

Keedy/Bittinger

Algebra and Trigonometry

Fifth Edition



Algebra and Trigonometry

5th Edition

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Algebra and Trigonometry

5th Edition

This volume is part of the Bittinger precalculus series for college algebra and trigonometry courses. Other texts in the series are the following.

BITTINGER/BEECHER

Algebra and Trigonometry covers precalculus algebra and trigonometry with an emphasis on functions. Trigonometric functions are presented first through a discussion of right-triangle trigonometry.

CONTENTS

Basic Concepts of Algebra. Equations, Inequalities, and Problem Solving. Relations, Functions, and Transformations. Linear and Quadratic Functions and Inequalities. Polynomial and Rational Functions. Exponential and Logarithmic Functions. The Trigonometric Functions. Trigonometric Identities, Inverse Functions, and Equations. Triangles, Vectors, and Applications. Systems of Equations and Matrices. Equations of Second Degree and Their Graphs. Sequences, Series, and Probability.

College Algebra covers the basic topics of college-level precalculus algebra.

CONTENTS

Basic Concepts of Algebra. Equations, Inequalities, and Problem Solving. Relations, Functions, and Transformations. Linear and Quadratic Functions and Inequalities. Polynomial and Rational Functions. Exponential and Logarithmic Functions. Systems of Equations and Matrices. Equations of Second Degree and Their Graphs. Sequences, Series, and Probability.

Trigonometry introduces the trigonometric functions and their properties, beginning with the trigonometry of the right triangle.

CONTENTS

Relations, Functions, and Transformations. The Trigonometric Functions. Trigonometric Identities, Inverse Functions, and Equations. Triangles, Vectors, and Applications. Imaginary and Complex Numbers. Exponential and Logarithmic Functions.

Preface

This text covers college-level algebra and trigonometry and is appropriate for a one- or two-term course in precalculus mathematics. Its approach is more intuitive and interactive than most precalculus texts, and is designed to help students achieve a greater understanding of sophisticated mathematical concepts than they might be able to achieve with other texts. Our goal is to provide as much support and help for students as we can, in order to ease the difficult transition into their first college-level mathematics course. At the same time, we want to give students solid preparation for calculus and maintain the appropriate topical coverage and level of presentation for a precalculus course.

CONTENT FEATURES

Algebra Review

- A lack of basic algebra skills is a common cause for student struggles in precalculus and calculus. This text contains a careful and detailed review of intermediate algebra topics in Chapters 1 and 2. (See pp. 1–129.)

Graphing and Functions

- Graphing and functions are both introduced in Chapter 3. We present graphing as a means of representing relations or correspondences visually, and use it throughout the text to give students an intuitive and visual understanding of the material they are studying. For example, whenever a new type of function is introduced, we immediately show its graph. (See pp. 136, 290, and 357.)
- Functions are presented in Chapter 3 after correspondences, relations, and

graphs have been introduced. We include a detailed presentation of topics that will be important in calculus: the algebra of functions, the composition of functions. (See pp. 145 and 181.)

Modeling, Problem Solving, and Applications

- Throughout the text, we try to motivate the material by providing meaningful real-life applications of the mathematics. We also include many problem-solving sections. (See pp. 155, 214, 315, and 499.)
- Since solving applied problems is one of the most difficult and important areas of algebra, we present a problem-solving algorithm in Section 2.3, and use this algorithm consistently throughout the text. The algorithm provides students with a starting point, and allows them to focus on the mathematics needed to solve the problem. To give students the background they need to translate problems themselves, we discuss extensively the “familiarization” and “translation” steps of the problem-solving process. (See pp. 79 and 557.)
- In Chapter 4, we include a section on mathematical models, which describes the characteristics of a mathematical model and discusses when a model will work, and, more important, when it will not work. (See pp. 235–242.)

