematical truth or falsehood comes as a revelation. Promoting this level of understanding is the goal of this text.

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Features of the Text

Parallel Structure Most linear algebra texts begin with a long, basically computational, unit devoted to solving systems of equations and to matrix algebra and determinants. Students find this fairly easy and even somewhat familiar. But, after a third or more of the class has gone by peacefully, the boom falls. Suddenly, the students are asked to absorb abstract concept after abstract concept, one following on the heels of the other. They see little relationship between these concepts and the first part of the course or, for that matter, anything else they have ever studied. By the time the abstractions can be related to the first part of the course, many students are so lost that they neither see nor appreciate the connection.

This text is different. We have adopted a parallel mode of development in which the abstract concepts are introduced right from the beginning, along with the computational. Each abstraction is used to shed light on the computations. In this way, the students see the abstract part of the text as a natural outgrowth of the computational part. This is not the "mention it early but use it late" approach adopted by some texts. Once a concept such as linear independence or spanning is introduced, it becomes part of the vocabulary to be used frequently and repeatedly throughout the rest of the text.

The advantages of this kind of approach are immense. The parallel development allows us to introduce the abstractions at a slower pace, giving students a whole semester to absorb what was formerly compressed into two-thirds of a semester. Students have time to fully absorb each new concept before taking on another. Since the concepts are utilized as they are introduced, the students see *why* each concept is necessary. The relation between theory and application is clear and immediate.

Gradual Development of Vector Spaces One special feature of this text is its treatment of the concept of vector space. Most modern texts tend to introduce this concept fairly late. We introduce it early because we need it early. Initially, however, we do not develop it in any depth. Rather, we slowly expand the reader's understanding by introducing new ideas as they are needed.

This approach has worked extremely well for us. When we used more traditional texts, we found ourselves spending endless amounts of time trying to explain what a vector space is. Students felt bewildered and confused, not seeing any point to what they were learning. With the gradual approach, on the other hand, the question of what a vector space is hardly arises. *With this approach, the vector space concept seems to cause little difficulty for the students.*

Treatment of Proofs It is essential that students learn to read and produce proofs. Proofs serve both to validate the results and to explain why they are true. For many students, however, linear algebra is their first proof-based course. They come to the subject with neither the ability to read proofs nor an appreciation for their importance.

Many introductory linear algebra texts adopt a formal "definition-theorem-proof" format. In such a treatment, a student who has not yet developed the ability to read abstract mathematics can perceive both the statements of the theorems and their proofs (not to mention the definitions) as meaningless abstractions. They wind up reading only the examples in the hope of finding "patterns" that they can imitate to complete the assignments. In the end, such students wind up only mastering the computational techniques, since this is the only part of the course that has any meaning for them. In essence, we have taught them to be nothing more than slow, inaccurate computers.

Our point of view is different. *This text is meant to be read by the student - all of it!* We always work from the concrete to the abstract, never the opposite. We also make full use of geometric reasoning, where appropriate. We try to explain "analytically, algebraically, and geometrically." We use carefully chosen examples to motivate both the definitions and theorems. Often, the essence of the proof is already contained in the example. Despite this, we give complete and rigorous *student readable* proofs of most results.

Conceptual Exercises Most texts at this level have exercises of two types: proofs and computations. We certainly do have a number of proofs and we definitely have lots of computations. The vast majority of the exercises are, however, "conceptual, but not theoretical." That is, *each exercise asks an explicit, concrete question which requires the student to think conceptually in order to provide an answer.* Such questions are both more concrete and more manageable than proofs and thus are much better at demonstrating the concepts. They do not require that the student already have facility with abstractions. Rather, they act as a bridge between the abstract proofs and the explicit computations.

Applications Sections Doable as Self-Study Applications can add depth and meaning to the study of linear algebra. Unfortunately, just covering the "essential" topics in the typical first course in linear algebra leaves little time for additional material, such as applications.

Most of our sections are followed by one or more application sections that use the material just studied. This material is designed to be read unaided by the student and thus may be assigned as outside reading. As an aid to this, we have provided two levels of exercises: self-study questions and exercises. The self-study questions are designed to beanswerable with a minimal investment of time by anyone who has carefully read and digested the relevant material. The exercises require more thought and a greater depth of understanding. They would typically be used in parallel with classroom discussions.

We feel that, in general, there is great value in providing material that the students are responsible for learning on their own. Learning to read mathematics is the first step in learning to do mathematics. Furthermore, there is no way that we can ever teach everything the students need to know; we cannot even predict what they need to know. Ultimately, the most valuable skill we teach is the ability to teach oneself. The applications form a perfect vehicle for this in that an imperfect mastery of any given application will not impede the student's understanding of linear algebra.

