



PETER P. WAKKER

# Prospect Theory

FOR RISK AND AMBIGUITY

CAMBRIDGE

# Prospect Theory

## For Risk and Ambiguity

---

---

Peter P. Wakker



**CAMBRIDGE**  
UNIVERSITY PRESS

## *Preface*

---

This book is the culmination of 14 years of teaching. In the 15th year, when for the first time I did not feel like rereading or rewriting, the time had come to publish it. The book received helpful comments from Han Bleichrodt, Arie de Wild, Itzhak Gilboa, Glenn Harrison, Amit Kothiyal, Gijs van de Kuilen, Georg Weizsäcker, and many students during the past 14 years. Thorough comments from Rich Gonzalez and Vitalie Spinu are especially acknowledged. I am most indebted to Stefan Trautmann for the numerous improvements he suggested. This book has also benefited from many inspiring discussions with Craig Fox, with whom I share the privilege of having collaborated with Amos Tversky on uncertainty during the last years of his life.

# Contents

---

<i>Preface</i>	<i>page xiii</i>
----------------	------------------

<b>Introduction</b>	<b>1</b>
Prospect theory	2
Behavioral foundations	2
Homeomorphic versus paramorphic modeling	3
Intended audience	3
Attractive feature of decision theory	4
Structure	4
Preview	5
Our five-step presentation of decision models	7

## **Part I Expected utility**

<b>1 The general model of decision under uncertainty and no-arbitrage (expected utility with known utilities and unknown probabilities)</b>	<b>11</b>
1.1 Basic definitions	11
1.2 Basic properties of preferences	15
1.3 Expected value	17
1.4 Data fitting for expected value	19
1.5 The bookmaking argument of de Finetti, or the no-arbitrage condition from finance	22
1.6 A behavioral foundation of subjective probabilities and expected value using no-arbitrage and no-book	27
1.7 Discussion of Structural Assumption 1.2.1 (Decision under uncertainty)	32
1.8 The general nature and usefulness of behavioral foundations	34
1.9 Appendix: Natural, technical, and intuitive preference conditions, and the problematic completeness condition	37
1.10 Appendix: The relation of Theorem 1.6.1 to the literature	39
1.11 Appendix: Proof of Theorem 1.6.1 and Observation 1.6.1'	41

<b>2</b>	<b>Expected utility with known probabilities – “risk” – and unknown utilities</b>	<b>44</b>
2.1	Decision under risk as a special case of decision under uncertainty	44
2.2	Decision under risk: basic concepts	47
2.3	Decision under risk as a special case of decision under uncertainty; continued	48
2.4	Choices under risk and decision trees	51
2.5	Expected utility and utility measurement	53
2.6	Consistency of measurement and a behavioral foundation of expected utility	57
2.7	Independence and other preference conditions for expected utility	63
2.8	Basic choice inconsistencies	65
2.9	Appendix: Proof of Theorem 2.6.3 and Observation 2.6.3'	67
<b>3</b>	<b>Applications of expected utility for risk</b>	<b>69</b>
3.1	An application from the health domain: decision tree analysis	69
3.2	Risk aversion	71
3.3	Applications of risk aversion	74
3.4	Indexes of risk aversion	77
3.5	Parametric families of utility	78
3.6	Data fitting for expected utility under risk	83
3.7	Multiattribute utility	85
3.8	Taxation and welfare theory with cardinal utility	91
3.9	Proofs for Chapter 3	92
<b>4</b>	<b>Expected utility with unknown probabilities and unknown utilities</b>	<b>94</b>
4.1	Fifteen preference questions	94
4.2	(Subjective) expected utility	100
4.3	Measuring utility and testing EU for §4.1	103
4.4	A decision process	107
4.5	General utility measurement and the tradeoff notation	108
4.6	A behavioral foundation of expected utility	110
4.7	Further discussion of Theorem 4.6.4 (Uncertainty-EU)	112
4.8	Further implications of expected utility	114
4.8.1	Savage's sure-thing principle	114
4.8.2	Utility analysis based on t-indifferences	115
4.8.3	Debreu's additive decomposability and state-dependent utility	117
4.9	A hybrid case: subjective probabilities and utilities when also objective probabilities are given	118
4.9.1	A general (“single-stage”) approach	119

