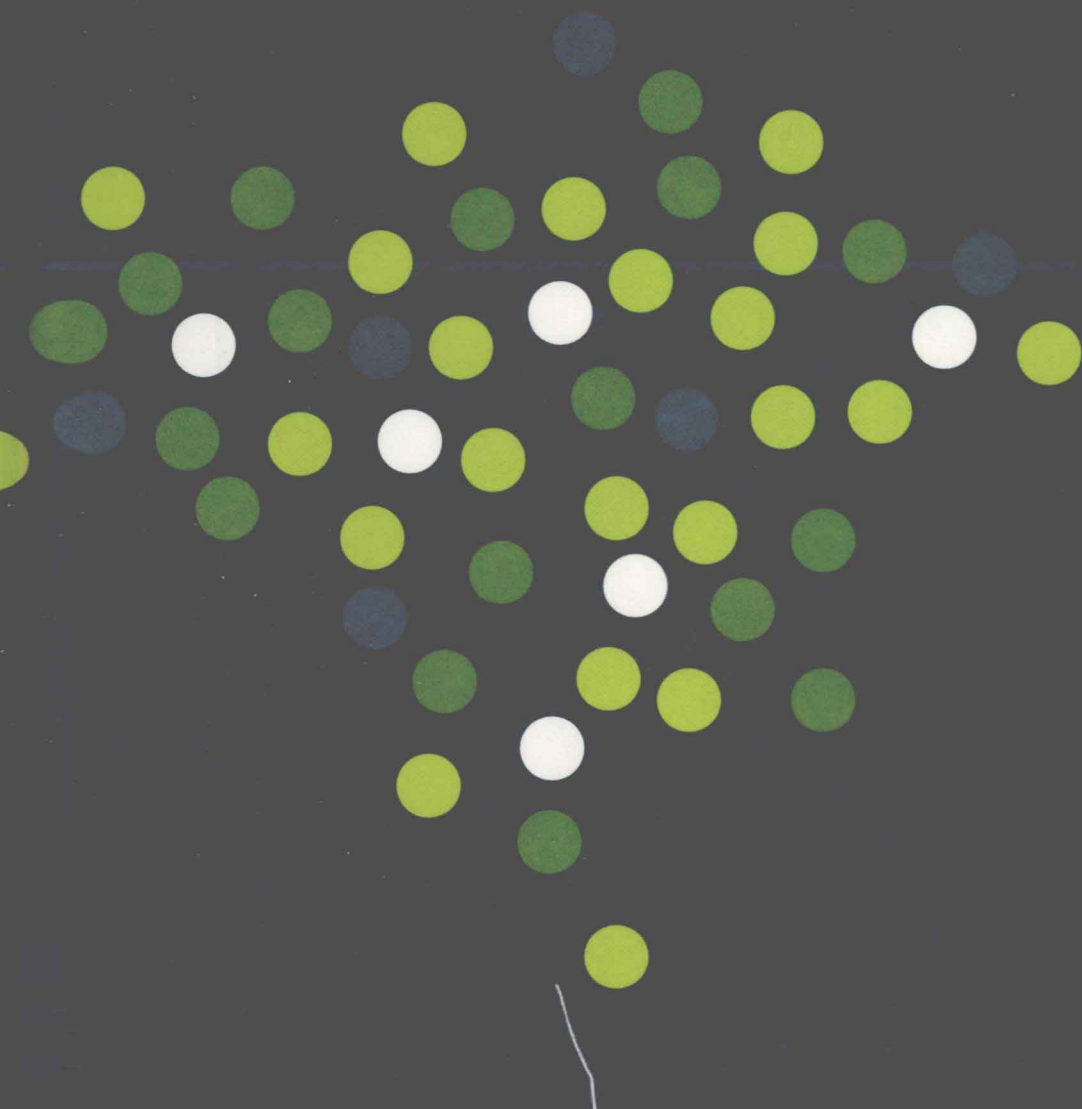


# ANOVA

An Analysis of Variance Primer

Richard J. Harris



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**Richard J. Harris**



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## Preface

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*An Analysis of Variance Primer* is intended to provide graduate and advanced undergraduate students with a thorough understanding of the statistical techniques that have been developed to examine differences in central tendency associated with one or more independent variables. These techniques are generally gathered under the rubric of *analysis of variance* (ANOVA), but include *t* tests as special cases, chi-square and log-linear techniques as extensions to situations where the effect of your independent variable is measured primarily in terms of the proportion of subjects making each of a number of possible choices, and correlation/regression techniques as a useful adjunct to measuring the magnitude of your effects and to disentangling the separate effects of the correlated independent variables that arise when unequal ANOVA designs are analyzed.

There are a number of excellent textbooks with this same basic goal of elucidating ANOVA techniques. Indeed, a colleague of mine at UNM is coauthor of one of the best and most recent of them (Maxwell & Delaney, 1989). Why, then, add yet another ANOVA text to this set? As John Fowles (1969) points out in *The French Lieutenant's Woman*, authors

...write for countless different reasons: for money, for fame, for reviewers, for parents, for friends, for loved ones; for vanity, for pride, for curiosity, for amusement: as skilled furniture makers enjoy making furniture, as drunkards like drinking, as judges like judging, as Sicilians like emptying a shotgun into an enemy's back. [One] could fill a book with reasons, and they would all be true, though not true of all. (p. 105)

