J. N. REDDY

PRINCIPLES OF CONTINUUM MECHANICS

A Study of Conservation Principles with Applications

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PRINCIPLES OF CONTINUUM MECHANICS

A Study of Conservation Principles with Applications

As most modern technologies are no longer discipline-specific but involve multidisciplinary approaches, undergraduate engineering students should be introduced to the principles of mechanics so that they have a strong background in the basic principles common to all disciplines and are able to work at the interface of science and engineering disciplines. This textbook is designed for a first course on principles of mechanics and provides an introduction to the basic concepts of stress and strain and conservation principles. It prepares engineers and scientists for advanced courses in traditional as well as emerging fields such as biotechnology, nanotechnology, energy systems, and computational mechanics. This simple book presents the subjects of mechanics of materials, fluid mechanics, and heat transfer in a unified form using the conservation principles of mechanics.

J. N. Reddy is a Distinguished Professor and holder of the Oscar S. Wyatt Endowed Chair in the Department of Mechanical Engineering at Texas A&M University (http://www.tamu.edu/acml).

Dr. Reddy is a renowned researcher and educator in the broad fields of mechanics, applied mathematics, and computational engineering science. Dr. Reddy's research areas include theory and finite element analysis of problems in structural mechanics (composite plates and shells), fluid dynamics, and heat transfer; theoretical modeling of stress and deformation of biological cells and soft tissues; nanocomposites; and development of robust computational technology (including the *K*-version finite element models based on the least-squares method in collaboration with Professor Karan Surana of the University of Kansas). He is the author of more than 400 journal papers and 16 books on these subjects. His books include *Mechanics of Laminated Plates and Shells: Theory and Analysis*, Second Edition, 2004; *An Introduction to Nonlinear Finite Element Analysis*, 2004; *Introduction to the Finite Element Method*, 2006; and *An Introduction to Continuum Mechanics*, 2008.

Dr. Reddy's outstanding research credentials have earned him wide international acclaim in the form of numerous professional awards; citations; fellowship in all major professional societies including AAM, AIAA, ASC, ASCE, ASME, IACM, and USACM; membership on two dozen archival journals; and numerous keynote and plenary lecture invitations at international conferences. Dr. Reddy is the editor-in-chief of *Applied Mechanics Reviews*, *Mechanics of Advanced Materials and Structures*, *International Journal of Computational Methods in Engineering Science and Mechanics*, and *International Journal of Structural Stability and Dynamics*.

The extent of Dr. Reddy's original and sustained contributions to education, research, and professional service is substantial. As a result of his extensive publications of archival journal papers and books on a wide range of topics in applied sciences and engineering, Dr. Reddy is one of the few researchers in engineering around the world who is recognized by *ISI Highly Cited Researchers*, with more than 10,000 citations with an H-index greater than 46. In February 2009 he was awarded a *Honoris Causa* (Honorary Doctorate) by the Technical University of Lisbon.

When even the brightest mind in our world has been trained up from childhood in a superstition of any kind, it will never be possible for that mind, in its maturity, to examine sincerely, dispassionately, and conscientiously any evidence or any circumstance which shall seem to cast a doubt upon the validity of that superstition.

Mark Twain

The fact that an opinion has been widely held is no evidence whatever that it is not utterly absurd; indeed in view of the silliness of the majority of mankind, a widespread belief is more likely to be foolish than sensible.

Bertrand Russell

Desire for approval and recognition is a healthy motive, but the desire to be acknowledged as better, stronger, or more intelligent than a fellow being or fellow scholar easily leads to an excessively egoistic psychological adjustment, which may become injurious for the individual and for the community.

Albert Einstein

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Preface

You cannot teach a man anything, you can only help him find it within himself.
Galileo Galilei

This book is a simplified version of the author's book, *An Introduction to Continuum Mechanics with Applications,* published by Cambridge University Press (New York, 2008), intended for use as an undergraduate textbook. As most modern technologies are no longer discipline-specific but involve multidisciplinary approaches, undergraduate engineering students should be educated to think and work in such environments. Therefore, it is necessary to introduce the subject of *principles of mechanics* (i.e., laws of physics applied to science and engineering systems) to undergraduate students so that they have a strong background in the basic principles common to all disciplines and are able to work at the interface of science and engineering disciplines. A first course on principles of mechanics provides an introduction to the basic concepts of stress and strain and conservation principles and prepares engineers and scientists for advanced courses in traditional as well as emerging fields such as biotechnology, nanotechnology, energy systems, and computational mechanics. Undergraduate students with such a background may seek advanced degrees in traditional (e.g., aerospace, civil, electrical or mechanical engineering; physics; applied mathematics) as well as interdisciplinary (e.g., bioengineering, engineering physics, nanoscience and engineering, biomolecular engineering) degree programs.