4.9.2 A behavioral foundation for the single-stage approach	120
4.9.3 The multi-stage approach of Anscombe & Aumann (1963)	122
4.9.4 Comparing the Anscombe & Aumann model with the single-stage approach	125
4.10 Data fitting for expected utility under uncertainty	127
4.11 A statistical analysis of the experiment of §4.1, revealing violations of expected utility	131
4.12 The Allais paradox: a well-known violation of expected utility	133
4.13 Appendix: Experimental economics' real-incentive principle and the tradeoff method	135
4.14 Appendix: Tradeoff consistency as a generalization of de Finetti's bookmaking	138
4.15 Appendix: Proofs for Chapter 4	139

## **Part II Nonexpected utility for risk**

<b>5 Heuristic arguments for probabilistic sensitivity and rank dependence</b>	<b>145</b>
5.1 Probabilistic sensitivity versus outcome sensitivity for single-nonzero-outcome-prospects	145
5.2 Probabilistic sensitivity for multi-outcome prospects, and the old way of transforming probabilities	149
5.3 A violation of stochastic dominance	153
5.4 Rank-dependent utility: discovering the formula using psychological intuition	155
5.5 Rank-dependent utility: discovering the formula using mathematical intuition	161
5.6 Calculating rank-dependent utility	165
5.7 Conclusion	168
<b>6 Probabilistic sensitivity and rank dependence analyzed</b>	<b>169</b>
6.1 Ranks, ranked probabilities, and rank-dependent utility defined	169
6.2 Ranks, ranked probabilities, and rank-dependent utility discussed	172
6.3 Where rank-dependent utility deviates from expected utility: optimism and pessimism	172
6.4 Further deviations of rank-dependent utility from expected utility	176
6.4.1 The certainty effect	176
6.4.2 Further discussion of rank dependence	178
6.5 What rank-dependent utility shares with expected utility	181
6.5.1 Measuring utility under rank-dependent utility	181
6.5.2 A behavioral foundation of rank-dependent utility	184
6.5.3 An elaborated example	185
6.5.4 Abdellaoui's method of measuring probability weighting	188
6.5.5 Further implications of rank-dependent utility	188

6.6 Appendix: Yet further deviations of rank-dependent utility from expected utility	190
6.7 Appendix: Ranking identical outcomes and collapsing outcomes	194
6.8 Appendix: An interpretation of rank-dependent utility using the derivative of $w$	197
6.9 Appendix: RDU for continuous distributions and figures for negative outcomes	199
<b>7 Applications and extensions of rank dependence</b>	<b>203</b>
7.1 Likelihood insensitivity and pessimism as two components of probabilistic risk attitudes	203
7.2 Parametric forms of weighting functions	206
7.3 Data fitting for rank-dependent utility under risk	211
7.4 Direct ways to test convexity, concavity, and likelihood insensitivity using violations of the sure-thing principle	214
7.5 An alternative direct way to directly investigate properties of nonlinear decision weights	217
7.6 Bad-news probabilities or loss-ranks	219
7.7 A formal definition of likelihood insensitivity (inverse-S)	222
7.8 The choice of insensitivity region	226
7.9 Discussion of likelihood insensitivity	227
7.10 Indexes of pessimism and likelihood insensitivity	229
7.11 Binary rank-dependent utility	230
7.12 Appendix: Alternative definition of likelihood insensitivity: first concave and then convex	231
7.13 Appendix: Proofs for Chapter 7	233
<b>8 Where prospect theory deviates from rank-dependent utility and expected utility: reference dependence versus asset integration</b>	<b>234</b>
8.1 A choice paradox	235
8.2 A discussion and the real culprit of the paradox	236
8.3 Deviations from a fixed reference point ("initial wealth") as nothing but an alternative way of modeling final wealth, and of incorporating some empirical improvements	237
8.4 Loss aversion defined	238
8.5 Deviations from a variable reference point as a major breakaway from final-wealth models	240
8.6 Rabin's paradox	242
8.7 Future directions for theories of reference dependence	245
8.8 Appendix: Empirical meaningfulness problems of loss aversion	247
8.9 Appendix: A formal model of initial wealth and reference points	249

