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THEORY AND PROBLEMS OF

MODERN PHYSICS

RONALD GAUTREAU

WILLIAM SAVIN

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SCHAUM'S OUTLINE OF

THEORY AND PROBLEMS

of

MODERN PHYSICS

by

RONALD GAUTREAU, Ph.D.
Associate Professor of Physics
New Jersey Institute of Technology

and

WILLIAM SAVIN, Ph.D.
Associate Professor of Physics
New Jersey Institute of Technology

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Dedicated to the memory of
Professor Marcus M. Mainardi

Preface

The area of Modern Physics embraces topics that have evolved since roughly the turn of this century. These developments can be mind-boggling, as with the effects on time predicted by Einstein's Special Theory of Relativity, or quite practical, like the many devices based upon semiconductors, whose explanation lies in the band theory of solids.

The scope of the present book may be gauged from the Table of Contents. Each chapter consists of a succinct presentation of the principles and "meat" of a particular subject, followed by a large number of completely solved problems that naturally develop the subject and illustrate the principles. It is the authors' conviction that these solved problems are a valuable learning tool. The solved problems have been made short and to the point, and have been ordered in terms of difficulty. They are followed by unsolved supplementary problems, with answers, which allow the reader to check his grasp of the material.

It has been assumed that the reader has had the standard introductory courses in general physics, and the book is geared primarily at the sophomore or junior level, although we have also included problems of a more advanced nature. While it will certainly serve as a supplement to any standard Modern Physics text, this book is sufficiently comprehensive and self-contained to be used by itself to learn the principles of Modern Physics.

We extend special thanks to David Beckwith for meticulous editing and for input that improved the final version of the book. Any mistakes are ours, of course, and we would appreciate having these pointed out to us. Finally, we are indebted to our families for their enormous patience with us throughout the long preparation of this work.

RONALD GAUTREAU
WILLIAM SAVIN

New Jersey Institute of Technology

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Contents

CONTENTS

CONTENTS

CONTENTS

CONTENTS

PART I: *The Special Theory of Relativity*

Chapter 1

Galilean Transformations

1.1 EVENTS AND COORDINATES

We begin by considering the concept of a physical event. The event might be the striking of a tree by a lightning bolt or the collision of two particles, and happens at a point in space and at an instant in time. The particular event is specified by an observer by assigning to it four coordinates: the three position coordinates x, y, z that measure the distance from the origin of a coordinate system where the observer is located, and the time coordinate t that the observer records with his clock.

Consider now two observers, O and O', where O' travels with a constant velocity v with respect to O along their common x-x' axis (Fig. 1-1). Both observers are equipped with metersticks and clocks so that they can measure coordinates of events. Further, suppose both observers adjust their clocks so that when they pass each other at $x = x' = 0$, the clocks read $t = t' = 0$. Any given event P will have eight numbers associated with it, the four coordinates (x, y, z, t) assigned by O and the four coordinates (x', y', z', t') assigned (to the same event) by O'.
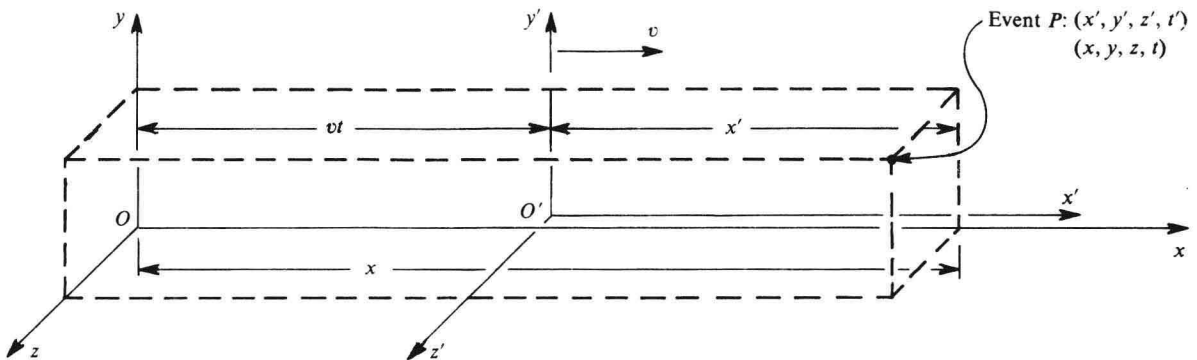
Fig. 1-1

