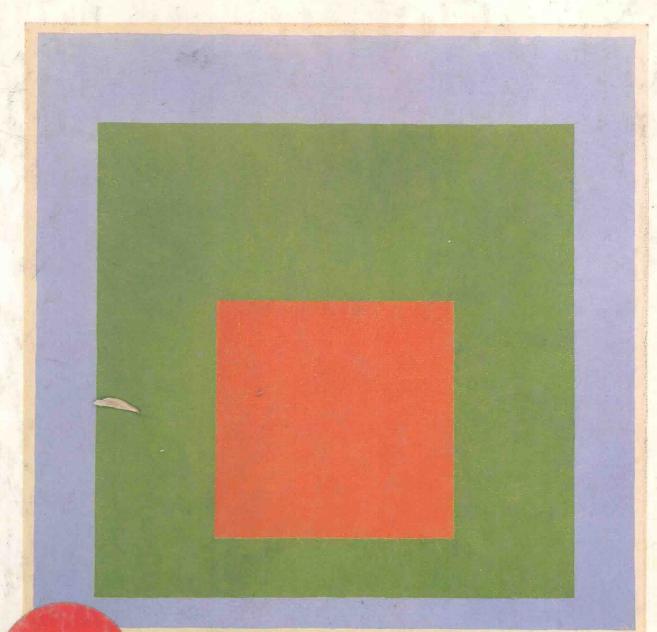
# STATISTICS

FOR BUSINESS AND ECONOMICS



International Edition Special Price!

ROBERT SANDY

# STATISTICS FOR BUSINESS AND ECONOMICS ROBERT SANDY

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# Preface

Historically, we must admit that the classical version of our key introductory college course has been an abysmal failure, semester after semester, year after year. It deserves a grade of F—yet we persist in teaching it more or less the same way we have been for forty years.

This disheartening quotation is from an article by Brian Joiner, one of the authors of MINITAB ("Let's Change How We Teach Statistics," *Chance*, 1988, no. 1). Joiner advocates two changes in the way introductory Statistics is taught: First, give students real data sets to manipulate throughout the course, and second, eliminate most of the standard material on parametric tests. This text reflects the first recommendation, but I chose not to perform major surgery on the course topics, as the second suggestion would require. I present the traditional topics, but I've done everything I can to make them intelligible and interesting.

My main objectives in writing this text were to:

- □ Explain the material in a highly intuitive and conversational manner with well-written and lively examples,
- □ Systematize the problem-solving process,
- □ Emphasize graphical methods to describe models and analyze data, and
- Provide real computer applications for more topics than any other text of this kind.

Wherever possible, explanations take the place of perfunctory statements such as "it can be shown that . . ." or "in advanced statistics texts it is shown that . . . ." Proofs are relegated to an appendix. The discussion of why the formula for the sample standard deviation has n-1 in the denominator (Chapter 3) illustrates this intuitive approach. Other examples are the explanations of why the distribution of sample means approaches a normal distribution

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tion as the sample size increases (Chapter 7) and why the chi-square test statistic is distributed as the sum of a series of independent, squared standard normal distributions (Chapter 11).

This text emphasizes the development of the skills needed to recognize which distribution applies in a given situation and to translate a problem from words into equations. The charts inside the front and back covers present clear-cut rules that lead students to the correct formula for almost every hypothesis test or confidence interval discussed in the text.

Coverage is extensive, yet some familiar topics have been omitted. The choice was guided by two criteria. First, each topic covered should have a realistic business application. The discrete uniform distribution and kurtosis are two of the topics that were excluded because they lacked obvious applications. On the other hand, the exponential distribution, ARIMA, and probit regression, which are generally not covered in other texts, are discussed because they have good business applications. The second criterion for deciding which topics to cover was my conviction that the text should reflect the widespread use of computers in the practice of statistics in business. I believe that virtually all business-related statistical computations will soon be done with computers and that there is no benefit to covering techniques aimed solely at reducing computation time. Assuming that students using this text have access to either a computer with a statistical package or a hand-held calculator that has functions such as summation, mean and standard deviation, and a memory register, there is no need to discuss short-cut formulas and coding.

The growing use of computers also led to the inclusion of some topics that usually are *not* covered in an introductory text. For example, most statistical packages present the pooled and the unpooled variance versions of a difference-in-means test, and many computer packages have graphing routines that simplify the creation of charts such as box-and-whisker plots. These and other topics are addressed so that students can handle the options presented by a computer package or take advantage of some easy-to-understand but computationally tedious techniques.

