COLLEGE ALGEBRA

William Bosch

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Brooks/Cole Publishing Company
Monterey, California

CONTEMPORARY UNDERGRADUATE MATHEMATICS SERIES
Consulting Editor: Robert J. Wisner

Brooks/Cole Publishing Company
A Division of Wadsworth, Inc.

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Library of Congress Cataloging in Publication Data

Bosch, William, [date]
College algebra.

Includes index.

1. Algebra. I. Title.
QA154.2.B67 1983 512.9 83-18953

ISBN 0-534-02866-7

Sponsoring Editor: *Craig F. Barth*
Production Editors: *Patricia E. Cain and Jane Stanley*
Manuscript Editor: *Dex Ott*
Interior and Cover Design: *Vernon T. Boes*
Cover Illustration: *David Aguero*
Art Coordinator: *Judith Macdonald*
Interior Illustration: *Lori Heckelman*
Typesetting: *Science Typographers, Long Island, New York*
Cover Printing: *Phoenix Color Corporation, Phoenix, Arizona*
Printing and Binding: *R. R. Donnelley and Sons Company, Crawfordsville, Indiana*

This book is dedicated to my wife, Peggy,
and to our parents, Mr. and Mrs. Wendelin W. Bosch
and Colonel and Mrs. Edward E. Robinson.

PREFACE

College Algebra was designed to accomplish the development of skills necessary in mathematics, and to facilitate the application of those skills to the sciences and other fields. In achieving these goals, we have written a text that differs in a number of ways from college algebra books currently available.

1. We include a number of new, interesting and challenging exercises. While we provide problems like

$$\text{Simplify:} \quad \left(\frac{a^2b^{-1}}{c^{-2}}\right)^3$$

to give the student a chance to develop confidence, we also include problems like the following:

$$\text{Simplify:} \quad \frac{s^{3/2} - b^2s^{-1/2}}{s + b}$$

$$\text{Simplify:} \quad \frac{(a^2 - x^2)^{1/2} - (a^2 + x^2)(a^2 - x^2)^{-1/2}}{a^2 - x^2}$$

We include a large number of verbal problems. In some sections, the set of verbal problems is divided into two groups according to level of challenge.

This book is structured so that the student works exercises designed to move him or her from the level of intermediate algebra to a level somewhat beyond the average college algebra text. This added competence should enable the student to more readily use algebra skills in other courses or in job situations.

The following exercise sets contain some exercises usually not found in a college algebra text but very common in later mathematics or science courses.

Exercise Set	Exercises
1.4	49–71
2.5	89–96
3.1	44–48
3.2	67–84
3.3	13–24
3.3	39–44
4.1	43–50

(and so on)

2. We have assumed that a student using this text has had some experience with algebra. Here are some consequences of this assumption:

a. The first section deals with the distributive law. The distributive law is the basis for the techniques used to expand and factor polynomials, simplify fractions, and solve equations. It is also the source of many of the difficulties students have with algebra. In order to address this problem we identify the distributive law as providing a unifying theme for the first two chapters and immediately present the student with significant algebraic problems. We feel that this approach will motivate the student more effectively than would the standard light review of the structure of the real number system, sets, order relations, and so on.

b. Definitions and concepts are not introduced until they serve a purpose. Concepts (such as the associative law) that generally do not cause difficulties for the student are simply assumed without comment. No attempt is made to review all of intermediate algebra.

3. In this book the exposition is based on examples. Our experience has convinced us that students are more likely to study examples than to read pages of prose. Many of the definitions and techniques are woven into the examples. Naturally not every concept is expressed or developed by example. Explanations before or after examples are also used.

We have tried to limit the number of definitions and terms. The algebra student is too often overwhelmed by detail and terminology–much of it unnecessary. Reducing the number of terms also unifies the subject. This should help the student maintain a sense of direction.

4. The topics in this book are not presented as a sequence of isolated tricks. A technique is presented and then used again and again. This spiral arrangement is developed as much as possible. It should help the student learn algebra as a unified subject rather than as a bag of tricks.

5. Some important applied mathematics techniques are covered in this text. These techniques include optimization of quadratic functions (Section 6.3) and fitting straight lines to data (Section 6.2).

The book is arranged so as to provide the instructor with considerable flexibility. Exercises and examples that are designed mainly for calculus or science bound students are designated by the labels "Preparing for calculus" or "Preparing for science". Students not planning to continue in mathematics or science can skip those topics and exercises without being handicapped later in the text.

