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James W. Morrison

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ARCO PUBLISHING COMPANY, INC.
NEW YORK

Published by Arco Publishing Co., Inc.
219 Park Avenue South, New York, N.Y. 10003

Library of Congress Cataloging in Publication Data

Morrison, James Warner, 1940-
Solid mechanics.

(Professional career examination series)
1. Statics. 2. Structures, Theory of.
3. Strength of materials. I. Title.

TA351.M67 620.1'12 77-18555
ISBN 0-668-04409-8 (Paper Edition)

Printed in the United States of America

PREFACE

The object of the text is to provide an easy, practical and comprehensive review of the solid mechanics, fully covering all the subjects as may be found in any college engineering textbook on statics and strength of materials.

Solid mechanics is concerned with the study of effects of forces on systems of material particles, i.e., solids. Since solids are rigid or deformable, the mechanics of solids is further classified as the "mechanics of rigid" or "deformable bodies". Statics is then, the investigation of the equilibrium of bodies under the action of stationary forces. In statics the resultant force acting on a body is zero, the linear acceleration is zero and the body is either stationary or moves with constant velocity. An equilibrium must exist between all the forces acting on the body, i.e., applied forces, frictional forces, gravitational forces and all others.

Strength of materials beginning with distributed forces is the mechanics of deformable bodies and requires an understanding of the principles of stress and strain, theory of elasticity, applications of beams, columns, and other topics.

While the treatment of the text is not intended to be exhaustive, enough theory is given to make sure that the reader has a thorough understanding of the basic principles involved.

The professional engineer and architect will find this volume an invaluable addition to his library of educational engineering literature for reference and also as a refresher on all practical problems that may arise in structural engineering.

All engineers and architects who must take a state examination for their professional license will find this book of great assistance in their preparations. Studying this book will be of greater help to them as a refresher than trying to re-study their standard theoretical textbooks.

The questions and answers in the text have been stated and analyzed so clearly and concisely, that the student or even the beginner, with a limited knowledge of mathematics will find the text very easy to follow and thus gain a considerable understanding of the underlying principles in the solution of structural engineering problems.

All numerical problems following every subject covered are provided with answers.

E. Viertels' work in SIMPLIFIED PROBLEMS IN STRENGTH OF MATERIALS AND STRUCTURAL DESIGN and the Federal Government publication numbers 426-9 and 428-8 of the U. S. Army Engineer School is acknowledged. The drawings by Cathryn F. Morrison and typing assistance of Ms. Janet Lord is appreciated.

J.W.M.

CONTENTS

Chapter 1

FUNDAMENTAL PRINCIPLES OF STATICS

1.0 INTRODUCTION

Solid mechanics is concerned with the effect of forces in and around a solid body. This area of study is called strength of material which deals with internal effects of forces acting on a body when subjected to externally applied loads. The mechanics of solids includes shafts, bars, beams, welded joints, columns, and structures with trusses and frames.

The study of solid mechanics begins with an understanding of the principles of statics which is concerned with the forces and effects upon rigid bodies at rest. The word statics comes from the Greek word "statikas," meaning "causing to stand." Ancient Greeks pioneered the study of mechanics: Aristotle spoke of the parallelogram law and Archimedes is well known for his work on balances and levers. Later, during the Renaissance, the ingenious designs of Leonardo da Vinci attest to his mastery of the concept of static moments. Galileo was the first to explain the behavior of structural materials on a rational basis. In 1678 Robert Hook's developed the relationship between force and deformation and leading the way for modern formulations of Bernoulli, Euler, Navier and others.

1.1 BASIC TERMS

Statics is that branch of Engineering Mechanics which deals with forces and effects upon rigid bodies at rest.

A rigid body is one that does not deform. Most of the bodies considered in statics are assumed to be rigid.

A force is an action that changes or attempts to change the state of motion of a body upon which it acts. A body also possesses internal forces which hold the component parts together.

A force system is comprised of all the forces in a given situation.

A scalar quantity possesses magnitude only.

A vector quantity possesses both magnitude and direction. A vector may be represented by an arrow whose head indicates direction and whose length represents its magnitude.

The resultant of a force system is the least number of forces which will replace the given system.

1.2 NEWTON'S THREE FUNDAMENTAL LAWS

Newton's first law of motion states that a body will continue in its state of rest or uniform motion if the resultant force acting on the body is zero.

Newton's second law of motion states that a body will have an acceleration proportional to the magnitude of the resultant and in the direction of the resultant of a force system acting on the body provided the resultant is not zero.