<b>9 Prospect theory for decision under risk</b>	<b>251</b>
9.1 A symmetry about 0 underlying prospect theory	251
9.2 The definition of prospect theory	252
9.3 Properties of the prospect theory formula, and calculations	254
9.4 What prospect theory shares with rank-dependent utility and with expected utility, and one more reanalysis of the experiment of § 4.1	260
9.4.1 Monotonicity, stochastic dominance, and the sure-thing principle	260
9.4.2 Measuring utility, event weighting, and loss aversion	262
9.5 Empirical findings on prospect theory	264
9.6 Analytical problems of power utility for analyzing loss aversion	267
9.6.1 The first problem: loss aversion depends on the monetary unit	267
9.6.2 The second problem: always $U(\alpha) > -U(-\alpha)$ for some outcome $\alpha > 0$	270
9.6.3 Ways to avoid the problems	271
9.7 Appendix: Some theoretical issues concerning prospect theory	271
9.8 Appendix: Original prospect theory of 1979	273

### **Part III Nonexpected utility for uncertainty**

<b>10 Extending rank-dependent utility from risk to uncertainty</b>	<b>279</b>
10.1 Probabilistic sophistication	279
10.2 Rank-dependent utility defined for uncertainty and without probabilistic sophistication	282
10.3 Rank-dependent utility and probabilistic sophistication once more	286
10.3.1 Ellsberg's violation of probabilistic sophistication	286
10.3.2 Further observations	287
10.4 Where rank-dependent utility under uncertainty deviates from expected utility in the same way as it did under risk	288
10.4.1 Optimism and pessimism	289
10.4.2 Likelihood insensitivity (Inverse-S)	290
10.4.3 Direct ways to test convexity, concavity, and likelihood insensitivity using violations of the sure-thing principle	292
10.5 What rank-dependent utility shares with expected utility for uncertainty (in the same way as it did for risk)	294
10.5.1 Rank-preference conditions	294
10.5.2 Measuring utility (and event weighting) under RDU	295
10.5.3 A behavioral foundation for RDU	297
10.6 Binary rank-dependent utility	298
10.7 A hybrid case: rank-dependent utility for uncertainty when also objective probabilities are given	299

