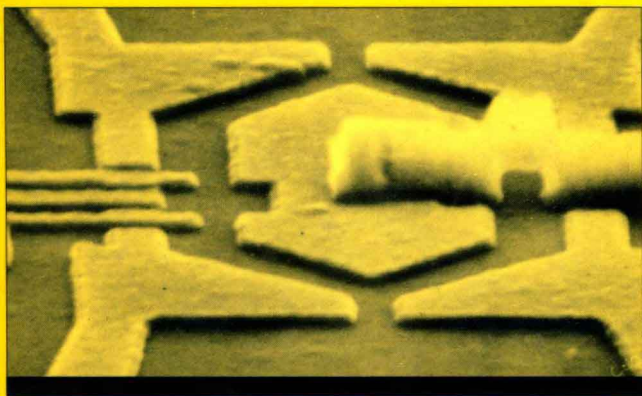


MESOSCOPIC PHYSICS AND NANOTECHNOLOGY



INTRODUCTION TO
MESOSCOPIC
PHYSICS

YOSEPH IMRY

INTRODUCTION TO MESOSCOPIC PHYSICS

Yoseph Imry

New York Oxford
Oxford University Press
1997

Oxford University Press

Oxford New York

Athens Auckland Bangkok Bogota Bombay Buenos Aires
Calcutta Cape Town Dar es Salaam Delhi Florence Hong Kong
Istanbul Karachi Kuala Lumpur Madras Madrid Melbourne
Mexico City Nairobi Paris Singapore Taipei Tokyo Toronto

and associated companies in

Berlin Ibadan

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Published by Oxford University Press, Inc.

198 Madison Avenue, New York, New York 10016

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Library of Congress Cataloging-in-Publication Data

Imry, Yoseph.

Introduction to mesoscopic physics / Yoseph Imry.

p. cm.—(Mesoscopic physics and nanotechnology)

Includes bibliographical references and index.

ISBN 0-19-510167-7

1. Mesoscopic phenomena (Physics) I. Title. II. Series.

QC176.8.M46147 1997

537.6—dc20

95-47793

9 8 7 6 5 4 3 2 1

Printed in the United States of America
on acid-free paper

Mesoscopic Physics and Nanotechnology

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1. Y. Imry. *Introduction to Mesoscopic Physics*

To CYLA

Preface

Mesoscopic physics is a rather young branch of science. It started about 15 years ago and has already had several exciting and instructive achievements. It enjoys the unique combination of being able to deal with and provide answers on fundamental questions of physics while being relevant for applications in the not-too-distant future. In fact, some of the experimental possibilities in this field have been developed with an eye to reducing the sizes of electronic components. It can be hoped that cross-fertilization between physics and technology will continue and go both ways. We now already understand much more about the realm intermediate between the microscopic and macroscopic. Basic questions about how the quantum rules operate and go over into the classical macroscopic regime have been and are being answered. It is hoped that the whole regime between man-made structures and naturally occurring molecules, with their modifications, will be approached and understood soon. Impressive nanoscale techniques for that future stage are being developed.

This book is written in an attempt to make these interesting issues clear to physicists, chemists, and electronic and optical engineers and technologists. The reader should have a solid background in physics, but not necessarily be conversant with advanced formal theoretical methods. The understanding of the underlying physical ideas and the ability to make quite accurate estimates should be of help to both experimental researchers and technologists. At the same time, the study of this material should be helpful to graduate physics and chemistry students for integrating and solidifying their studies of quantum mechanics, statistical mechanics, electromagnetism, and condensed-matter physics.

The author is indebted to many colleagues for collaborations related to these subjects over the years, from which much was learned and the results obtained from which constitute much of the material covered. These colleagues include: Y. Aharonov, A. Aharony, B. L. Altshuler, N. Argaman, the late A. G. Aronov, M. Ya Azbel, D. J. Bergman, M. Büttiker, G. Deutscher, O. Entin-Wohlman, B. Gavish, Y. Gefen, L. Gunther, C. Hartzstein, I. Kander, R. Landauer, N. Lang, I. Lerner, Y. Levinson, S. Mohlecke, G. Montambaux, M. Murat, Z. Ovadyahu, J. L. Pichard, S. Pinhas,

E. Pytte, A. Shalgi, D. J. Scalapino, A. Schwimmer, N. S. Shiren, N. Shmueli, U. Sivan, U. Smilansky, A. Stern, A. D. Stone, M. Strongin, D. J. Thouless, A. Yacoby, and N. Zanon.