The third law of motion states that to every action there is an equal and opposite reaction.

1.3 VECTORS AND FORCES

Vectors are used to represent forces, displacements, velocities and other quantities possessing both magnitude and direction.

A unit vector is one which has a length of one unit.

A free vector is one which may be moved anywhere in space, maintaining its original magnitude and direction.

A fixed vector is one which acts upon a fixed point of application.

Collinear vectors or forces have a common line of action.

Concorrent vectors or forces have a common point of application or a common intersection of their lines of action.

Parallel vectors or forces have parallel lines of action.

Coplanar vectors or forces have their lines of action in a common plane.

1.4 ADDITION OF VECTORS

The parallelogram law states that the resultant of two concurrent vectors is the diagonal of the parallelogram formed by using the two given vectors as adjacent sides.

Example 1.1

In figure 1-1 (a), two forces acting at point O are represented by vectors A and B. The lengths of the vectors are proportional to the respective force magnitudes, the direction (orientation or sense) of each vector being the direction in which each force acts, as indicated. In figure 1-1 (b), a parallelogram has been completed by adding dotted lines as shown. The diagonal, vector R, represents the resultant force which would be equivalent to the combined effect of the two original forces.

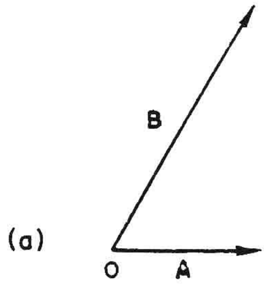
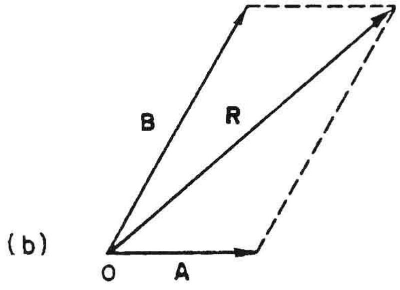


Figure 1-1



The triangle law, which is a corollary to the parallelogram law, states that the resultant of two concurrent vectors is the geometric sum of the two given vectors. One of the given vectors is moved in space as a free vector so that its tail is at the tip of the other vector. The resultant vector extends from the tail of the fixed vector to the tip of the free vector in its new position.

Example 1.2

In figure 1-2 (a) two forces acting at point O are represented by vectors A and B. In figure 1-2 (b) vector B has been moved in space so that its tail touches the tip of vector A. The resultant force is represented by vector R which extends from point O to the tip of vector B.

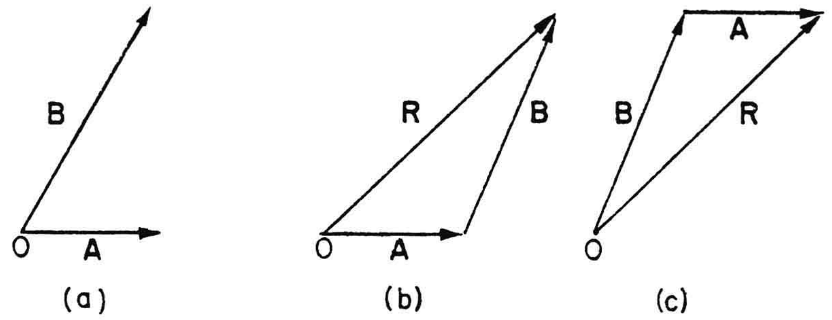


Figure 1-2

Vector addition is commutative, that is A + B = B + A.

Example 1.3

Observe carefully figures 1-1 and 1-2. Note that figure 1-2 (b) corresponds to half of the parallelogram in figure 1-1 (b) and that figure 1-2 (c) corresponds to the other half of the parallelogram in figures 1-1 (b). The resultant R has the same magnitude and direction in both figure 1-2 (b) which represents A + B and figure 1-2 (c) which represents B + A.

The subtraction of a vector may be defined as the addition of a corresponding negative vector. We may write P - Q = P + (-Q).

A zero vector (sometimes called a null vector) is obtained by the subtraction of a vector from itself, A - A = 0. It may also be obtained by multiplying a vector by its zero scalar, 0A = 0.

The sum of three or more vectors is the resultant of a vector polygon formed by arranging the given vectors in tip-to-tail manner and connecting the tail of the first vector with the tip of the last one.