10.7.1	A general (“single-stage”) approach	299
10.7.2	A behavioral foundation for the single-stage approach	300
10.7.3	Schmeidler’s and Jaffray’s two-stage approaches	301
10.8	Bad-news events or loss-ranks	304
10.9	Further observations regarding rank dependence	305
10.10	Appendix: An integral representation	309
10.11	Appendix: Ranking states and collapsing outcomes	310
10.12	Appendix: Comonotonicity	310
10.13	Appendix: Proofs for Chapter 10	314
10.14	Appendix: Proof of Theorem 6.5.6	316
<b>11</b>	<b>Ambiguity: where uncertainty extends beyond risk</b>	<b>317</b>
11.1	The Ellsberg paradox and the home bias as within-subject between-source comparisons	317
11.2	Using rank dependence to analyze ambiguity	318
11.3	Elaborated examples for ranks, ambiguity premiums, and hedging in finance	322
11.4	The CORE	323
11.5	Multiple priors (Maxmin expected utility and $\alpha$ -maxmin): basic results	324
11.6	Approaches in the literature that we will not take	327
11.7	Indexes of ambiguity aversion	328
11.8	Indexes of ambiguity aversion and sensitivity	332
11.9	Appendix: Discussion of multiple priors and other models	335
11.10	Appendix: Violations of monotonicity for multiple priors	338
11.11	Appendix: Möbius transforms and belief functions	339
<b>12</b>	<b>Prospect theory for uncertainty</b>	<b>342</b>
12.1	Prospect theory defined	342
12.2	Where prospect theory deviates from rank-dependent utility and expected utility	344
12.3	What prospect theory shares with rank-dependent utility and expected utility under uncertainty	344
12.3.1	Sign-preference conditions	344
12.3.2	Measuring utility (and event weighting) under prospect theory	345
12.3.3	Measuring loss aversion under prospect theory	346
12.3.4	A behavioral foundation for prospect theory	347
12.4	A hybrid case: prospect theory for uncertainty when also objective probabilities are given	349
12.5	Loss aversion versus ambiguity aversion	349
12.6	Violations of prospect theory and rank dependence	350
12.7	Indexes of ambiguity attitudes under prospect theory	354

12.8 Appendix: Some theoretical issues concerning prospect theory for uncertainty	356
12.9 Appendix: Proofs for Chapter 12	357
<b>13 Conclusion</b>	<b>358</b>
<b>Appendices</b>	<b>359</b>
Appendix A Models never hold perfectly: how to handle their deficiencies?	359
A.1 Nonparametric measurements and parametric fittings for imperfect models: general discussion	359
A.2 Our distance measure for parametric fitting	361
A.3 Discussion of the distance measure	363
Appendix B Choosing from multiple prospects and binary choice: the principles of revealed preference	366
B.1 Examples	367
B.2 A behavioral foundation of preference relations derived from choice behavior	371
B.3 Assumptions underlying revealed preference	374
B.4 The history of revealed preference	378
Appendix C Dynamic decisions	380
Appendix D Applications other than decision under uncertainty	384
Appendix E Bisymmetry-based preference conditions	387
Appendix F Nonmonotonic rank-dependent models and the Fehr–Schmidt model of welfare evaluation	391
Appendix G Extensions of finite-dimensional results to infinite-dimensional results: a meta-theorem	393
Appendix H Measure theory	395
Appendix I Related textbooks and surveys	397
Appendix J Elaborations of exercises	399
Appendix K Skipping parts and interdependencies between sections	455
 <i>References</i>	 461
<i>Author index</i>	492
<i>Subject index</i>	500

# Introduction

---

This book has been written and organized especially for readers who do not want to read all of its contents, but want to skip parts and select the material of their own interest. This has been achieved by an organization of exercises explained later, and by an Appendix K that describes the interdependencies between sections. Because of this organization, this book can be used by readers with different backgrounds.

We will examine theories of individual decision making under uncertainty. Many of our decisions are made without complete information about all relevant aspects. This happens for instance if we want to gamble on a horse race and have to decide which horse to bet on, or if we are in a casino and have to decide how to play roulette, if at all. Then we are uncertain about which horse will win or how the roulette wheel will be spun. More serious examples include investments, insurance, the uncertain results of medical treatments, and the next move of your opponent in a conflict. In financial crises, catastrophes can result from the irrational attitudes of individuals and institutions towards risks and uncertainties.

Two central theories in this book are expected utility theory and prospect theory. For all theories considered, we will present ways to empirically test their validity and their properties. In many applications we require more than just qualitative information. We may want to know exactly that spending 1 percent more on a new medicine will generate a 3 percent increase in quality of life for the patient group affected, rather than just knowing that spending more money improves the quality of life. Similarly, we may want to know that a person is willing to pay a maximum of \$350 extra tax so as to avoid a 1:100 risk of losing savings to the value of \$30,000 in case a bank goes bankrupt. Hence, for all the theories presented in this book, methods will be provided for obtaining precise quantitative measurements concerning those theories and their parameters. Thus precise quantitative predictions can be made. The possibility of obtaining tractable quantitative measurements was a selection criterion for the theories presented in this book.