The instructor with a lot of time may be able to cover most or all of the text in a semester. At the other extreme, an instructor could omit Chapters 1-3, Sections 4.5, 4.6, 5.5, 6.2, 6.4, 7.3, 8.4, 9.5, 9.6, 9.7, and Chapter 11. This could be done without serious loss of continuity. Most classes will be served best by a selection of topics between these two extremes.

Finally we want to thank those who reviewed various manuscripts of this book: James E. Arnold, Jr., University of Wisconsin; Louis Chatterly, Brigham Young University; Ferdinand Haring, North Dakota State University; L. Kim Jones, Southern Utah State College; Herbert E. Kasube, Bradley University; James W. Maxwell, Oklahoma State University; Thomas L. Shiflett, Southwest Missouri State University; John Spellman, Southwest Texas State University; H. Keith Stumpff, Central Missouri State University; Claudia Taylor, Brevard Community College; Warren W. Watson, Louisiana Tech. University; Clifton Whyburn, University of Houston.

We also want to thank Mathematics Editor Craig Barth and the staff at Brooks/Cole for their patience and assistance. We extend a special thanks to Consulting Editor Bob Wisner and Production Editor Jane Stanley.

William Bosch

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CONTENTS

1

REVIEW OF SOME TOPICS OF ALGEBRA

1.1 THE DISTRIBUTIVE LAW

If you are asked to multiply 106 by 5, you probably give the answer 530 immediately. And if you are asked to multiply 95 by 5, you quickly give 475 as the answer. If you diagram the thinking process you use to work these problems, it may look something like this:

$$5 \cdot 106 = 500 + 30 = 530$$
$$5 \cdot 95 = 500 - 25 = 475$$

If we insert additional steps, the diagrams become

$$5 \cdot 106 = 5(100 + 6) = 5 \cdot 100 + 5 \cdot 6 = 500 + 30 = 530$$
$$5 \cdot 95 = 5(100 - 5) = 5 \cdot 100 - 5 \cdot 5 = 500 - 25 = 475$$

These problems and diagrams illustrate the use of a fundamental rule of arithmetic and algebra known as the **distributive law**. Using symbols to write the abstract forms of this law, we obtain

$$a(b + c) = ab + ac$$
$$a(b - c) = ab - ac$$

or

$$(b + c)a = ba + ca$$
$$(b - c)a = ba - ca$$

It is helpful to remember that $ab = ba$ (**commutative law**) and that subtraction is the addition of the negative of a number. This knowledge allows us to condense the four rules above into just one:

Distributive Law

$$a(b + c) = ab + ac$$

This rule is known as the distributive law because it shows that multiplication distributes over addition and subtraction. Notice that multiplication does not distribute over multiplication:

$$44(2) = 44(2 \cdot 1) \neq 88 \cdot 44$$

Keeping numerical examples in mind will help you remember how to use the distributive law correctly in abstract settings.

Expanding, Factoring, Factors, Terms

When we use the distributive law to equate the expression $a(b + c)$ to $ab + ac$, we are **multiplying** or **expanding**. When we do the reverse, we are **factoring**.

The distributive law can be verbalized by introducing the concepts of **factors** and **terms**. Here is an informal but adequate definition of these words. *Factors make up a product or quotient; terms make up a sum or difference*. For example, the expression

$$2x(x + y)$$

is a single term since it is made up of the factors 2, x, and $(x + y)$. These factors are connected by multiplication. The factor $(x + y)$ consists of the two terms x and y connected by addition. In

$$2x + y(x + y)$$

there are two terms, $2x$ and $y(x + y)$, connected by addition. The term $2x$ consists of the factors 2 and x, and the term $y(x + y)$ consists of the factors y and $(x + y)$. The factor $(x + y)$ of the term $y(x + y)$ itself consists of the two terms x and y.

The distributive law can now be verbalized as follows: *When multiplying a sum or difference of terms by an expression, multiply every term in the sum or difference by that expression.*

Example 1 Multiply $a - x + y$ by 2.

Solution

$$2(a - x + y) = 2a - 2x + 2y$$

■

Example 2 Multiply $2(x + y)$ by 3.

Solution

$$3[2(x + y)] = 6(x + y)$$

or

$$3[2(x + y)] = 2(3x + 3y)$$

The first form of the answer is preferable. ■

Example 3 Factor $ax + ay$.

Solution

Using the distributive law in reverse, we have

$$ax + ay = a(x + y)$$

■

Example 4 Factor $2ax - 4ay$.