Example 1.4

The resultant R of the coplanar forces whose vectors are drawn in figure 1-3 (a) is found by constructing the vector polygon in figure 1-3 (b). The lengths of the vectors are drawn to a common scale and the directions of corresponding force vectors are unchanged. The dotted line vector R from the tail of the first force vector to the tip of the last force vector is the resultant vector.

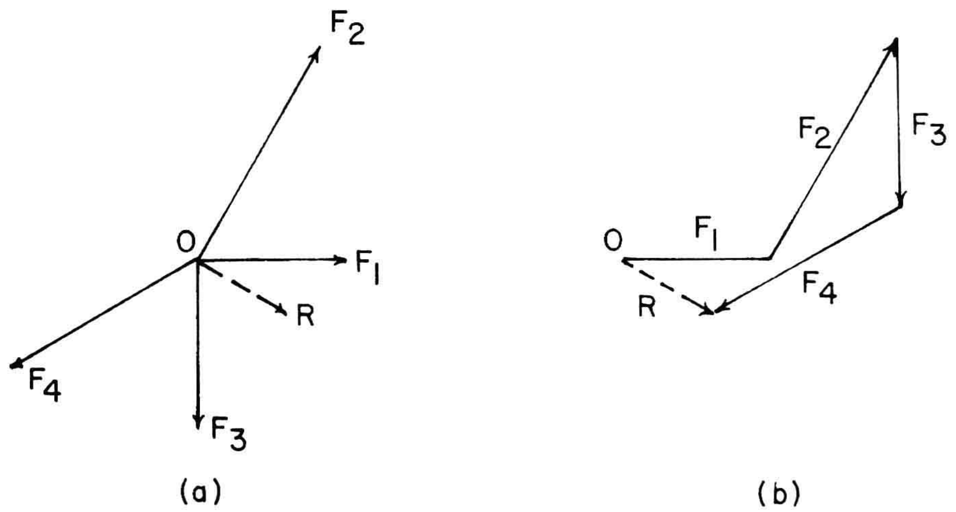


Figure 1-3

1.5 COMPOSITIONS AND RESOLUTION OF FORCES

Composition of forces is the process of replacing a force system with its resultant.

Conversely, a single force may be replaced by two or more forces which have the same effect as the original force. This process is known as resolving a force into components.

The rectangular components of a force in space are found by resolving the force into three components respectively parallel to the x, y, and z axes. In vector form, we may write $F = F_x + F_y + F_z$.

Example 1.5

In figure 1-4 (a) force F is applied at the origin 0 of the rectangular coordinate axes. F_x is the component in the x direction, F_y is the component in the y direction, and F_z is the component in the z direction.

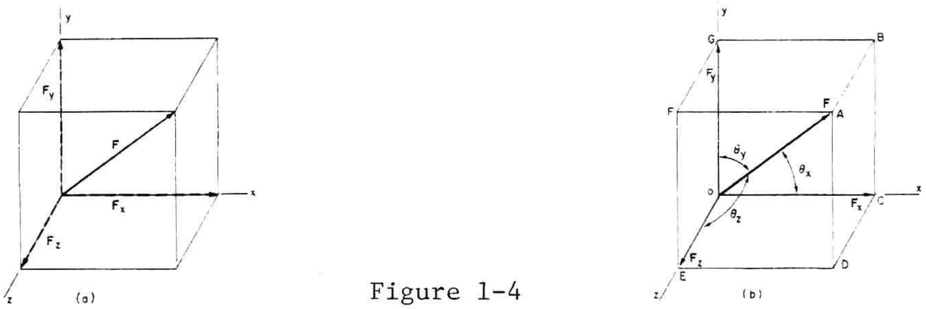


Figure 1-4

The corresponding scalar components of a given force F in a rectangular coordinate system are F_x, F_y, and F_z, where $F_x = F \cos \theta_x$, $F_y = F \cos \theta_y$ and $F_z = F \cos \theta_z$; and where θ_x, θ_y and θ_z are the angles between the force F and the respective coordinate axes.

Example 1.6

In figure 1-4 (b), the magnitude of the rectangular component F_x is equal to the product of the magnitude of the force F and the cosine of angle AOC, ($F_x = F \cos \theta_x$).

The magnitude of force F is the square root of the sum of the squares of its scalar components, $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

When using unit vectors i, j, and k directed respectively along the x, y, and z axes, we may express the force F as the sum of the products of its scalar components and its unit vectors, $F = F_x i + F_y j + F_z k$.