Typical for the analyses in this book is the interaction between, on the one hand, theoretical and algebraic tools, and, on the other hand:

- prescriptive considerations as relevant for consultancies, policy decisions, and your own decision making;
- descriptive considerations as relevant in psychology and other empirical disciplines.

### **Prospect theory**

Until the end of the 1970s, irrational behavior was believed to be chaotic and unsuited for modeling. The normative expected utility model was taken to be the best approximation of descriptive behavior (Arrow 1951a p. 406; Tversky & Kahneman 1981 opening sentence). Kahneman & Tversky's (1979) prospect theory provided a major breakaway. It was the first descriptive theory that explicitly incorporated irrational behavior in an empirically realistic manner (Kahneman 2003 p. 1456), while at the same time being systematic and tractable. It was the first rational theory of irrational behavior, so to say.

Tversky & Kahneman (1992) introduced an improved version of prospect theory. First, they used Quiggin's (1982) rank dependence to correct a theoretical problem in probability weighting. Second, and more importantly, they extended the theory from risk (known probabilities) to uncertainty and ambiguity (unknown probabilities), using Schmeidler's (1989) rank dependence. In this manner, for the first time a theory has resulted that combines empirical realism with theoretical soundness and tractability. Prospect theory comprises the happy marriage between the empirical insights of Kahneman & Tversky (1979) and the theoretical insights of Gilboa (1987) and Schmeidler (1989).

At this moment of writing, 30 years after its invention, prospect theory is still the only theory that can deliver the full spectrum of what is required for decision under uncertainty, with a natural integration of risk and ambiguity. Therefore, a textbook on the theory is useful. The main purpose of this book is to make this theory accessible to a wide audience by presenting it in a manner as tractable as possible.

### **Behavioral foundations**

Behavioral foundations will play a central role in this book. For a particular decision model, a behavioral foundation gives a list of conditions, stated directly in terms of observable preferences, that hold if and only if the decision model holds. Preference foundations translate the meaning of quantitative decision models and their subjective parameters ("theoretical constructs"), such as subjective probabilities or utilities, into observables. Descriptively, they show how to verify or falsify decision models. Normatively, they provide the terms to justify or criticize models. When de Finetti (1931a), von Neumann & Morgenstern (1944), and Savage (1954) provided behavioral foundations for expected utility, this gave a big boost to the popularity of this theory in many fields. Those fields include economics and game theory (Mas-Colell, Whinston, & Green 1995), management science under the name decision analysis

(Keeney & Raiffa 1976), medicine (Weinstein *et al.* 1980) where utilities are often referred to as QALYs, and statistics (reviving the Bayesian approach; DeGroot 1970). Behavioral foundations ensure the intrinsic soundness of a decision model, preventing historical accidents such as happened for what is known as the separate-probability-transformation model (details in Chapter 5).

### Homeomorphic versus paramorphic modeling

A model is *paramorphic* if it describes the empirical phenomena of interest correctly, but the processes underlying the empirical phenomena are not matched by processes in the model (Harré 1970). For example, as emphasized by Milton Friedman (1953; see Bardsley *et al.* 2010 Box 2.4), market models can make correct predictions even if their assumptions about consumers do not match actual consumers' behavior. A model is *homeomorphic* if not only its empirical phenomena match reality, but also its underlying processes do so. We will seek homeomorphic models of decision making. Not only do the decisions predicted by the model match the decisions observed, but also we want the theoretical parameters in the model to have plausible psychological interpretations.