Many other colleagues contributed by instructive discussions for which the author is extremely grateful. They include: E. Abrahams, E. Akkermans, S. Alexander, E. L. Altshuler, A. Altland, V. Ambegaokar, T. Ando, Y. Avishai, Y. Bar-Joseph, A. Baratoff, C. Beenakker, E. Ben-Jacob, A. Benoit, M. Berry, the late F. Bloch, H. Bouchiat, E. Brezin, M. Brodsky, C. Bruder, J. Chalker, P. Chaudhari, C.-s. Chi, M. Cyrot, D. Divincenzo, V. Eckern, K. B. Efetov, A. L. Efros, W. A. B. Evans, A. Finkel'stein, A. Fowler, E. Fradkin, H. Fukuyama, N. Garcia, L. Glazman, G. Grinstein, D. Gubser, B. I. Halperin, M. Heiblum, S. Hikami, A. Houghton, A. Kameneev, M. A. Kastner, D. E. Khmel'nitskii, S. Kirkpatrick, S. Kivelson, S. Kobayashi, W. Kohn, B. Kramer, A. Krichevsky, the late R. Kubo, J. Langer, A. I. Larkin, D.-H. Lee, P. A. Lee, A. J. Leggett, S. Levit, L. P. Levy, H. J. Lipkin, D. Loss, the late S.-k Ma, A. MacDonald, A. MacKinnon, D. Mailly, R. S. Markiewicz, Y. Meir, P. A. Mello, U. Meirav, M. Milgrom, J. E. Mooij, B. Mühlischlegel, D. Mukamel, D. Newns, Y. Ono, D. Orgad, the late I. Pelah, J. P. Pendry, M. Pepper, D. Prober, N. Read, H. Rohrer, T. M. Rice, M. Sarachik, M. Schechter, A. Schmid, G. Schön, T. D. Schultz, Z. Schuss, M. Schwartz, S. Shapiro, B. I. Shklovskii, N. Sivan, C. M. Soukoulis, B. Z. Spivak, F. Stern, C.-c. Tsuei, D. C. Tsui, B. van Wees, D. Vollhardt, K. von Klitzing, S. von Molnar, R. Voss, S. Washburn, R. Webb, F. Wegner, H. Weidenmüller, R. Wheeler, P. Wiegman, J. Wilkins, N. Wingreen, S. Wolf, and P. Wölfle.

Special thanks are due to the author's most recent four Ph.D. students (in chronological order): Yuval Gefen, Uri Sivan, Ady Stern, and Nathan Argaman, and to Amir Yacoby. All of them quickly became colleagues and friends and contributed immensely to the work and to the physical understanding of the subject. The collaborations with the late A. G. Aronov, S.-k. Ma, and I. Pelah are especially remembered. The person whose ideas and insights have contributed the most to the author's understanding of the related physics is Rolf Landauer, who deserves special thanks and whose contribution is deeply appreciated. The responsibility for errors, omissions, and misunderstandings rests solely on the author. R. Landauer, C. Bruder, M. Heiblum, D. Orgad, U. Sivan, U. Smilansky and A. Stern are also thanked for pertinent comments on the manuscript.

Various phases of this research were done in a number of laboratories and institutes for which the author is grateful for support. These include Soreq Nuclear Research Center, Cornell University, Tufts University, Tel-Aviv University, the University of California at Santa Barbara and San Diego, Brookhaven National Laboratory, the IBM Yorktown Research Center, CEN Saclay, the University of Karlsruhe and the Humboldt Foundation, the Wissenschaftskolleg of Berlin, Ecole Normale Supérieure and, last but not least, the Weizmann Institute of Science. The following agencies are

thanked for recently supporting parts of this research: BSF (U.S.–Israel Binational Science Foundation), GIF (German–Israeli Binational Science Foundation), the Israel Academy of Sciences, and the Minerva Foundation. Mrs. Naomi Cohen is thanked for expert typing of the manuscript.