Although the text is designed to be used in courses with or without computers, it offers many advantages if students have access to a computer. The computer applications start with Chapter 2 and continue through Chapter 17. The sections of the chapters that have computer applications are marked with either one or two computer disks in the table of contents and in the text. It is recommended that the double-disk sections be covered only in classes that have access to a computer. The rest of the text does not require use of a computer.

A computer manual is bound into the text as Appendix A. This manual contains printouts and instructions for four widely used computer packages: DATA DESK, MINITAB, SAS, and SPSSx. MYSTAT, a student version of SYSTAT, is available from McGraw-Hill either separately or in a package with the text. The documentation and examples are packaged with the MYSTAT

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disk rather than bound into the text. This extensive coverage is intended to help instructors integrate computers in the introductory statistics course by accommodating a wide variety of computer resources. DATA DESK works on Macintosh computers. MINITAB, SAS, and SPSSx are usually available on university mainframe computers. MYSTAT works on IBM PCs and compatibles, as well as Macintosh computers. The five packages were chosen to reflect current mainframe usage in statistics courses as well as the emerging transition to microcomputers.

Other significant features of the text include the following:

- □ Over 800 exercises. Within each chapter the exercises increase in difficulty up to the highly challenging starred exercises at the end. Each section includes several drill exercises that review the mechanics of applying a formula. Exercises at the end of each section and at the end of the chapter provide a realistic context for the business use of a technique. Adding interest are exercises relating to student life and exercises having a humorous twist.
- □ Solutions to the odd-numbered exercises show the intermediate steps. Alternative solutions to a problem, given the computational resources available and the validity of parametric assumptions, are detailed.
- □ Optional sections within most chapters which allow instructors to tailor the course to special interests and to students' varied abilities.
- □ Non-calculus proofs of most theorems in the text (Appendix C).
- □ Carefully refined statistical notation. For example, all parameters are symbolized by Greek letters and all statistics are symbolized by italic Latin letters.

A *Study Guide* to accompany this text is available from the publisher. Other supplements, which are available to instructors, include a sophisticated test bank with class-tested questions. The computerized test generator prints graphs, Greek letters, and statistical notation and randomizes the order of multiple choice questions. The test bank is also available in printed form. In addition, there are over 30 data sets on disk for use with the computer applications, transparency masters, and an instructor's manual. The transparency masters include the figures in the text as well as additional worked-out examples. The instructor's manual consists of solutions to all exercises in the text plus suggested outlines for one- and two-semester courses with and without access to a computer.

### **ACKNOWLEDGMENTS**

To a textbook author, reviewers retained by the publisher are invisible antagonists who have to be mentally wrestled into compliance with the author's goals and limitations of the publication process. I struggled with most of the reviewers' suggestions, repeatedly asking whether a given change fit my concept of the text. In the end, I accepted most of their ideas. Nevertheless, the reviewers

bear no responsibility for any errors that remain. My appreciation goes to the following reviewers: William Cooke, University of Wyoming; Larry Cornwell, Bradley University; Robert Kowalczyk, Southeastern Massachusetts University; Ralph Miller, California State University—Pomona; Amitava Mitra, Auburn University; Stephen K. Pollard, California State University—Los Angeles; Ruby Ramirez, Loyola University; Michael Sklar, University of Georgia; and Carol Stamm, Western Michigan University. Robert Kowalczyk and Lee Ing-Tong, both at Southeastern Massachusetts University, reviewed the computer appendixes. Susan Reiland did an accuracy check of the manuscript. Her reputation as a meticulous and expert reviewer is entirely deserved. The McGraw-Hill Business Series Advisor, David Dannenbring of Bernard Baruch College, reviewed the manuscript at all stages, reviewed the computer appendixes and checked the solutions to the exercises. I thank him for all of his help.

The first thirteen chapters of this text have been used at IUPUI for three years. I thank my colleagues who class-tested the manuscript: Robert Calhoun, Martin Spechler, Chris Starkey, Shah Towfighi, and Gang Yi. I am also grateful to the IUPUI students who offered suggestions. I thank David Bivin, Gang Yi, and Monte Juillerat for advice on specific topics. Martin Spechler's insightful advice and unflagging encouragement were well beyond any reasonable expectation.

In conclusion, I think that my treatment as an author by my publisher has been exemplary. Courtesy, patience, and consistency were the hallmarks of all of their actions. My contacts were with Seib Adams, editorial director of the college division; June Smith, director of editing and development; Johanna Schmid, acquisitions editor; Judith Kromm, project manager; Dan Alpert, senior development editor; Anne Mahony, development editor; and Stacey Alexander, production manager. Betty Binns of Binns & Lubin and Associates and Karl Illg of IUPUI provided invaluable input on design and graphics problems. Finally, my most earnest appreciation goes to Susan Badger who worked closely with me throughout the project.