Friedman's arguments in favor of paramorphic models are legitimate if all that is desired is to explain and predict a prespecified and limited domain of phenomena. It is, however, usually desirable if concepts are broadly applicable, also for future and as yet unforeseen developments in research. Homeomorphic models are best suited for this purpose. In recent years, economics has been opening up to introspective and neuro-imaging data. It is to be expected that the concepts of prospect theory, in view of their sound psychological basis, will be well suited for such future developments and for connections with other domains of research. Behavioral foundations with plausible preference conditions support the homeomorphism of a model.

### Intended audience

No particular mathematical background knowledge is required, besides a basic knowledge of probability theory and calculus. A willingness to work with formal models and to follow abstract trains of thought is needed for this book though. The measurement methods and behavioral foundations presented in this book will be as simple and transparent as possible, so as to be accessible to as many readers as possible.

Mathematically sophisticated readers may be interested in this book, and will perhaps be surprised by it, from a didactic perspective. For example, Gilboa's (1987) and Schmeidler's (1989) rank-dependent utility theory, and Tversky & Kahneman's (1992) new prospect theory have often been considered to be complex, with presentations based on a comonotonicity concept. These theories can, however, be presented and derived in an elementary manner if we use ranks instead of comonotonicity, as will be done in this book.

Decisions under uncertainty are relevant in many fields, including finance, economics, psychology, management science, medicine, computer science, Bayesian statistics, and engineering. Readers from any of these fields can rest assured that no advanced concepts will appear from any of the other fields because the author does not have a bachelor's degree in any of the fields mentioned.

### Attractive feature of decision theory

An attractive feature of decision theory is that the reader can always imagine that he or she is the decision maker. For each preference condition presented in the text, you can ask yourself whether you would want to satisfy this condition in your own decisions. It is easiest to read this book with this question in mind. Hence, the decision maker and the readers will usually be referred to as “you.”

### Structure

The material in this book has been structured so as to be accessible to readers with different backgrounds, levels, and interests. Many results that will be relevant to some readers but not to all have been stated in exercises, whose elaborations are in Appendix J. This structure gives different readers the chance to skip and select different parts. Italicized superscripts *a*, *b*, *c* indicate which exercises are suited for which readers. The superscript *a* refers to exercises that are easiest, and the superscript *c* refers to exercises that are most difficult and that will be of interest only to the most theoretically oriented readers. Many readers, especially empirically oriented readers who are not used to working with formal models, will want to skip almost all exercises. Typically, psychology students interested in formal models will be *a*-students who will study the empirical parts of this book; mathematical students are *c*-students who are required to study the theoretical parts; and economics students are somewhere in between, so that they are usually *b* students.

The best way to completely master the material in this book – if there are no time restrictions – is to stop reading after every exercise and then first do that exercise. Readers who are satisfied with a less thorough and time-consuming study can use the exercises flexibly. *Sometimes an exercise contains results that are needed to understand the rest of the text. This is indicated by an exclamation! as superscript. Then every reader, even those not doing the exercise, should read its results.*

Exercises are interspersed throughout the text, and are located where they are most relevant. Some sections conclude with assignments. These are further exercises that serve to grade students and/or to practice. Their results play no role in the theory development in the main text, and no elaborations of assignments are given in this book. On the author's homepage, further exercises and assignments are provided. This serves teachers who wish to have more exercises without solutions available to the students. Teachers can obtain solutions to assignments from the publisher. Proofs of theorems are collected in appendices at the end of chapters.

For the use of this book, with comprehensive theoretical discussions and comprehensive discussions of empirical implications, Appendix K is instrumental. It illustrates how sections depend on preceding sections. By using this appendix, you need not read the book from start to finish. In a backward approach, you pick out any topic of interest, and then use Appendix K to see which preceding material you need to read for it. In a forward approach, you skip whatever you do not like. If needed later after all, then Appendix K will show you so. If you are interested in only part of the book, this organization allows you to use the book efficiently. In particular, teachers can easily select the material targeted at the interests of specific students.