Certainly my reasons for writing the *ANOVA Primer* include some of those Fowles lists. However, my primary reason was simply the frustration of having to teach ANOVA to my students from textbooks whose treatment of a number of issues seemed to be badly misguided and/or out of date. This frustration was further added to by a paucity of published references to give colleagues (including students) to cite in their theses as authority for some approach to ANOVA issues that was clearly called for but that also departed from those propounded in existing textbooks. Year after year I waited for a new ANOVA textbook or a new edition of one of the classics to become available that would point out that your analysis is not complete until you have spelled out (via single-*df* contrasts) one or more specific patterns of differences among your means that had led to your statistically significant overall *F*; that would quit trying to "patch up" the univariate approach to

within-subjects designs and simply recommend testing each within-subjects contrast against its own variance; that would recognize that determining sample size doesn't require carrying a 900-paged book of power tables or tables of noncentral distributions around with you; that would notice that the precise numerical values of the multiplying coefficients in expected mean squares had no impact on selection of error terms and could be dispensed with; and so forth. Year after year I prepared separate handouts for my Intermediate Statistics students correcting or supplementing their textbook on these and other issues. As have so many other instructors over the years, I finally decided that the only way to get a textbook whose approaches to ANOVA I was happy with was to write it myself. Whence—several drafts tested on Psych 300 students and one bought-out, warehoused, and finally abrogated contract (prior to F.E. Peacock) later—cometh the *ANOVA Primer*.

That tells you why I intend to use this *Primer*, but it leaves open the question of why other instructors should adopt it and why their students should be glad they did. The two primary reasons are that, unlike any other available ANOVA textbook at the time I wrote this preface, the *ANOVA Primer* (a) integrates multiple-comparison procedures with overall tests, deriving the latter from the former, and (b) emphasizes the correct, multivariate approach to within-subjects designs. These two points may need some expansion, especially for those of you who may currently be using the alternatives to these approaches.