ROBERT SANDY Indianapolis, Indiana

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- means that a computer application is provided for that topic in Appendix A, but it is not necessary to have the use of a computer in order to cover the topic.
- means that a computer application is provided at the end of the chapter for that topic and that it is recommended that the topic be covered only if a computer is available for students' use.

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hen the first American Indians visited London, England, in the early seventeenth century, they came with instructions to count the men in the city and report their findings back to their tribe. They took sticks along, intending to keep count by their usual method—making a scratch on a stick for every man they saw. Given the number of men in London at that time, the method was hopeless. Be-

sides quickly running out of sticks, they had no idea if they were counting the same men twice. Clearly the methods of recording and analyzing data that are adequate at one time or in one environment can be totally inadequate at another time or in another place.



### 1.1 DEFINING STATISTICS

Today the scope of the information required to make sound decisions to resolve problems in finance, marketing, production, and other aspects of business demands the use of modern Statistics. Statistics—with a capital "S"—is the branch of mathematics devoted to the collection, display, and analysis of data to aid in decision making. We could estimate the population of a large city, for example, by mapping the city's blocks and sampling houses in blocks selected at random to find the number of residents. We could then project these sample results upward to estimate the population of the entire city. The word "data" (plural of "datum," something given), which was used in the definition of Statistics, also requires a definition. **Data** consist of a set of numbers that represent the measurements of a numerical characteristic or characteristics of some group. The number of men, women, and children in each sampled block are data. Their heights, weights, and incomes are also data. A data set can also contain terms that describe nonnumerical characteristics associated with some group. The race of each person interviewed is a nonnumerical characteristic that could be designated by the word "white," "black," "Oriental," or "other."

The following examples will give you a better idea of why the scope of information needed for sound business decisions requires the use of modern Statistics.

Consider the problem of determining relative prices in competing supermarkets. Fifty years ago a grocery store owner could stroll through a competitor's store and get a good impression of relative prices in a few minutes. Supermarkets today carry between 15,000 and 25,000 items and in a typical week the prices of

1,000 of them will be changed. Supermarkets do occasionally check the price of every item in their competitors' stores, but this procedure is too expensive and time-consuming to do as frequently as it is needed. A monthly or even weekly pulse of relative prices is essential. Clearly, a sample of items must be taken to represent the average level of prices in a supermarket. The science of Statistics deals with such questions as how the particular items that make up the sample should be selected, how many items need to be in the sample, and how accurately the average price of the sample data will reflect the storewide average level of prices.

Ten years ago the process of brazing or joining the tubes that make up a bicycle frame was done by hand. The alignment of the tubes is critical because a bicycle with a misaligned frame will pull to the right or left when it is ridden. A skilled craftsman would check the alignment after each tube was brazed. The whole process took several hours. Today most bicycle frames are brazed by robots and the process takes a few minutes. When the frames are measured and deviations from a perfect alignment are observed, one of two factors may be responsible: the settings on the brazing robots may have drifted away from their specified values or there was an unavoidable small random variation. The tolerances are so close, in thousandths of an inch, that when a deviation is found, it is uncertain whether the robots have to be reset or if settings are correct and the deviation is unavoidable. Statistical process control is used to track the alignments of each frame as it is completed, and when the trend of measurements from frame to frame is analyzed, it is possible to identify with a high degree of certainty when the settings have drifted even slightly, so that the robots can be stopped and recalibrated.

The marketing of a new toy is much more expensive today than it was a few years ago. Instead of just relying on television commercials to make children aware of a new toy, many toy manufacturers subsidize the creation of a Saturday-morning cartoon show based on the toy. Sometimes even these efforts fail: the kids simply don't like the toy. To reduce the uncertainty inherent in marketing a new toy, manufacturers sample children to gauge their reactions. There are many millions of children in the potential market for a new toy. The science of Statistics is used to estimate the proportion of this market that would purchase the toy on the basis of the reactions of a small sample of children.

In all of these examples, samples were taken from a population and the sample was used to obtain an estimate of a parameter of the population. A **parameter** is *an overall summary measure applied to a population*, such as the average price of all items in a supermarket, the average alignment of all frames made under the current calibration of the brazing robots, or the proportion of all children who will buy the toy. A **population** is *every element in a group that is the subject of analysis*. The three populations in the above examples are the prices of all items in a supermarket, the dimensions of all future bicycle frames brazed by the robot under the current settings, and the decision to buy or not to buy a particular toy by every child in a certain age range. The word "popu-