CORE MATHS for A-level

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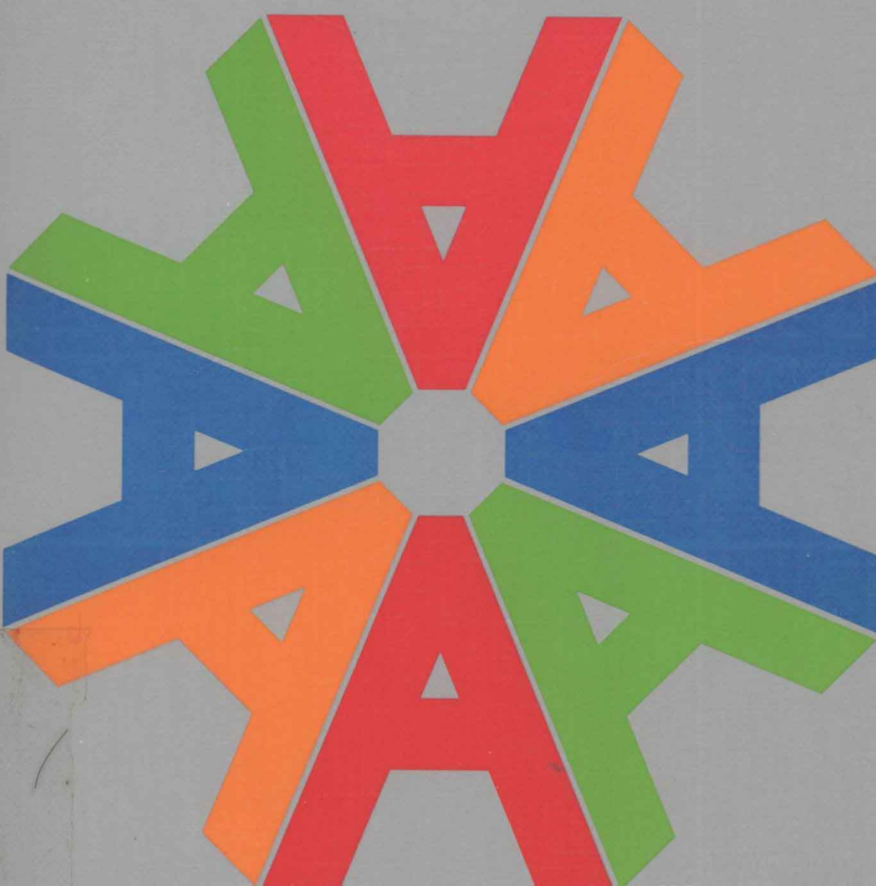

CORE MATHS for A-level

By the same authors

MATHEMATICS – THE CORE COURSE FOR A-LEVEL
FURTHER PURE MATHEMATICS – with C. Rourke
MATHEMATICS – MECHANICS AND PROBABILITY
FURTHER MECHANICS AND PROBABILITY
APPLIED MATHEMATICS I
APPLIED MATHEMATICS II
PURE MATHEMATICS I
PURE MATHEMATICS II

PREFACE

Why produce yet another A-level Maths textbook?

Now that GCSE courses have been introduced it can no longer be assumed that all students enter an A-level course with the algebraic skills and geometric knowledge that used to be expected. Many more students now move in to sixth-form colleges to do A-levels and hence come from a variety of backgrounds, including those who wish to embark on an A-level course from intermediate level GCSE.

In much the same way as multiplication tables are the tools needed to build a mathematics course from 11 to 16, skill in algebraic techniques are the tools necessary for building a body of mathematical knowledge beyond the 16+ level. This book starts with work designed to help those students acquire a facility in using algebra. To interest those students who already have these skills, new work is included in all chapters. Chapter 2 for example, includes an introduction to simple partial fractions.

All too many students regard A-level mathematics as being intrinsically difficult - an opinion with which we strongly disagree. Part of the reason for this myth may be that students, at an early stage in their course, tackle problems that are too sophisticated. The exercises in this book are designed to overcome this problem, all starting with straightforward questions. The more sophisticated A-level type questions are given in consolidation sections which appear at regular intervals throughout the book. These are intended for use at a later date to give practice in examination type questions when confidence and sophistication have been developed. The consolidation sections also include a summary of the work in preceeding chapters and a set of multiple choice questions, which are very useful for self-testing even if they do not form part of the examination to be taken.

There are many computer programs that aid in the understanding of mathematics. In particular, a good graph drawing package is invaluable for investigating graphical aspects of functions. In a few places we have indicated a program that we think is relevant. This is either *Super Graph* or a program from *132 Short Programs for the Mathematics Classroom.*

Super Graph by David Tall is a flexible graph drawing package and is available from Glentop Press Ltd.
132 Short Programs for the Mathematics Classroom is published in book form by Stanley Thornes (Publishers) Ltd.