The ultimate goal of any ANOVA is to be able to make specific statements about the magnitude and sign of particular patterns of differences among your means. It certainly is *not* simply to tell us what we already know, namely that the means of the populations from which we drew our samples are not all identical to some common value. There are no true null hypotheses except by construction (e.g., for Monte Carlo runs testing the performance of some statistic were  $H_0$  actually true). If you doubt this, consider the thought experiment of carrying out any study with an  $N$  of 35,000,000. Given that the overall null hypothesis is simply a straw man we must be able to reject before proceeding to more interesting hypotheses about what processes are generating our data (after all, if we don't have enough evidence to reject the hypothesis of exact equality, we also won't have enough evidence to rule out either small positive or small negative values, so that we can't be confident of the sign of our effect), it follows that the only real utility of an overall test is as an indication of whether there are any specific contrasts to be uncovered that will be statistically significant (i.e., the sign of whose population value we can specify with confidence). And from this it follows that the choice of an overall test should be tailored to the set of specific comparisons of potential interest to you. Thus, the *Primer* begins its discussion of the one-way, independent-means design (chapter 3) by focusing on the great variety of questions (specific contrasts) one can ask about the ways in which three or more means differ from each other, moving on to the issue of how one controls for the potential inflation of experimentwise Type I error rate occasioned by examining many (potentially an infinite number) such contrasts. This is then used as the justification for employing overall tests such as the traditional overall  $F$  ratio or the Studentized Range test as (a) preliminary tests

to tell you whether there are any statistically significant specific contrasts to be found and (b) bases for post hoc critical values for those specific contrasts. Thus, rather than throwing the overall  $F$  test at students and treating specific comparisons as an unrelated issue covered in a separate chapter, the *ANOVA Primer* thoroughly integrates overall tests and “followup” contrasts.

There is a large literature on ways of adjusting the degrees of freedom with which you look up the critical value for the overall  $F = MS_T/MS_{S \times T}$  computed in the univariate approach to within-subjects designs so as to have a test whose actual and nominal Type I error rates match up fairly well. Most of the currently available ANOVA textbooks point out the need for this epsilon-adjustment procedure when—as is almost always the case—the homogeneity-of-treatment-difference-variance assumption on which the validity of the univariate approach is predicated is violated. (At least one popular textbook still recommends the unadulterated univariate approach.) However, what current textbook treatments (with the exception of Maxwell & Delaney, 1989) fail to point out is that (a) epsilon adjustment is of no use in handling the often enormous inflation or deflation of actual alpha levels for specific contrasts when the pooled error term is used in testing all specific contrasts, no matter how large or small their own particular variance across subjects is, and (b) the overall test may be inconsistent with appropriate (own-variance, multivariate-approach) tests of within-subjects contrasts, in that a significant epsilon-adjusted  $F$  is no guarantee that **any** statistically significant contrast can be found, and a nonsignificant epsilon-adjusted  $F$  may be hiding one or more contrasts that **would** be statistically significant even by the fully post hoc critical value had the nonsignificant overall  $F$  not kept you from looking for them. Finally, Boik (1981) has demonstrated that this problem of wildly inflated (for some contrasts) and wildly deflated (for other contrasts) alpha levels is *not* eliminated by a finding of a nonsignificant departure from the h.o.t.d.v. assumptions, since even degrees of departure that are too small to be detected by the available tests of significance of epsilon can drastically affect the validity of univariate-approach tests of contrasts. Given our earlier agreement that it is specific comparisons that are the ultimate goal of our analysis, it follows that the univariate-approach overall  $F$  is of no use. (Maxwell and Delaney recognize the inconsistency between overall test and follow-up contrasts, but still condone use of the univariate overall  $F$  followed by multivariate-approach contrasts. Most other textbook authors either present only the univariate approach to contrasts or don't discuss specific contrasts at all, confining their attention to the overall test.)