1.6 CONDITIONS FOR EQUILIBRIUM

Equilibirum exists when the external forces acting on a rigid body form a system of forces equivalent to zero.

The vector polygon of a concurrent force system in equilibrium is a closed polygon, i.e., the beginning and end points are coincident and resultant is equal to zero.

If the respective forces of a force system in equilibrium are resolved into their respective components along standard axes of reference, the summations of components along such axes must also be zero.

Thus:

$\Sigma F_x = 0$ (vector components parallel to x-axis)

$\Sigma F_y = 0$ (vector components parallel to y-axis)

$\Sigma F_z = 0$ (vector components parallel to z-axis)

If a system is known to be in equilibrium, but all the forces therein are not known, then there must exist unknown forces which will have a net effect of balancing the system. The above equations then can be used to solve for unknown forces.

If there is an unbalanced force, or component of a force, the system cannot be in equilibrium and the body will be accelerated according to Newton's second law: $F = ma$ In such case the problem is no longer one of statics but lies in the field of dynamics

1.7 TRANSMISSIBILITY

The principle of transmissibility states that force may be considered as acting at any point along the line of action of the force.

Example 1.7

In figure 1-5, the force F may be considered as acting anywhere along the line AB.

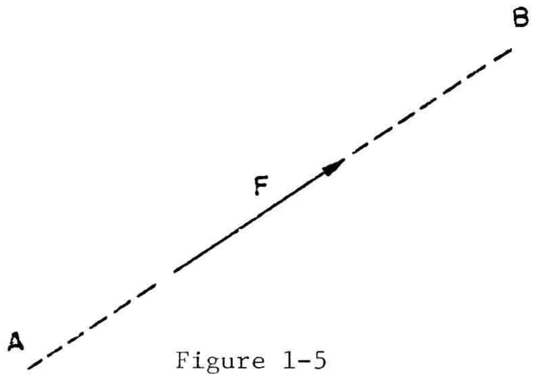


Figure 1-5

When two or more forces act on a rigid body along the same line of action (collinear), the resultant is the algebraic sum of their scalar quantities and acts in that line.

1.8 SUPERPOSITION

The law of superposition states that a force system in equilibrium may be added to or subtracted from another force system acting on a body without changing the effect of the latter on the body.

Example 1.8

Figure 1-6 (a) represents a simple force system in equilibrium which may be added to the force system represented in figure 1-6 (b) to produce the force system in figure 1-6 (c).

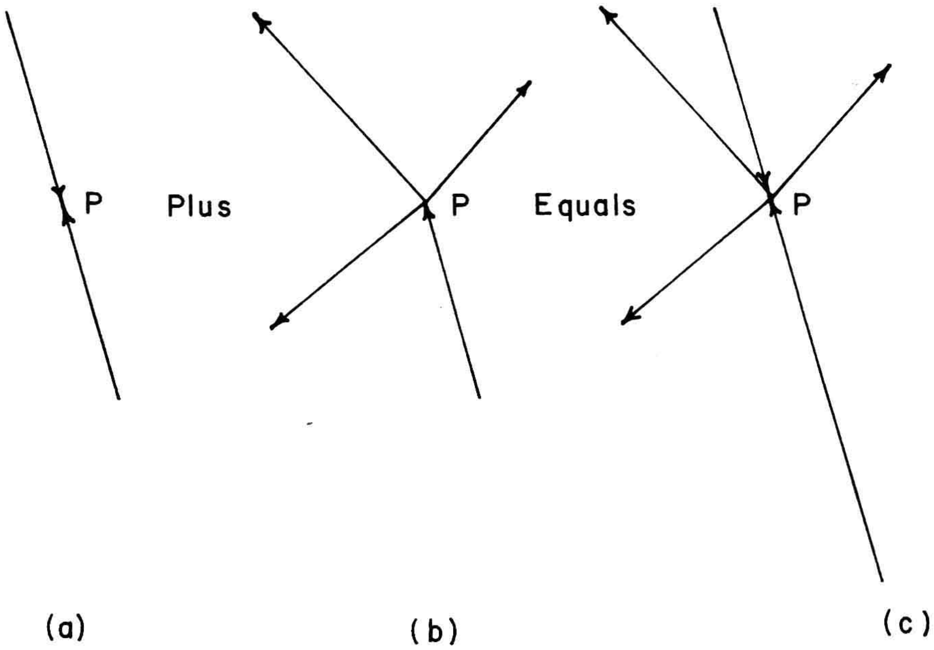


Figure 1-6