If you want to know the definition of prospect theory for unknown probabilities in §12.1, then you can select the texts depicted in Figure K.1 in Appendix K. The sections listed there comprise about 46 pages to be read. If you are not interested in the tradeoff technique of §4.1 and §4.5, then you can skip all of Chapter 4 except for §4.2 and §4.9, and then skip §§6.5, 9.4, 10.5, and 12.3. If you are interested only in decision under risk, then you can learn about the definition of prospect theory in §9.2, using the same method and the same figure, skipping virtually all sections on decision under uncertainty, and reading approximately 34 pages. If you want to learn about a pragmatic index of ambiguity aversion under prospect theory, then you can similarly use Figure K.2. If you want to understand as quickly as possible how the popular value at risk (VaR) for measuring the reliability of banks is a special case of prospect theory and rank dependence (Exercise 6.4.4), then you can find the shortest path: §§6.4, 6.3, 6.1, 3.2, 2.5, 2.4, 2.2, 2.1, 1.3, 1.2, 1.1.

For 10 meetings of three hours each, a typical timetable may be: *meeting 1*: §1.1–§1.8; *meeting 2*: §2.1–§2.9; *meeting 3*: §3.1–§3.6, §4.1; *meeting 4*: §4.2–§4.7, §4.9.1, §4.11, §4.12; *meeting 5*: §5.1–§5.7, §6.1, §6.3–§6.5; *meeting 6*: §7.1–§7.4, §7.6–§7.11; *meeting 7*: §8.1–§8.5, §9.1–§9.5; *meeting 8*: §10.1–§10.6, §10.7.1, §10.8; *meeting 9*: §11.1, §11.4–§11.8; and *meeting 10*: §12.1–§12.3, §12.7. I have used this book in teaching advanced master's students in economics who had digested large parts of Mas-Colell, Whinston, & Green (1995). I would then cover the material allocated to the first four meetings above in about two meetings, after which I would follow the above timetable. The total workload of this selection for students is about 120 hours of full-time work.

A nice way to teach only part of this book is by restricting all models only to binary (two-outcome) prospects. This domain is rich enough to measure and define all components of risk attitude, utility, probability- or event-weighting, and loss aversion. Rank dependence and prospect theory are considerably simplified on this domain. This is how I taught this course to business students. They are particularly interested in prescriptive applications of decision theory.

## Preview

The book consists of three parts. Part I deals with the classical expected utility theory, and Parts II and III consider deviations. In Part I, §1.1 and §1.2 present the basics of decision under uncertainty. The rest of Chapter 1 presents the famous bookmaking

condition of de Finetti, developed in the 1930s to justify the use of subjective probabilities for one-shot events. This condition is equivalent to the no-arbitrage condition in finance, which implies that market prices of financial derivatives have to be based on what are called as-if risk neutral evaluations. That is, these conditions imply expected utility when probabilities are unknown but utility is known (linear). Chapter 2 deals with expected utility when probabilities are known ("decision under risk") but utilities are unknown. There are so many applications of this long-existing theory that they are presented separately in Chapter 3. Chapter 4 turns to the more complex topic of expected utility when both probabilities and utilities are unknown, using tradeoffs between outcomes as a tool to measure utility differences. It ends with some empirical violations of expected utility, preparing for the parts to follow.

Part II deals with deviations from expected utility for decision under risk, where probabilities are known. We present rank-dependent utility, which generalizes expected utility by adding a new dimension of risk attitude: probabilistic sensitivity – i.e., the nonlinear ways in which people may process probabilities. This dimension is descriptively as relevant for risk attitudes as the nonlinear ways in which people process outcomes (utility), and had been sorely missing in the models used before. In 1982 John Quiggin introduced a correct theoretical manner of modeling such nonlinear processing, the rank-dependent formula. It was only then that a serious descriptive analysis of risk attitudes could begin.