Consistent with all this, chapter 5 of the *ANOVA Primer* derives tests of contrasts among the levels of your within-subjects factor as special cases of the single-sample  $t$  test (since a score on any within-subjects contrast can be computed for each subject, and our hypothesis that the contrast among the means is zero in the population is thus equivalent to the hypothesis that these individual-subject contrast scores have a population mean of zero), and the multivariate overall test (based on Hotelling's  $T^2$  statistic) is explained as the natural outcome of asking how often the maximum possible  $F_{\text{contr}}$  for any within-subjects contrast would ex-

ceed a particular value. This multivariate approach is compared with the more traditional univariate approach to within-subjects designs, the primary point being that tests of contrasts in the univariate approach compare each contrast to a pooled, average variance term that may grossly underestimate or overestimate the actual consistency (or lack thereof) of that effect from subject to subject. In short, the *ANOVA Primer* guides the reader into the correct, internally consistent approach to analyzing within-subjects designs, rather than continuing to try to “patch up” the inappropriate univariate approach.

These two issues—integration of specific comparisons and overall tests and appropriate handling of within-subjects designs—are the *raison d’être* of the *ANOVA Primer*. Naturally enough, however, the advantage of hindsight has made it possible to incorporate a number of other improvements that can be considered bonuses for using this textbook.

For instance, the *ANOVA Primer* draws a clear distinction between the concepts of *orthogonal* contrasts (those for which the two patterns of means are uncorrelated with each other) and *uncorrelated* contrasts (those whose values tend to be uncorrelated across replications of the study).

And, instead of diving straight into overall main-effect and interaction tests, chapter 4 derives main-effect and interaction *contrasts*, and from them overall *F*s for main effects and interactions, as a consequence of the sets of questions one is inevitably led to ask about differences among the various means obtained when one administers all possible combinations of the levels of two or more independent variables (factors). Similarly, the *ANOVA Primer* makes it clear (as earlier texts do not) that simple-main-effect contrasts do *not* constitute explanations of interaction (since they are confounded with main-effect contrasts). Instead, a main-effects-and-interactions breakdown of your design and a simple, main-effects-and-interactions breakdown are presented as equally legitimate responses (together with treating your design as a one-way ANOVA with  $k_A \cdot k_B$  cells) to a statistically significant interaction. Given that you’re interested in explaining your interaction, it is highly likely that your verbal explanation will imply a *contrast of contrasts* (a contrast across the levels of one factor with respect to the magnitude of a contrast across the levels of the other factor). The *ANOVA Primer* is the only textbook that discusses and presents a table of the lower critical values Boik has derived from such contrasts of contrasts. Chapter 4 also provides a much simpler approach to selecting error terms for designs involving random factors (dropping the “excess baggage” of precise computation of multiplying coefficients in expected mean squares in favor of a focus on which sources of variance are included) than is to be found in other textbooks.

As you may have inferred from some of the earlier descriptions, a mildly Socratic approach is employed throughout the *ANOVA Primer*: each new technique is gradually developed from fundamental goals—but this step-by-step development is supplemented by end-of-chapter “Menus” for modal analyses. This bottom-up approach is, I believe, a more effective way of introducing students to the fundamentals of ANOVA, though a general-linear-model, models-comparison approach

(such as that provided by Maxwell & Delaney) can be quite effective in helping a student *who already has a basic understanding of ANOVA* see the full generality (majesty?) of ANOVA techniques.

Chapter 7 introduces the student to incomplete factorial designs, that is, designs in which some of the cells of a complete factorial design are missing either because of the rarity of some combinations of factor levels in the population from which subjects have been sampled or because the researcher has deliberately decided to omit certain cells so as to permit allocating more subjects to other cells—at the cost of ambiguity as to whether, for example, a significant  $F$  for a main effect is really due to a nonzero population value of that main effect or due to one or more nonzero interactions with which it is confounded. The basic approach is to examine the contrasts among means that would have been estimated from the complete factorial design and then note which of these contrasts become indistinguishable when certain cells are dropped. Thus, while the *ANOVA Primer's* coverage of incomplete factorial designs is not nearly so comprehensive as the several chapters devoted to it by Kirk (1982), I believe that students will derive from chapter 7 a much better understanding of what is gained and what is lost in using an incomplete factorial design than they're likely to get from the modular-arithmetic treatments of other texts.