There are not many books on principles of mechanics that are written that keep the undergraduate engineering or science student in mind. A vast majority of books on the subject are written for graduate students of engineering and tend to be more mathematical and too advanced to be of use for third-year or senior undergraduate students. This book presents the subjects of mechanics of materials, fluid mechanics, and heat transfer in unified form using the conservation principles of mechanics. It is hoped that the book, which is simple, will facilitate in presenting the main concepts of the previous three courses under a unified framework.

With a brief discussion of the concept of a continuum in Chapter 1, a review of vectors and tensors is presented in Chapter 2. Because the analytical language of applied sciences and engineering is mathematics, it is necessary for all students of this course to familiarize themselves with the notation and operations of vectors, matrices, and tensors that are used in the mathematical description of physical phenomena. Readers who are familiar with the topics of this chapter may refresh or skip and go to the next chapter. The subject of kinematics, which deals with geometric changes without regard to the forces causing the deformation, is discussed in Chapter 3. Measures of engineering normal and shear strains and definitions of mathematical strains are introduced here. Both simple one-dimensional systems as well as two-dimensional continua are used to illustrate the strain and strain-rate measures introduced. In Chapter 4, the concept of stress vector and stress tensor are introduced. It is here that the readers are presented with entities that require two directions – namely, the plane on which they are measured and the direction in which they act – to specify them. Transformation equations among components of stress tensor referred to two different orthogonal coordinate systems are derived, and principal values and principal planes (i.e., eigenvalue problems associated with the stress tensor) are also discussed.

Chapter 5 is dedicated to the derivation of the governing equations of mechanics using the conservation principles of continuum mechanics (or laws of physics). The principles of conservation of mass, linear momentum, angular momentum, and energy are presented using one-dimensional systems as well as general three-dimensional systems. The derivations are presented in invariant (i.e., independent of a coordinate system) as well as in component form. The equations resulting from these principles are those governing stress and deformation of solid bodies, stress and rate of deformation of fluid elements, and transfer of heat through solid media. Thus, this chapter forms the heart of the course. Constitutive relations that connect the kinematic variables (e.g., density, temperature, deformation) to the kinetic variables (e.g., internal energy, heat flux, stresses) are discussed in Chapter 6 for elastic materials, viscous fluids, and heat transfer in solids.

Chapter 7 is devoted to the application of the field equations derived in Chapter 5 and constitutive models presented in Chapter 6 to problems of heat conduction in solids, fluid mechanics (inviscid flows as well as viscous incompressible flows), diffusion, and solid mechanics (e.g., bars, beams, and plane elasticity). Simple boundary-value problems are formulated and their solutions are discussed. The material presented in this chapter illustrates how physical problems are analytically formulated with the aid of the equations resulting from the conservation principles.

As stated previously, the present book is an undergraduate version of the author's book *An Introduction to Continuum Mechanics* (Cambridge University Press, New York, 2008). The presentation herein is limited in scope when compared to the author's graduate-level textbook. The major benefit of a course based on this book is to present the governing equations of diverse physical phenomena from a unified point of view, namely, from the conservation principles (or laws of physics), so that students of applied science and engineering see the physical principles as well as the mathematical structure common to diverse fields.

Readers interested in advanced topics may consult the author's continuum mechanics book or other titles listed in references therein.

The author is pleased to acknowledge the fact that the manuscript was tested with the undergraduate students in the College of Engineering at Texas A&M University as well as in the Engineering Science Programme at the National University of Singapore. The students, in general, have liked the contents and the simplicity with which the concepts are introduced and explained. They also expressed the feeling that the subject is more challenging than most at the undergraduate level but a useful prerequisite to graduate courses in engineering.

The author wishes to thank Drs. Vinu and Ginu Unnikrishnan and Ms. Feifei Cheng for their help with the proofreading of the manuscript of this book during the course of its preparation and production. The book contains so many mathematical expressions that it is hardly possible not to have typographical and other kinds of errors. The author wishes to thank in advance those who are willing to draw the author's attention to typos and errors, using the e-mail address jn_reddy@yahoo.com.

Contents

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1 Introduction

One thing I have learned in a long life: that all our science, measured against reality, is primitive and childlike—and yet it is the most precious thing we have.

Albert Einstein

1.1 Continuum mechanics

Matter is composed of discrete molecules, which in turn are made up of atoms. An atom consists of electrons, positively charged protons, and neutrons. Electrons form chemical bonds. An example of mechanical (i.e., has no living cells) matter is a carbon nanotube (CNT), which consists of carbon molecules in a certain geometric pattern in equilibrium with each other, as shown in Figure 1.1.1.