We are grateful to the following Examination Boards for permission to reproduce questions from their past examination papers (part questions are indicated by the suffix p):

University of London (U of L)
Joint Matriculation Board (JMB)
University of Cambridge Local Examinations Syndicate (C)
The Associated Examining Board (AEB)

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NOTES ON USE OF THE BOOK

Notation

$=$	is equal to	$:$	is such that
$\equiv$	is identical to	$\mathbb{N}$	the natural numbers
$\approx$	is approximately equal to*	$\mathbb{Z}$	the integers
$>$	is greater than	$\mathbb{Q}$	the rational numbers
$\geqslant$	is greater than or equal to	$\mathbb{R}$	the real numbers
$<$	is less than	$\mathbb{R}^+$	the positive real numbers excluding zero
$\leqslant$	is less than or equal to		
∞	infinity; infinitely large	$\mathbb{C}$	the complex numbers
$\Rightarrow$	implies	$[a, b]$	the interval $\{x : a \leqslant x \leqslant b\}$
$\Leftarrow$	is implied by	$(a, b]$	the interval $\{x : a < x \leqslant b\}$
$\Leftrightarrow$	implies and is implied by	(a, b)	the interval $\{x : a < x < b\}$
$\in$	is a member of		

A stroke through a symbol negates it, e.g. $\neq$ means 'is not equal to'

Abbreviations

$\parallel$	is parallel to	w.r.t.	with respect to
+ve	positive	exp	exponential, e.g. $\exp x$ means e^x
−ve	negative		

Useful Formulae

For a cone with base radius r, height h and slant height l

volume $= \frac{1}{3}\pi r^2 h$ curved surface area $= \pi r l$

For a sphere of radius r

volume $= \frac{4}{3}\pi r^3$ surface area $= 4\pi r^2$

For any pyramid with height h and base area a

volume $= \frac{1}{3}ah$

*Practical problems rarely have exact answers. Where numerical answers are given they are correct to two or three decimal places depending on their context, e.g. π is 3.142 correct to 3 d.p. and although we write $\pi = 3.142$ it is understood that this is not an exact value. We reserve the symbol $\approx$ for those cases where the approximation being made is part of the method used.

Computer Program References

Marginal symbols indicate a computer program which is helpful, programs being identified in the following manner,

47 Program No. 47 from *132 Short Programs for the Mathematics Classroom*

SG *Super Graph*

Instructions for Answering Multiple Choice Exercises

These exercises are included in each consolidation section. The questions are set in groups, each group representing one of the variations that may arise in examination papers. The answering techniques are different for each group and are classified as follows:

TYPE I

These questions consist of a problem followed by several alternative answers, only *one* of which is correct.

Write down the letter corresponding to the correct answer.

TYPE II

In this type of question some information is given and is followed by a number of responses. *One or more* of these follow(s) directly and necessarily from the information given.

Write down the letter(s) corresponding to the correct response(s).
e.g. PQR is a triangle

A $\angle P + \angle Q + \angle R = 180°$
B PQ + QR is less than PR
C if $\angle P$ is obtuse, $\angle Q$ and $\angle R$ must both be acute.
D $\angle P = 90°$, $\angle Q = 45°$, $\angle R = 45°$

The correct responses are **A** and **C**.
B is definitely incorrect and **D** may or may not be true of triangle PQR, i.e. it does not follow directly and necessarily from the information given. Responses of this kind should not be regarded as correct.

TYPE III

A single statement is made. Write T if it is true and F if it is false.

CONTENTS

CHAPTER 1

ALGEBRA 1

The ability to manipulate algebraic expressions is an essential base for any mathematics course beyond GCSE. Applying the processes involved needs to be almost as instinctive as the ability to manipulate simple numbers. This and the next two chapters present the facts and provide practice necessary for the development of these skills.

MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

The multiplication sign is usually omitted, so that, for example,

$$2q \text{ means } 2 \times q$$

and $\quad x \times y$ can be simplified to xy

Remember also that if a string of numbers and letters are multiplied, the multiplication can be done in any order, for example

$$\begin{aligned} 2p \times 3q &= 2 \times p \times 3 \times q \\ &= 6pq \end{aligned}$$

Powers can be used to simplify expressions such as $x \times x$,

i.e. $\quad x \times x = x^2$

and $\quad x \times x^2 = x \times x \times x = x^3$

But remember that a power refers only to the number or letter it is written above, for example

$2x^2$ means that x is squared, but 2 is not.

Example 1a

Simplify (a) $(4pq)^2 \times 5$ (b) $\frac{ax^2}{y} \div \frac{x}{ay^2}$

(a) $$(4pq)^2 \times 5 = 4pq \times 4pq \times 5$$
$$= 80p^2q^2$$

(b) $$\frac{ax^2}{y} \div \frac{x}{ay^2} = \frac{ax^{\not 2}}{\not y} \times \frac{ay^{\not 2}}{\not x}$$
$$= a^2xy$$

EXERCISE 1a

Simplify

1. $3 \times 5x$	**2.** $x \times 2x$	**3.** $(2x)^2$
4. $5p \times 2q$	**5.** $4x \times 2x$	**6.** $2pq \times 5pr$
7. $(3a)^2$	**8.** $7a \times 9b$	**9.** $8t \times 3st$
10. $2a^2 \times 4a$	**11.** $25x^2 \div 15x$	**12.** $12m^2 \div 6m$
13. $b^2 \times 4ab$	**14.** $25x^2y \div 5x$	**15.** $(7pq)^2 \times (2p)^2$
16. $\frac{22ab}{11b}$	**17.** $\frac{18ax^2}{3x}$	**18.** $\frac{36xy}{18y}$
19. $\frac{72ab^2}{40a^2b}$	**20.** $\frac{2}{5} \div \frac{1}{x}$	**21.** $\frac{x^2}{y} \div \frac{y}{x}$

ADDITION AND SUBTRACTION OF EXPRESSIONS

The *terms* in an algebraic expression are the parts separated by a plus or minus sign.

Like terms contain the same combination of letters; like terms can be added or subtracted.

For example, $2ab$ and $5ab$ are like terms and can be added,

i.e. $$2ab + 5ab = 7ab$$

Unlike terms contain different algebraic expressions; they cannot be added or subtracted. For example, ab and ac are unlike terms and $ab + ac$ cannot be simplified.