Computer use (with a focus on *SPSS*<sup>x</sup>, but examples of *SAS* and sometimes *BMDP* setups are also provided) is introduced as an option in chapter 2, though the first analysis for which computer use is necessary doesn't come until chapter 5.

Throughout the text a simple, easily understood method of computing necessary sample size [the MIDS criterion, cf. Harris & Quayle (1992)] is presented. This method requires only that you be able to specify what you would consider a minimally important population effect size and that you be able to compute on hypothetical data the same significance test you intend to carry out when the data are in hand. Tables of noncentral distributions are not required, neither are difficult decisions as to whether to try for .8 or .9 or .9165 power for the effect size you believe holds in the population. The other benefit of traditional power analyses—distinguishing between failure to achieve statistical significance because the population effect is small versus because your study had insufficient power—is provided in the *ANOVA Primer* by an emphasis on confidence intervals.

Clear, internally consistent positions are stated with respect to controversial issues in ANOVA (e.g., one-tailed vs. two-tailed tests, the relationship between overall tests and tests of specific comparisons, how to interpret statistically significant interactions, hierarchical vs. full-model approaches to unequal- $n$  factorial ANOVA, and univariate vs. multivariate approach to within-subjects designs), but the arguments for other positions are also presented.

I attempt to enliven the presentation by occasional intrusions of tongue-in-cheek humor. (See, e.g., Question (c) in the practice problems at the end of chapter 1, and the opening paragraphs of chapter 2.) Of course, this stylistic advantage must be balanced against my less laudatory tendency toward single-sentence paragraphs.



My first “thanks” go to my wife and sometime colleague, Mary B. Harris, Professor of Educational Foundations at UNM, for her attempts to ameliorate this and other stylistic deficiencies, as well as for her patience with the time working on the *ANOVA Primer* took away from family responsibilities, for her willingness to listen to my harangues about statistical issues and to tolerate the stares of other patrons when these conversations took place at restaurants or during theater intermissions, and for bearing up under the special burden of having to try to slip her husband’s approaches to statistical issues into her own many manuscripts despite editors’ and referees’ persistent conservatism. I also wish to thank the generations of Intermediate Statistics students at UNM who served as guinea pigs for drafts of the *ANOVA Primer*. At least now they’ll have an index to work with. Thanks, too, to all the anonymous referees who provided feedback during the process of finding a publisher (and then another) and making this book as accurate and as widely acceptable as possible, including Cynthia J. Thomsen of Tufts University (Massachusetts), Richard T. Campbell of the University of Illinois at Chicago, and William P. Dunlap of Tulane University. Especially valued have been the suggestions and other feedback of Kevin O’Grady, Psychology Department, University of Maryland (formerly at UNM), who I know served as one of the very early referees. I hope Kevin and the other referees will take my occasional failure to adopt a suggestion as a reflection of my stubborn streak, rather than as indicating anything other than the highest regard for their professional expertise.

Thanks to Robert Boik, Professor of Statistics at the University of Montana, for being that rare species of mathematical statistician with a real feel for what developments are most needed by us practitioners. I can’t imagine more important contributions to ANOVA practice than Robert’s work on consequences of violation of h.o.t.d.v. assumptions for specific comparisons and on analysis of interactions via contrasts of contrasts.

Finally, thanks to my younger son, Alexander N. Harris. Alec hadn’t yet joined our family when the *Primer of Multivariate Statistics* was published, so he couldn’t appear in that preface with his brother and sister. [Christopher and Jennifer were off at college or working during most of the preparation of the *ANOVA Primer*, but they contributed numerous opportunities for BIRGing—*Basking In Reflected Glory*, Cialdini, Borden, Thorne, Walker, & Freeman (1976)—to help sustain my morale during that period.] Alec’s acceptance of Dad’s presence at the computer terminal in the study but unavailability for Monopoly games and wrestling during the crucial few weeks of preparation of the final manuscript was an enormous help.

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