1.2 GALILEAN COORDINATE TRANSFORMATIONS

The relationship between the measurements (x, y, z, t) of O and the measurements (x', y', z', t') of O' for a particular event is obtained by examining Fig. 1-1:

$$x' = x - vt \qquad y' = y \qquad z' = z$$

In addition, in classical physics it is implicitly assumed that

$$t' = t$$

These four equations are called the *Galilean coordinate transformations.*

1.3 GALILEAN VELOCITY TRANSFORMATIONS

In addition to the coordinates of an event, the velocity of a particle is of interest. Observers O and O' will describe the particle's velocity by assigning three components to it, with (u_x, u_y, u_z) being the velocity components as measured by O, and (u'_x, u'_y, u'_z) being the velocity components as measured by O'.

1

The relationship between (u_x, u_y, u_z) and (u'_x, u'_y, u'_z) is obtained from the time differentiation of the Galilean coordinate transformations. Thus, from $x' = x - vt$,

$$u'_x = \frac{dx'}{dt'} = \frac{d}{dt}(x - vt)\frac{dt}{dt'} = \left(\frac{dx}{dt} - v\right)(1) = u_x - v$$

Altogether, the *Galilean velocity transformations* are

$$u'_x = u_x - v \qquad u'_y = u_y \qquad u'_z = u_z$$

1.4 GALILEAN ACCELERATION TRANSFORMATIONS

The acceleration of a particle is the time derivative of its velocity, i.e. $a_x = du_x/dt$, etc. To find the *Galilean acceleration transformations* we differentiate the velocity transformations and use the facts that $t' = t$ and $v = $ constant to obtain

$$a'_x = a_x \qquad a'_y = a_y \qquad a'_z = a_z$$

Thus the measured acceleration components are the same for all observers moving with uniform relative velocity.

1.5 INVARIANCE OF AN EQUATION

By *invariance* of an equation it is meant that the equation will have the same form when determined by two observers. In classical theory it is assumed that space and time measurements of two observers are related by the Galilean transformations. Thus, when a particular form of an equation is determined by one observer, the Galilean transformations can be applied to this form to determine the form for the other observer. If both forms are the same, the equation is invariant under the Galilean transformations. See Problems 1.11 and 1.12.

Solved Problems

1.1. A passenger in a train moving at 30 m/s passes a man standing on a station platform at $t = t' = 0$. Twenty seconds after the train passes him, the man on the platform determines that a bird flying along the tracks in the same direction as the train is 800 m away. What are the coordinates of the bird as determined by the passenger?

The coordinates assigned to the bird by the man on the station platform are

$$(x, y, z, t) = (800 \text{ m}, 0, 0, 20 \text{ s})$$

The passenger measures the distance x' to the bird as

$$x' = x - vt = 800 \text{ m} - (30 \text{ m/s})(20 \text{ s}) = 200 \text{ m}$$

Therefore the bird's coordinates as determined by the passenger are

$$(x', y', z', t') = (200 \text{ m}, 0, 0, 20 \text{ s})$$

1.2. Refer to Problem 1.1. Five seconds after making the first coordinate measurement, the man on the platform determines that the bird is 850 m away. From these data find the velocity of the bird (assumed constant) as determined by the man on the platform and by the passenger on the train.