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INTRODUCTION TO MESOSCOPIC PHYSICS

Introduction and a Brief Review of Experimental Systems

1. GENERALITIES

Much of solid state theory and statistical physics is concerned with the properties of macroscopic systems. These are often considered while using the “thermodynamic limit” (system’s volume, Ω , and particle number, N , tending to infinity with $n = N/\Omega$ kept constant), which is a convenient mathematical device for obtaining bulk properties. Usually, the system approaches the macroscopic limit once its size is much larger than some correlation length ξ (or, more generally, than all such relevant lengths). In most cases, ξ is on the order of a microscopic length (e.g., $\sim n^{-1/3}$), but in some special cases, such as in the vicinity of a second-order transition, ξ can become very large and one may observe behavior which is different from the macroscopic limit for a large range of sample sizes (Imry and Bergman 1971). Another case where the effective length scale dividing microscopic from macroscopic behavior is very large is that of small conducting (or semiconducting) systems at low temperatures. Here, two important new elements occur: First, the spectrum of electronic states is discrete (although the interaction with the outside world may broaden the levels enough to make that less relevant, see below). Second, the motion is coherent in the sense that once an electron can propagate across the whole system without inelastic scattering, its wavefunction will maintain a definite phase. The electron will thus be able to exhibit a variety of novel and interesting interference phenomena. In what follows, we shall concentrate on the study of the latter type of systems.

The interest in studying systems in the intermediate size range between microscopic and macroscopic (sometimes referred to as “mesoscopic”—a word coined by Van Kampen 1981) is not only in order to understand the macroscopic limit and how it is achieved by, say, building up larger and larger clusters to go from the “molecule” to the “bulk.” Many novel phenomena exist that are intrinsic to mesoscopic systems. A mesoscopic system is really like a large molecule, but it is always, at least weakly, coupled to a much larger, essentially infinite, system—via phonons, many-body excitations, and so on. Sometimes such a coupling can be controlled. Ideally, one would like to interpolate between open and closed systems, as far as energy, particle number, and so on are concerned, by varying some coupling strengths. The special phenomena that exist in this range are of great interest by themselves. We shall see how fundamental principles of quantum mechanics (related to the concept of the phase of the wavefunction) and statistical physics (brought about by the small specimen size and by the slowness of inelastic scattering and thermalization) appear and are amenable to theoretical clarification and experimental examination in these systems.

An important concept is that of the “impurity ensemble”—the collection of systems having the same *macroscopic* parameters (e.g., average concentrations of various defects) but differing in the detailed arrangement of the resulting disorder. In the macroscopic limit an average over this ensemble is usually performed, which restores various symmetries on average. However, a principal interesting aspect of mesoscopics is the distinction (Landauer 1970, Azbel 1973, Imry 1977, Anderson et al. 1980, Azbel and Soven 1983, Gefen et al. 1984 personal communication) between ensemble-averaged properties, and those specific to a particular given small system taken from the ensemble. The specific “fingerprint” of such a small system is of interest and may sometimes be used to obtain some statistical information on the particular arrangement of the constituents in the system (Azbel 1973). Alternatively, changes with time (usually on long scales) of the disorder configuration may lead to low-frequency noise (Feng et al. 1986).

Many of the usual rules that one is used to in macroscopic physics may not hold in “mesoscopic” systems. For example, the rules for addition of resistances, both in series (Landauer 1970, Anderson et al. 1980) and in parallel (Gefen et al. 1984a,b) are different and more complicated. The electronic motion is wavelike and it is not dissimilar to that of electromagnetic radiation in waveguide structures, except for very interesting complications due to disorder. These effects may set fundamental limits on how small various electronic devices can eventually be made. On the other hand, ideas for new devices such as those operating in analogy with various optical and waveguide ones, as well as with SQUIDS (superconducting quantum interference devices) and other Josephson-effect systems (see, e.g., Hahlbohm and Lübbig 1985) may emerge for small normal conductors.

The technology (see, e.g., Howard and Prober 1982, Prober 1983, Laibowitz 1983, Broers 1989) for the fabrication of supersmall structures is

progressing very quickly and has reached the stage where many theoretical predictions can now be confronted by experiments. One uses controlled growth, such as in the MBE method (see, e.g., Herman and Sitter 1989), and advanced optical, x-ray, or electron-beam lithographic techniques. In semi-conducting systems based on quantum-well concepts, an excellent restriction in one direction exists (Ando et al. 1982), so that creating small structures parallel to the 2D (two-dimensional) layer may achieve systems with a rather small number of active electrons or quantum states. (See the next section for a brief summary and references on available systems and fabrication methods.) On the other hand, we have the recent STM breakthrough (Binnig et al. 1982), which provides a novel tool for atomic-scale fabrication, analysis, and measurement (for a recent example see Crommie et al. 1993). One may soon reach the stage of having large conducting artificial molecules on which macroscopic-type experiments can be performed, in the same size range as ordinary macromolecules, or smaller. The latter may be addressed and are of course also of great interest.