Early Orthogonality Option We have designed the text so that the chapter on orthogonality, with the exception of the last three sections, may be done immediately following Chapter 3 rather than after the section on eigenvalues.

True-False Questions We have included true-false questions for most sections.

Chapter Summaries At the end of each chapter there is a chapter summary that brings together major points from the chapter so students can get an overview of what they just learned.

Student Tested This text has been used over a period of years by numerous instructors at both Purdue and other universities nationwide. We have incorporated comments from instructors, reviewers, and (most important) students.

Technology Most sections of the text include a selection of computer exercises under the heading Computer Projects. Each exercise is specific to its section and is designed to support and extend the concepts discussed in that section.

These exercises have a special feature: They are designed to be "freestanding." In principle, the instructor should not need to spend any class time at all discussing computing. Everything most students need to know is right there. In the text, the discussion is based on MATLAB. However, translations of the exercises into Maple are contained in the Student Resource Manual. They are also available free on the World Wide Web at http://www.math.purdue.edu/~rcp/LAText.

Meets LACSG Recommendations The Linear Algebra Curriculum Study Group (LACSG) recommended that the first class in linear algebra be a "student-oriented" class that considers the "client disciplines" and that makes use of technology. The above comments make it clear that this text meets these recommendations. The LACSG also recommended that the first class be "matrix-oriented." We emphasize matrices throughout.

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Systems of Linear Equations

1.1 THE VECTOR SPACE OF $m \times n$ MATRICES

It is difficult to go through life without seeing matrices. For example, the 2008 annual report of Acme Squidget might contain the table 1.1, which shows how much profit (in millions of dollars) each branch made from the sale of each of the company's three varieties of squidgets in 2008.

Table 1.1 Profits: 2008

	Red	*Blue*	*Green*	*Total*
Kokomo	11.4	5.7	6.3	23.4
Phili	9.1	6.7	5.5	21.3
Oakland	14.3	6.2	5.0	25.5
Atlanta	10.0	7.1	5.7	22.8
Total	44.8	25.7	22.5	93.0

If we were to enter this data into a computer, we might enter it as a rectangular array without labels. Such an array is called a **matrix**. The Acme profits for 2008 would be described by the following matrix. This matrix is a 5×4 matrix (read "five by four") in that it has five rows and four columns. We would also say that its "size" is 5×4. In general, a matrix is said to have **size** $m \times n$ if it has m rows and n columns.

$$P = \begin{bmatrix} 11.4 & 5.7 & 6.3 & 23.4 \\ 9.1 & 6.7 & 5.5 & 21.3 \\ 14.3 & 6.2 & 5.0 & 25.5 \\ 10.0 & 7.1 & 5.7 & 22.8 \\ 44.8 & 25.7 & 22.5 & 93.0 \end{bmatrix}$$

Definition 1. *The set of all $m \times n$ matrices is denoted $M(m, n)$.*

Each row of an $m \times n$ matrix may be thought of as a $1 \times n$ matrix. The rows are numbered from top to bottom. Thus, the second row of the Acme profit matrix is the 1×4 matrix

$$[9.1, 6.7, 5.5, 21.3]$$

This matrix would be called the "profit vector" for the Phili branch. (In general, any matrix with only one row is called a **row vector**. For the sake of legibility, we usually separate the entries in row vectors by commas, as above.)

Similarly, a matrix with only one column is called a **column vector**. The columns are numbered from left to right. Thus, the third column of the Acme profit matrix is the column vector

$$\begin{bmatrix} 6.3 \\ 5.5 \\ 5.0 \\ 5.7 \\ 22.5 \end{bmatrix}$$

This matrix is the "green squidget profit vector."

If $A_1, A_2, \ldots, A_n$ is a sequence of $m \times 1$ column vectors, then the $m \times n$ matrix A that has the A_i as columns is denoted

$$A = [A_1, A_2, \ldots, A_n]$$

Similarly, if $B_1, B_2, \ldots, B_m$ is a sequence of $1 \times n$ row vectors, then the $m \times n$ matrix B that has the B_i as rows is denoted

$$B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_m \end{bmatrix}$$

In general, if a matrix is denoted by an uppercase letter, such as A, then the entry in the ith row and jth column may be denoted by either A_{ij} or a_{ij}, using the corresponding lowercase letter. We shall refer to a_{ij} as the "(i, j) entry of A." For example, for the matrix P above, the $(2, 3)$ entry is $p_{23} = 5.5$. Note that the row number comes first. Thus, the most general 2×3 matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

We will also occasionally write "$A = [a_{ij}]$," meaning that "A is the matrix whose (i, j) entry is a_{ij}."

At times, we want to take data from two tables, manipulate it in some manner, and display it in a third table. For example, suppose that we want to study the performance