Chapter 5 presents mathematical and psychological arguments to show that the rank-dependent model naturally captures probabilistic sensitivity. Chapter 6 defines the theory formally, and shows how it can be used to tractably capture prevalent phenomena regarding risk attitude. We use ranks, introduced by Abdellaoui & Wakker (2005), as a tool to measure probability weight differences. We can then define ranked probabilities, which are the analogs in the probability dimension of the tradeoffs in the outcome dimension used in Chapter 4. Ranked probabilities facilitate the analyses of the rank-dependent model and are more tractable than the comonotonicity concepts that have been used in the literature. Chapter 7 presents empirical findings and special cases of rank dependence.

In Chapters 8 and 9 we turn to prospect theory. In 1992, Tversky and Kahneman incorporated Quiggin's idea of rank dependence to solve a theoretical problem of their original prospect theory of 1979. It led to the present version of prospect theory, also called cumulative prospect theory. To prepare for prospect theory, Chapter 8 introduces another generalization of expected utility beyond rank dependence: reference dependence. Outcomes are reinterpreted as changes with respect to a reference point (often the status quo). With reference dependence introduced, all ingredients are now available to define and analyze prospect theory for risk (Chapter 9).

Part III concerns decision under uncertainty, where probabilities need not be known. Ambiguity attitudes, which deviate from expected utility in fundamental ways and may not even admit the existence of (subjective) probabilities, are analyzed. Chapter 10 starts by extending Quiggin's definition of rank dependence from risk to the more subtle context of uncertainty, for which Schmeidler (1989, first version

1982) conceived it independently. Chapter 11 presents the main novelties of uncertainty, namely source dependence, which includes ambiguity aversion. We show how rank dependence can be used to analyze uncertainty, and to provide tractable measures of ambiguity aversion and sensitivity to ambiguity. These measures encompass the currently popular  $\alpha$ -maxmin model.

Chapter 12 presents the most important model of this book, namely prospect theory for uncertainty. This model entails a common generalization of all the models presented up till then. Relative to Chapters 10 and 11, it allows ambiguity attitudes for losses to be different than for gains. This generalization is desirable because empirical studies have shown that such differences are pronounced. Prospect theory is the first theory for decision under uncertainty that is both theoretically sound and empirically realistic. It means that only since 1992 do we have a satisfactory theory that can deliver the full spectrum of what is needed for decision under risk and uncertainty. Chapter 13 concludes the main text.

Appendices A–K complete the book. I will only discuss a few here. §A.1 in Appendix A contains a general methodological discussion of models being imperfect with inconsistencies in data, and of the nonparametric measurements that are central to this book. Appendix B presents some general issues of the revealed-preference paradigm. Appendix F shows that the influential Fehr–Schmidt model for welfare evaluations is a special case of rank dependent utility, and Appendix J contains the elaborations of the exercises in the book.

### Our five-step presentation of decision models

We usually present decision theories in five steps that serve to make the empirical meaning of the theories tangible for the readers. The first step is, simply, to define the decision model. We specify the subjective parameters of the model and give a formula describing how these parameters imply preferences. In expected utility with given probabilities (risk), the subjective parameter is utility. In prospect theory for risk, the subjective parameters are utility, probability weighting, and loss aversion. In the second step, it is demonstrated how decisions can be derived from the model using simple numerical exercises.

Although we do not endorse Lord Kelvin's maxim "science is measurement" as a universal principle, measurement is central to this book. Thus, the third step in our analysis presents what is called the elicitation method. It demonstrates how the subjective parameters of a decision theory can be measured from observed preferences in as simple and direct a way as possible. The third step reverses the second step. Now, preferences are not derived from the subjective parameters but the subjective parameters are derived from preferences.

To illustrate the third step for readers who know the expected utility model, assume expected utility with utility  $U$  for given probabilities. Assume a scaling  $U(\$0) = 0$  and  $U(\$100) = 1$  (such scaling is always possible as we will see later). Then, for any  $\$x$  between  $\$0$  and  $\$100$ , an indifference between receiving  $\$x$  for sure or receiving