Solution $$2ax - 4ay = 2a(x - 2y)$$ ■

We also want to point out that the distributive law can be used in stages. For example,

$$\begin{aligned}(44\tfrac{1}{2})(2\tfrac{1}{2}) &= (44 + \tfrac{1}{2})(2 + \tfrac{1}{2})\\ &= (44 + \tfrac{1}{2})2 + (44 + \tfrac{1}{2})\tfrac{1}{2}\\ &= 44 \cdot 2 + \tfrac{1}{2} \cdot 2 + 44 \cdot \tfrac{1}{2} + \tfrac{1}{2} \cdot \tfrac{1}{2}\\ &= 88 + 1 + 22 + \tfrac{1}{4}\\ &= 111\tfrac{1}{4}\end{aligned}$$

This formulation of the distributive law can be expressed literally as

$$(a + b)(c + d) = ac + ad + bc + bd$$

Factoring by Grouping

You should not make an effort to memorize this formula but simply prepare yourself to use the distributive law repeatedly. Repeated use of the distributive law is the basis for **factoring by grouping**. Example 5 illustrates this process.

Example 5 Factor $ax + bx + 2a + 2b$.

Solution $$\begin{aligned}ax + bx + 2a + 2b &= x(a + b) + 2(a + b)\\ &= (x + 2)(a + b)\end{aligned}$$ ■

EXERCISE SET 1.1

I

Apply the distributive law to compute the following without using paper, pencil, or calculator.

1. $6 \cdot 410$
2. $6 \cdot 390$
3. $2 \cdot 675$
4. $(-6) \cdot 410$
5. $(-2)(-675)$
6. $5(1.25)$
7. $99 \cdot 99$ (*Hint*: $99 = 100 - 1$)
8. $59 \cdot 59$
9. $(1.1)(2.1)$
10. $99 \cdot 101$
11. $29 \cdot 31$
12. $49 \cdot 51$
13. $(52)(2.75)$
14. $4 \cdot 4\frac{3}{4}$
15. $12 \cdot 20\frac{3}{4}$
16. $1\frac{1}{2} \cdot 22\frac{3}{4}$

II

Expand using the distributive law.

17. $6(2 + 1)$
18. $3(a + b)$
19. $4(x + y)$
20. $2(x - y)$
21. $2(x - 1)$
22. $4(2x + y)$
23. $7(2x + 3)$
24. $a(bc + d)$
25. $(-2)(-6x + 1)$
26. $a[2 + b(x + 1)]$
27. $3(x + y - 1)$
28. $4(x + y + w - z)$
29. $2(u - z - w)$
30. $(x - y)(x + y)$
31. $(x + y)(x + y)$
32. $(x - y)(x - y)$
33. $(x + 1)(x - 2)$
34. $(x - 1)(x - 2)$
35. $(x + a)(x + b)$
36. $(x - 2a)(x + 2a)$
37. $(2x - y)(x - 2y)$

III

Factor by using the distributive law and grouping.

38. $3x + 3y$
39. $3x + wx$
40. $2w - 2$
41. $3x + 6u$
42. $3x + 6$
43. $ty - t$
44. $4 - 2e$
45. $18 + 9t$
46. $2u + 4x + 6y$
47. $4 - 8x - 16y$
48. $2 - 6w - 10u$
49. $6 - 3x - 3y$
50. $ax + aby - az$
51. $2a - 2ax + 2ay$
52. $ax + ay + bx + by$
53. $ab + bx + ay + xy$
54. $a^2 + ab + ac + bc$
55. $2r - yc - 2c + yr$
56. $ab + ac - 2b - 2c$
57. $ax + bx - ay - by$
58. $xr + xs + yr + ys + r + s$
59. $t^2 - t$
60. $x - x^2a$

1.2 EXPANDING AND FACTORING

In this section we increase our ability to expand and factor algebraic expressions. Although expanding an expression can be accomplished by applying the distributive law again and again, a knowledge of special products makes the expansion of many products faster and more enjoyable and is also very important in factoring. *We must always remember that factoring is just the reverse of expanding; thus, everything we learn about the one process will reinforce our ability to perform the other.*

We now list some special products you should verify (see Exercise 1.2) and commit to memory. We include the distributive law in this list because it too is a method for expanding a product and is of such basic importance.