It should be noted that photons may also be well "guided" in such systems and similar phenomena may thus occur for these electromagnetic waves, not to mention ideas for electron-photon coupling in various combinations. Both subjects of mesoscopies and light propagation in disordered media (John et al. 1983, Feng and Lee 1991, Sheng 1995) have undergone real advances due to analogies and mutual fertilization; see van Haeringen and Lenstra (1991) for pertinent papers.

This book will deal mostly with theory. However, an attempt will be made to present the most economical explanation/semiquantitative calculation for physical effects, rather than to "demonstrate the power" of some formal method. It is hoped that the qualitative understanding of the physical principles involved that is gained might be useful in conceiving new experiments.

One of the powerful and useful concepts which appear is that of "universality," namely, that various measurable quantities do not depend on most microscopic details of the system. This was originally introduced by Kadanoff in the context of critical phenomena, where large-scale physical properties do not depend on most microscopic details. Older examples are: (a) the Hall coefficient, which does not depend in the simplest picture on the effective mass and on the scattering time; and (b) the Debye T^3 specific heat. The latter is a good example of universality due to general properties of the spectrum (here density of states (DOS)) of certain operators (the phonon Hamiltonian, for the Debye law). Such universalities are very relevant to our subject. In a sense, it is even more remarkable that dirty systems display such universalities, compared to the ultra-clean and perfect systems needed for studies of critical phenomena.

Experimental techniques will be referred to very briefly, mainly in order to understand what can be done and what the limitations are. The theoretical task is becoming more difficult (and interesting) as the understanding of single-electron properties has advanced and one now has to delve into many-body

physics, where interactions are important and the theory is much more sophisticated (for a recent account, see Imry and Sivan 1994). Our emphasis will be on equilibrium as well as on various electronic transport phenomena. For a good review on optical effects, we refer to Schmitt-Rink et al. (1989). Bastard et al. (1991) contains a good review of related aspects of electronic properties of semiconducting heterostructures and some of their optical properties. Analogies between optics and electronic phenomena are discussed in van Haeringen and Lenstra (1991). We shall treat only some aspects of the interesting case of ballistic transport (Heiblum et al. 1985, Beenakker and van Houten 1991d) and just mention briefly the topic of "Coulomb blockade" (see problem 5 of chapter 5 and references in Grabert and Devoret 1992, Hekking et al. 1994). A recent hydrodynamic analogy (de Jong and Molenkamp 1995) of electronic transport is also worth noting.

In the next section, systems and fabrication techniques will be briefly described. In Chapter 2 the limitations of the ordinary quasiclassical transport will be discussed and the opposite limit, of Anderson localization, introduced. Since phase coherence is so important in mesoscopics, we shall devote Chapter 3 to elucidating what it takes to destroy phase coherence, with some examples. Chapter 4 will consider equilibrium properties and Chapter 5 will discuss transport phenomena in mesoscopic systems. Chapter 6 will be devoted to high magnetic fields and the quantum Hall effect, Chapter 7 to mesoscopics with superconducting components and Chapter 8 to various noise phenomena. Concluding remarks will be given in Chapter 9. Various details are discussed in the appendices.

2. A BRIEF DESCRIPTION OF SYSTEMS AND FABRICATION METHODS

For experiments in conducting mesoscopic systems, one may in principle use members of the three principal classes of conductors:

1. *Metals*, having high charge-carrier densities in the range of $10^{22}/\text{cm}^3$ and a wide range of purities and mean free paths. Many metals become superconductors at low temperatures—which provides another interesting degree of freedom.
2. *Semiconductors*, where the carrier densities can range practically between 10^{15} and $10^{19}/\text{cm}^3$ and may be controlled, including the type of carrier, by doping, optical excitation, or electrostatic "gates." Special methods may be used to produce high mobilities and heterojunctions—interfaces between different semiconductors with interesting properties (see below).
3. In special cases, *semimetals* having intermediate carrier densities of 10^{19} to $10^{20}/\text{cm}^3$ are useful. They contain electrons and holes concurrently. In some cases, notably bismuth on which many of the quantum oscillation effects have very early been demonstrated, these materials can have