The coordinates assigned to the bird at the second position by the man on the platform are

$$(x_2, y_2, z_2, t_2) = (850 \text{ m}, 0, 0, 25 \text{ s})$$

Hence, the velocity u_x of the bird as measured by the man on the platform is

$$u_x = \frac{x_2 - x_1}{t_2 - t_1} = \frac{850 \text{ m} - 800 \text{ m}}{25 \text{ s} - 20 \text{ s}} = +10 \text{ m/s}$$

The positive sign indicates the bird is flying in the positive x-direction. The passenger finds that at the second position the distance x_2' to the bird is

$$x_2' = x_2 - vt_2 = 850 \text{ m} - (30 \text{ m/s})(25 \text{ s}) = 100 \text{ m}$$

Thus, $(x_2', y_2', z_2', t_2') = (100 \text{ m}, 0, 0, 25 \text{ s})$, and the velocity u_x' of the bird as measured by the passenger is

$$u_x' = \frac{x_2' - x_1'}{t_2' - t_1'} = \frac{100 \text{ m} - 200 \text{ m}}{25 \text{ s} - 20 \text{ s}} = -20 \text{ m/s}$$

so that, as measured by the passenger, the bird is moving in the negative x'-direction. Note that this result is consistent with that obtained from the Galilean velocity transformation:

$$u_x' = u_x - v = 10 \text{ m/s} - 30 \text{ m/s} = -20 \text{ m/s}$$

1.3. A sample of radioactive material, at rest in the laboratory, ejects two electrons in opposite directions. One of the electrons has a speed of $0.6c$ and the other has a speed of $0.7c$, as measured by a laboratory observer. According to classical velocity transformations, what will be the speed of one electron as measured from the other?

Let observer O be at rest with respect to the laboratory and let observer O' be at rest with respect to the particle moving with speed $0.6c$ (taken in the positive direction). Then, from the Galilean velocity transformation,

$$u_x' = u_x - v = -0.7c - 0.6c = -1.3c$$

This problem demonstrates that velocities greater than the speed of light are possible with the Galilean transformations, a result that is inconsistent with Special Relativity.

1.4. A train moving with a velocity of 60 mi/hr passes through a railroad station at 12:00. Twenty seconds later a bolt of lightning strikes the railroad tracks one mile from the station in the same direction that the train is moving. Find the coordinates of the lightning flash as measured by an observer at the station and by the engineer of the train.

Both observers measure the time coordinate as

$$t = t' = (20 \text{ s})\left(\frac{1 \text{ hr}}{3600 \text{ s}}\right) = \frac{1}{180} \text{ hr}$$

The observer at the station measures the spatial coordinate to be $x = 1$ mi. The spatial coordinate as determined by the engineer of the train is

$$x' = x - vt = 1 \text{ mi} - (60 \text{ mi/hr})\left(\frac{1}{180} \text{ hr}\right) = \frac{2}{3} \text{ mi}$$

1.5. A hunter on the ground fires a bullet in the northeast direction which strikes a deer 0.25 miles from the hunter. The bullet travels with a speed of 1800 mi/hr. At the instant when the bullet is fired, an airplane is directly over the hunter at an altitude of one mile and is traveling due east with a velocity of 600 mi/hr. When the bullet strikes the deer, what are the coordinates as determined by an obse₁ver in the airplane?

Using the Galilean transformations,

$$t' = t = \frac{0.25 \text{ mi}}{1800 \text{ mi/hr}} = 1.39 \times 10^{-4} \text{ hr}$$

$$x' = x - vt = (0.25 \text{ mi}) \cos 45° - (600 \text{ mi/hr})(1.39 \times 10^{-4} \text{ hr}) = 0.094 \text{ mi}$$

$$y' = y = (0.25 \text{ mi}) \sin 45° = 0.177 \text{ mi}$$

$$z' = z - h = 0 - 1 \text{ mi} = -1 \text{ mi}$$

1.6.   An observer, at rest with respect to the ground, observes the following collision.   A particle of
mass $m_1 = 3$ kg moving with velocity $u_1 = 4$ m/s along the x-axis approaches a second particle
of mass $m_2 = 1$ kg moving with velocity $u_2 = -3$ m/s along the x-axis.   After a head-on
collision the ground observer finds that m_2 has velocity $u_2^* = 3$ m/s along the x-axis.   Find
the velocity u_1^* of m_1 after the collision.

$$\text{initial momentum} = \text{final momentum}$$
$$m_1 u_1 + m_2 u_2 = m_1 u_1^* + m_2 u_2^*$$
$$(3\text{ kg})(4\text{ m/s}) + (1\text{ kg})(-3\text{ m/s}) = (3\text{ kg})u_1^* + (1\text{ kg})(3\text{ m/s})$$
$$9\text{ kg}\cdot\text{m/s} = (3\text{ kg})u_1^* + 3\text{ kg}\cdot\text{m/s}$$

Solving, $u_1^* = 2$ m/s.