Special Products

Special Products and Expanding

1. $$a(b + c) = ab + ac$$
2. $$(a + b)(a - b) = a^2 - b^2$$
3. $$(a + b)^2 = a^2 + 2ab + b^2$$
4. $$(a - b)^2 = a^2 - 2ab + b^2$$
5. $$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
6. $$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
7. $$(a + b)(a^2 - ab + b^2) = a^3 + b^3$$
8. $$(a - b)(a^2 + ab + b^2) = a^3 - b^3$$

Note that each of the eight identities just given consists of an indicated product on the left side and the product or expansion on the right side. These rules are properly called *identities* because they are valid for all values of a and b. Be careful not to confuse Identities 2 and 4, or 5 and 7, or 6 and 8. What are the differences? It will help if you put these rules into words. For example, Identity 5 gives us the "cube of a sum" whereas the use of Identity 7 results in the "sum of two cubes." This list does not exhaust the possibilities for special products; however, it will be sufficient for the problems we will encounter.

Expanding The distributive law, Identity 1, was found in a variety of forms in Section 1.1. Identities 2–8 can also be found in a variety of forms. Keep this in mind as you examine the following examples.

Example 1 Expand $(2x - y)^2$.

Solution

We use Identity 4 to write

$$\begin{aligned}(2x - y)^2 &= (2x)^2 - 2(2x)y + y^2 \\ &= 4x^2 - 4xy + y^2\end{aligned}$$

■

Example 2 Expand $(x - 2y)(x + 2y)$.

Solution

Here we use Identity 2 to write

$$\begin{aligned}(x - 2y)(x + 2y) &= x^2 - (2y)^2 \\ &= x^2 - 4y^2\end{aligned}$$

■

Example 3 Expand $(a - 3)(a^2 + 3a + 9)$.

Solution

Identity 8 applies, so we write

$$\begin{aligned}(a - 3)(a^2 + 3a + 9) &= a^3 - 3^3 \\ &= a^3 - 27\end{aligned}$$

■

We now list again the eight identities above. However, we interchange the left and right sides because that is how they are used in the process of factoring. We also wish once again to emphasize that factoring is the reverse of multiplying or expanding. Thus, we can always check a factoring procedure by expanding the result and then comparing it to the original expression.

Special Products

Special Products and Factoring

1′. $ab + ac = a(b + c)$

2′. $a^2 - b^2 = (a + b)(a - b)$

3′. $a^2 + 2ab + b^2 = (a + b)^2$

4′. $a^2 - 2ab + b^2 = (a - b)^2$

5′. $a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$

6′. $a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$

7′. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

8′. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Note that the right side of these identities appears as an indicated product; that is, the right side is the factored version of the left side. Again you should be careful not to confuse Identities 5′ and 7′, and so on. The processes indicated by Identities 1′–4′, 7′, and 8′ traditionally have been called "factoring out a common monomial," "factoring the difference of two squares," "factoring a perfect square trinomial," "factoring a perfect square trinomial," "factoring the sum of two cubes," and "factoring the difference of two cubes," respectively.

Prime Factors

You may be wondering what the final answer is to a factoring problem. Answering that question is more difficult than one might suppose. Technically, an expression is completely factored if it is written as a product of **prime factors**. The question then becomes "What is a prime factor?" A factor is prime if it cannot itself be factored (except in some trivial way), which depends in turn on the field or set of numbers over which the operations are performed. This explanation may seem somewhat involved, but the examples below will clarify.

In this book we will factor over the field of rational numbers or over the ring of integers. In plain words, we will use fractions to factor an expression if the expression has fractions as coefficients. If the expression has only integer coefficients, we will restrict ourselves to integer coefficients in the factors. We will also deviate from this rule when it suits our purpose. These points will become clear as you study the examples and work the exercises in this and later sections.

Factoring

Whenever we begin a factoring process, we should first factor out monomials. This factoring never complicates later steps and often simplifies them.

Example 4 Factor $xy - xw$.

Solution

$$xy - xw = x(y - w)$$ ■

Example 5 Factor $x^2y^2 - x^2w^2$.

Solution

First factor out x^2 to obtain

$$x^2y^2 - x^2w^2 = x^2(y^2 - w^2)$$

Then note that $y^2 - w^2$ is a difference of two squares and factor accordingly.

$$\begin{aligned} x^2y^2 - x^2w^2 &= x^2(y^2 - w^2) \\ &= x^2(y - w)(y + w) \end{aligned}$$ ■

Example 6 Factor $(y - w)$.

Solution

$y - w$ is prime. We could write $(y - w) = 1 \cdot (y - w)$ or $(y - w) = 2 \cdot \frac{1}{2} \cdot (y - w)$, but these are trivial factorizations. ■