Trigonometry

- We begin the trigonometry material with a discussion of distances on the unit circle in Section 6.1, to give the student a thorough understanding of where points like π and $-\pi/3$ are located. We then develop trigonometric functions of the real numbers on the unit circle in Sections 6.2 and 6.3, before introducing angles, radians, and degrees in Section 6.4. Trigonometric functions of the right triangle are introduced in Section 6.5 and applied in Section 8.1. (See pp. 348–399 and 494.)
- In Chapter 7, we spend an entire section discussing and practicing techniques for proving trigonometric identities. We present two different methods for proving identities and discuss the merits of each one. (See pp. 461–466.)

Calculus Preparation

- Throughout the text, we give careful attention to the presentation of concepts that will become important in later calculus courses. For example, in the introduction to relations and functions, we emphasize the idea of domain, discussing it three times: first in the context of relations, then with graphs of relations, and finally with functions. In addition, the material on polynomials and rational functions includes an intuitive and graphical introduction to the idea of limit. (See pp. 134, 142, 151, and 688.)

Exponential and Logarithmic Functions

- The material on exponential and logarithmic functions is introduced in the context of inverse functions, with careful emphasis placed on the relationship between exponents and logarithms. (See pp. 276 and 296.)

WHAT’S NEW IN THE FIFTH EDITION?

We have rewritten many key topics in response to user feedback, and have made significant improvements to several chapters. Detailed information about the changes made to this material is available in the form of a *Conversion Guide*.

Please ask your local Addison-Wesley sales representative for more information. Following is a list of the major organizational changes in this revision.

- Many applications and exercises have been added throughout the text.
- Sets and set-builder notation are now introduced in Chapter 1.
- The introduction to complex numbers is now covered in Chapter 2, and complex solutions of quadratic equations are now introduced in this chapter. The material on polar notation and DeMoivre's theorem has been moved into the third trigonometry chapter.
- Chapter 3, "Relations, Functions, and Transformations," has been extensively rewritten in response to user feedback and now includes many new applications as well as material on the algebra of functions and the composition of functions.
- The material on systems of equations and matrices has been combined and moved after the trigonometry chapters. The introduction to polynomial functions and some material on graphs of polynomials has been moved to the end of the chapter on linear and quadratic functions. This new organization provides a logical and unbroken flow of the functions material through the trigonometric functions.
- The material on circles from the chapter on conic sections has been moved earlier in the text.
- The material on exponential and logarithmic functions has been rewritten to provide a clearer understanding of the relationship between the exponential and the logarithmic functions. Many applications have also been added to this material.
- The introduction to trigonometric functions has been reorganized so that it flows more logically, and more time is spent introducing students to distances on the unit circle.
- The material on trigonometric identities has been reorganized and includes the material on cofunction identities. We now also include two methods for proving trigonometric identities with a discussion of the relative advantages and disadvantages of each method.

PEDAGOGICAL FEATURES

Intuitive and Interactive Presentation

- Whenever possible, we base the presentation of a concept on students' prior experience. For example, the introduction to relations and functions is based on correspondences that students are familiar with from daily life. (See pp. 132 and 145.)
- Throughout each section, students are directed to do exercises in the margin; these have answers at the back of the text. Margin exercises involve students interactively in the development of the material and provide a built-in study guide. Many instructors use the margin exercises for in-class reinforcement of the material. (See p. 552.)

Section Objectives

- Objectives for each section are stated in the margin at the beginning of the section, and keyed with a symbol like **2**. This symbol is used throughout the text to key sections of the exposition, exercises, and answers at the back of the text to specific objectives, allowing students to easily find appropriate material for review. (See p. 206.)

Variety of Exercises

- In addition to regular computational and applied exercises, most exercise sets contain two levels of challenge exercises. *Synthesis exercises* require students to synthesize objectives from several sections and help to develop their critical thinking skills. *Challenge exercises* go beyond the section objectives and are designed to challenge the brightest students. (See pp. 216 and 224–226.)

Tests and Review

- Answers to review sections and chapter tests are at the back of the text, together with section references so that students can easily find the correct material to restudy if they miss an exercise. (See p. A-1.)