1.7.   A second observer, O', who is walking with a velocity of 2 m/s relative to the ground along
the x-axis observes the collision described in Problem 1.6.   What are the system momenta
before and after the collision as determined by him?

Using the Galilean velocity transformations,
$$u_1' = u_1 - v = 4\text{ m/s} - 2\text{ m/s} = 2\text{ m/s}$$
$$u_2' = u_2 - v = -3\text{ m/s} - 2\text{ m/s} = -5\text{ m/s}$$
$$u_1^{*\prime} = u_1^* - v = 2\text{ m/s} - 2\text{ m/s} = 0$$
$$u_2^{*\prime} = u_2^* - v = 3\text{ m/s} - 2\text{ m/s} = 1\text{ m/s}$$
$$(\text{initial momentum})' = m_1 u_1' + m_2 u_2' = (3\text{ kg})(2\text{ m/s}) + (1\text{ kg})(-5\text{ m/s}) = 1\text{kg}\cdot\text{m/s}$$
$$(\text{final momentum})' = m_1 u_1^{*\prime} + m_2 u_2^{*\prime} = (3\text{ kg})(0) + (1\text{ kg})(1\text{ m/s}) = 1\text{kg}\cdot\text{m/s}$$

Thus, as a result of the Galilean transformations, O' also determines that momentum is conserved (but at
a different value from that found by O).

1.8.   An open car traveling at 100 ft/s has a boy in it who throws a ball upward with a velocity of
20 ft/s.   Write the equation of motion (giving position as a function of time) for the ball as
seen by (a) the boy, (b) an observer stationary on the road.

(a)   For the boy in the car the ball travels straight up and down, so
$$y' = v_0 t' + \tfrac{1}{2}at'^2 = (20\text{ ft/s})t' + \tfrac{1}{2}(-32\text{ ft/s}^2)t'^2 = 20t' - 16t'^2$$
$$x' = z' = 0$$

(b)   For the stationary observer, one obtains from the Galilean transformations
$$t = t'$$
$$x = x' + vt = 0 + 100t \qquad y = y' = 20t - 16t^2 \qquad z = z' = 0$$

1.9.   Consider a mass attached to a spring and moving on a horizontal, frictionless surface.   Show,
from the classical transformation laws, that the equations of motion of the mass are the same
as determined by an observer at rest with respect to the surface and by a second observer
moving with constant velocity along the direction of the spring.

The equation of motion of the mass, as determined by an observer at rest with respect to the surface,
is $F = ma$, or
$$-k(x - x_0) = m\frac{d^2x}{dt^2} \tag{1}$$

To determine the equation of motion as found by the second observer we use the Galilean transforma-
tions to obtain
$$x = x' + vt' \qquad x_0 = x_0' + vt' \qquad \frac{d^2x}{dt^2} = \frac{d^2x'}{dt'^2}$$

Substituting these values in (1) gives
$$-k(x' - x_0') = m\frac{d^2x'}{dt'^2} \tag{2}$$

Because (*1*) and (*2*) have the same form, the equation of motion is invariant under the Galilean transformations.

1.10. Show that the electromagnetic wave equation,

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 0$$

is not invariant under the Galilean transformations.