ACKNOWLEDGMENTS

The authors wish to express their appreciation to the many people who helped with the development of this book, particularly John K. Baumgart and Donna DeSpain for their precise proofreading and checking of the manuscript.

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M.L.K.
 M.L.B.

SUPPLEMENTS FOR THE INSTRUCTOR

Addison-Wesley is committed to providing the best possible service and support for your classroom needs. If you have any questions about the extensive supplements package that accompanies this text for you or your students, please feel free to call 1-800-447-2226.

Instructor's Resource Guide

This includes the following:

- Six forms of each chapter test and six final examinations with answers;
- Transparency masters;
- BASIC computer programs for each chapter designed to give students an idea of the power and utility of the computer in solving exercises encountered in this text;
- User notes for *Cactusplot*, one of Addison-Wesley's function-graphing software packages (see below for more information), and suggestions for integrating *Cactusplot* into your course;
- Videotape indexes cross-referenced to each section of the text.

Instructor's Solutions Manual by Judith A. Penna and John K. Baumgart

This contains worked-out solutions to every exercise in the text.

Printed Test Bank by Betty P. Givan

This is a collection of over 3000 multiple-choice test items organized by text section and objective.

Videotapes

Using the blackboard and manipulative aids, Professor John K. Baumgart of North Park College gives a careful, detailed, and polished presentation of the text material in 50 videotapes ranging from 30 minutes to 45 minutes in length. Professor Baumgart's use of the blackboard and step-by-step discussion brings the concepts of algebra and trigonometry to life and provides ideal review or supplementary coverage for classroom lectures.

Transparency Masters

This is a booklet of 120 black-line masters for key rules, definitions, theorems, proofs, and figures taken from the text. Twenty of these are available on four-color acetates for qualifying adopters.

SUPPLEMENT FOR THE STUDENT

Student's Solutions Manual by Judith A. Penna

This contains completely worked-out solutions to the odd-numbered exercises and answers to the even-numbered exercises in the text.

SOFTWARE SUPPLEMENTS

Testing Software

AWTest. Addison-Wesley's random-number test-generating system is available for the Apple II Series with this text. Using *AWTest*, you can generate up to 99 variations of any particular test with a few keystrokes. You can also choose test items by number from a bank of over 320 multiple-choice algorithm-driven test items or request tests to be printed out in chapter-test format.

AWTestEdit. This is a computerized test item bank containing over 3000 multiple-choice test items for the IBM PC. The program also allows you to edit existing test items and enter your own as easily as using a word processor. You can create tests with both multiple-choice and open-ended questions, save them, and create multiple versions of each test by scrambling question order and multiple-choice distractors.

Utilities Software

Cactusplot: Addison-Wesley Student Version. This is an easy-to-use software utilities package for the Apple II series and IBM PC by John Losse of Scottsdale Community College (IBM adaptation by David Yunker). *Cactusplot* can perform many mathematical operations, such as graphing functions, creating tables, solving equations, and finding the area under curves. Suggestions for integrating *Cactusplot* into your course are included in the Instructor's Manual for this text. A site license to the Student Version of *Cactusplot* is available from Addison-Wesley and is free to qualifying adopters.

The Student Version of *Cactusplot* is available only from Addison-Wesley. A professional version of *Cactusplot* with many enhanced capabilities is available directly from

John Losse
The Cactusplot Company
4712 E. Osborn
Phoenix, AZ 85281
(602) 945-1667.

Related Addison-Wesley Software Titles

Demana/Waits Master Grapher and *3D Grapher Software.* These two utilities packages are available for the IBM PC, MacIntosh, and Apple II series computers. *Master Grapher* allows you to graph and manipulate functions in two dimensions. You can change function parameters, rotate axes, overlay one graph with another, and perform a variety of transformations. The *3D Grapher* graphs complex functions in three dimensions. You can zoom in on the function, change the resolution of its graph, rotate the axis of the graph horizontally and vertically, change its scale, and perform other operations designed to help students visualize graphics in three dimensions more clearly. Contact your Addison-Wesley sales representative for more information.

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