The equation will be invariant if it retains the same form when expressed in terms of the new variables x', y', z', t'. We first find from the Galilean transformations that

$$\frac{\partial x'}{\partial x} = 1 \qquad \frac{\partial x'}{\partial t} = -v \qquad \frac{\partial t'}{\partial t} = \frac{\partial y'}{\partial y} = \frac{\partial z'}{\partial z} = 1$$

$$\frac{\partial x'}{\partial y} = \frac{\partial x'}{\partial z} = \frac{\partial y'}{\partial x} = \frac{\partial t'}{\partial x} = \cdots = 0$$

From the chain rule and using the above results we have

$$\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial x'} \frac{\partial x'}{\partial x} + \frac{\partial \phi}{\partial y'} \frac{\partial y'}{\partial x} + \frac{\partial \phi}{\partial z'} \frac{\partial z'}{\partial x} + \frac{\partial \phi}{\partial t'} \frac{\partial t'}{\partial x} = \frac{\partial \phi}{\partial x'} \qquad \text{and} \qquad \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial x'^2}$$

Similarly,

$$\frac{\partial^2 \phi}{\partial y^2} = \frac{\partial^2 \phi}{\partial y'^2} \qquad \frac{\partial^2 \phi}{\partial z^2} = \frac{\partial^2 \phi}{\partial z'^2}$$

Moreover,

$$\frac{\partial \phi}{\partial t} = -v \frac{\partial \phi}{\partial x'} + \frac{\partial \phi}{\partial t'} \qquad \frac{\partial^2 \phi}{\partial t^2} = \frac{\partial^2 \phi}{\partial t'^2} - 2v \frac{\partial^2 \phi}{\partial x' \partial t'} + v^2 \frac{\partial^2 \phi}{\partial x'^2}$$

Substituting these expressions in the wave equation gives

$$\frac{\partial^2 \phi}{\partial x'^2} + \frac{\partial^2 \phi}{\partial y'^2} + \frac{\partial^2 \phi}{\partial z'^2} - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t'^2} + \frac{1}{c^2} \left(2v \frac{\partial^2 \phi}{\partial x' \partial t'} - v^2 \frac{\partial^2 \phi}{\partial x'^2} \right) = 0$$

Therefore the wave equation is *not* invariant under the Galilean transformations, for the form of the equation has changed.

The electromagnetic wave equation follows from Maxwell's equations of electromagnetic theory. By applying the procedure described here to Maxwell's equations, one finds that Maxwell's equations also are *not* invariant under Galilean transformations. Compare with Problem 6.23.

Supplementary Problems

1.11. A man (O') in the back of a 20-ft flatcar moving at 30 ft/s records that a flashbulb is fired in the front of the flatcar two seconds after he has passed a man (O) on the ground. Find the coordinates of the event as determined by each observer.   *Ans.* $(x', t') = (20 \text{ ft}, 2 \text{ s})$; $(x, t) = (80 \text{ ft}, 2 \text{ s})$

1.12. A boy sees a deer run directly away from him. The deer is running with a speed of 20 mi/hr. The boy gives chase and runs with a speed of 8 mi/hr. What is the speed of the deer relative to the boy?
Ans.   12 mi/hr

1.13. A boy in a train throws a ball in the forward direction with a speed of 20 mi/hr. If the train is moving with a speed of 80 mi/hr, what is the speed of the ball as measured by a man on the ground?
Ans.   100 mi/hr

1.14.   A passenger walks backward along the aisle of a train with a speed of 2 mi/hr as the train moves along a straight track at a constant speed of 60 mi/hr with respect to the ground.   What is the passenger's speed as measured by an observer standing on the ground?   *Ans.*   58 mi/hr

1.15.   A conductor standing on a railroad platform synchronizes his watch with the engineer in the front of a train traveling at 60 mi/hr.   The train is 1/4 mile long.   Two minutes after the train leaves the platform a brakeman in the caboose lights a cigarette.   What are the coordinates of the brakeman, as determined by the engineer and by the conductor, when the cigarette is lit?
Ans. $(x', t') = (-\frac{1}{4}$ mi, 2 min$)$; $(x, t) = (1\frac{3}{4}$ mi, 2 min$)$

1.16.   A man sitting in a train lights two cigarettes, one ten minutes after the other.   The train is moving in a straight line with a velocity of 20 m/s.   What is the distance separation as measured by a man on the ground?   *Ans.*   12,000 m

1.17.   A one-kilogram ball is constrained to move to the north at 3 m/s.   It makes a perfectly elastic collision with an identical second ball which is at rest, and both balls move on a north-south axis after the collision.   Compute, in the laboratory system, the total momentum before and after the collision.
Ans.   3 kg · m/s

1.18.   For Problem 1.17 calculate the total energy before and after the collision.   *Ans.*   4.5 J

1.19.   Refer to Problem 1.17.   Calculate the total momentum before and after the collision as measured by an observer moving northwards at 1.5 m/s.   *Ans.*   0

1.20.   For the observer in Problem 1.19 calculate the total energy before and after the collision.
Ans.   2.25 J

1.21.   Repeat Problems 1.19 and 1.20 for an observer moving eastwards at 2 m/s.
Ans.   5 kg · m/s 37° north of west; 8.5 J

1.22.   A person is in a boat moving eastwards with a speed of 15 ft/s.   At the instant that the boat passes a dock a person on the dock throws a rock northwards.   The rock strikes the water 6 s later at a distance of 150 ft from the dock.   Find the coordinates of the splash as measured by the person in the boat.
Ans. $(x, y, t) = (-90$ ft, 150 ft, 6 s$)$

1.23.   Consider a one-dimensional, elastic collision that takes place along the *x*-axis of *O*.   Show, from the classical transformation equations, that kinetic energy will also be conserved as determined by a second observer, *O'*, who moves with constant velocity *u* along the *x*-axis of *O*.

Chapter 2

The Postulates of Einstein

2.1 ABSOLUTE SPACE AND THE ETHER

A consequence of the Galilean velocity transformations is that if a certain observer measures a light signal to travel with the velocity $c = 3 \times 10^8$ m/s, then any other observer moving relative to him will measure the same light signal to travel with a velocity different from c. What determines the particular reference frame such that if an observer is at rest relative to this frame, this privileged observer will measure the value c for the velocity of light signals?

Before Einstein it was generally believed that this privileged observer was the same observer for whom Maxwell's equations were valid. Maxwell's equations describe electromagnetic theory and predict that electromagnetic waves will travel with the speed $c = 1/\sqrt{\epsilon_0 \mu_0} = 3 \times 10^8$ m/s. The space that was at rest with respect to this privileged observer was called "absolute space." Any other observer moving with respect to this absolute space would find the speed of light to be different from c. Since light is an electromagnetic wave, it was felt by 19th century physicists that a medium must exist through which the light propagated. Thus it was postulated that the "ether" permeated all of absolute space.

2.2 THE MICHELSON-MORLEY EXPERIMENT

If an ether exists, then an observer on the earth moving through the ether should notice an "ether wind." An apparatus with the sensitivity to measure the earth's motion through the hypothesized ether was developed by Michelson in 1881, and refined by Michelson and Morley in 1887. The outcome of the experiment was that *no motion through the ether was detected*. See Problems 2.4, 2.5 and 2.6.

2.3 LENGTH AND TIME MEASUREMENTS— A QUESTION OF PRINCIPLE

The one element common to both the null result of the Michelson-Morley experiment and the fact that Maxwell's equations hold only for a privileged observer is the Galilean transformations. These "obvious" transformations were reexamined by Einstein from what might be termed an "operational" point of view. Einstein took the approach that any quantity relevant to physical theories should, at least in principle, have a well-defined procedure by which it is measured. If such a procedure cannot be formulated, then the quantity should not be employed in physics.

Einstein could find no way to justify operationally the Galilean transformation $t' = t$, i.e. the statement that two observers *can* measure the time of an event to be the same. Consequently, the transformation $t' = t$, and with it the rest of the Galilean transformations, was rejected by Einstein.

2.4 THE POSTULATES OF EINSTEIN

Einstein's guiding idea, which he called the *Principle of Relativity*, was that *all* nonaccelerating observers should be treated equally in all respects, even if they are moving (at constant velocity) relative to each other. This principle can be formalized as follows:

Postulate 1: The laws of physics are the same (invariant) for all inertial (nonaccelerating) observers.

7