Another example of matter is a biological cell, which is a fundamental unit of any living organism. There are two types of cells: prokaryotic and eukaryotic cells. Eukaryotic cells are generally found in multicellular organs and have a true nucleus, distinct from a prokaryotic cell. Structurally, cells are composed of a large number of macromolecules, or large molecules. These macromolecules consist of large numbers of atoms and form specific structures, like chromosomes and plasma membranes in a cell. Macromolecules occur as four major types: carbohydrates, proteins, lipids, and nucleic acids. To highlight the hierarchical nature of the structures formed by the macromolecule in a cell, let us analyze a chromosome.

Chromosomes, which are carriers of hereditary traits in an individual, are found inside the nucleus of all eukaryotes. Each chromosome consists of a single nucleic acid macromolecule called deoxyribonucleic acid (DNA), each 2.2–2.4 nanometers wide. These nucleic acids are in turn formed from the specific arrangement of monomers called mononucleotides, each 0.3–0.33 nanometers wide. The fundamental units of nucleotides are formed again by a combination of a specific arrangement of a phosphate radical, nitrogenous base, and a carbohydrate sugar. The hierarchical nature of the chromosome is shown in Figure 1.1.2(a). Similar to the chromosomes, all the structures in a cell are formed from a combination of the macromolecules.

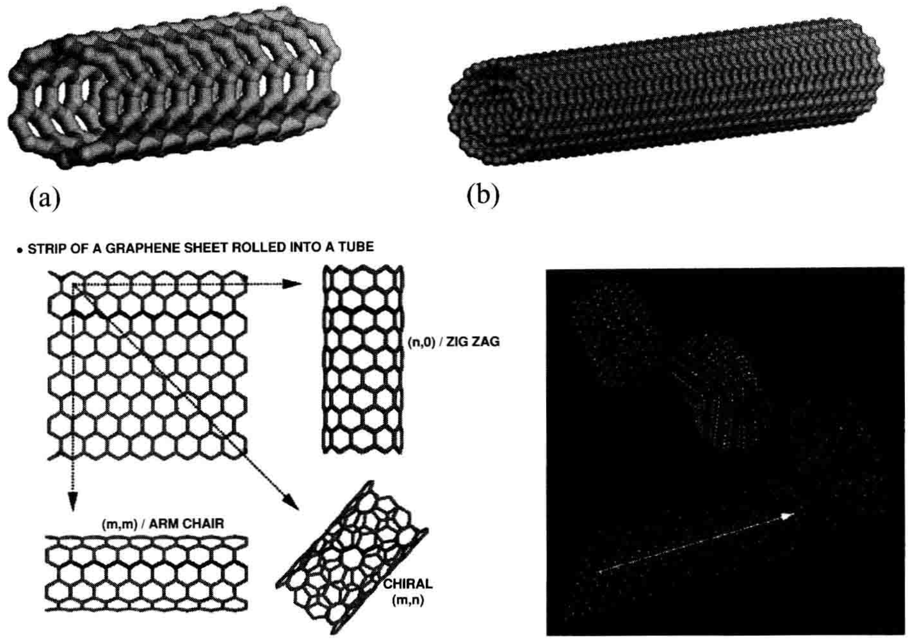

Figure 1.1.1 Carbon nanotubes (CNTs) with different chiralities.

At the macroscopic scale, eukaryotic cells can be divided into three distinct regions: nucleus, plasma membrane, and a cytoplasm having a host of other structures, as shown in Figure 1.1.2(b). The nucleus consists of the chromosomes and other protein structures and is the control center of the cell determining how the cell functions. The plasma membrane encloses the cell and separates the material outside the cell from inside. It is responsible for maintaining the integrity of the cell and also acts as channels for the transport of molecules to and from the cell. The cell membrane is made up of a double layer of phospholipid molecules (macromolecules) having embedded transmembrane proteins. The region between the cell membrane and the nucleus is the cytoplasm, which consists of a gel-like fluid called cytosol, the cytoskeleton, and other macromolecules. The cytoskeleton forms the biomechanical framework of the cell and consists of three primary protein macromolecule structures of actin filaments, intermediate filaments, and microtubules. Growth, cell expansion, and replication are all carried out in the cytoplasm.

The interactions between the different components of the cell are responsible for maintaining the structural integrity of the cell. The analysis of these interactions to obtain the response of the cell when subjected to an external stimulus (mechanical, electrical, or chemical) is studied systematically under cell mechanics. The structural framework of primary macromolecular structures in a cell is shown in Figure 1.1.2(c).

The study of matter at molecular or atomistic levels is very useful for understanding a variety of phenomena, but studies at these scales are not useful to solve common engineering problems. The understanding gained at the molecular

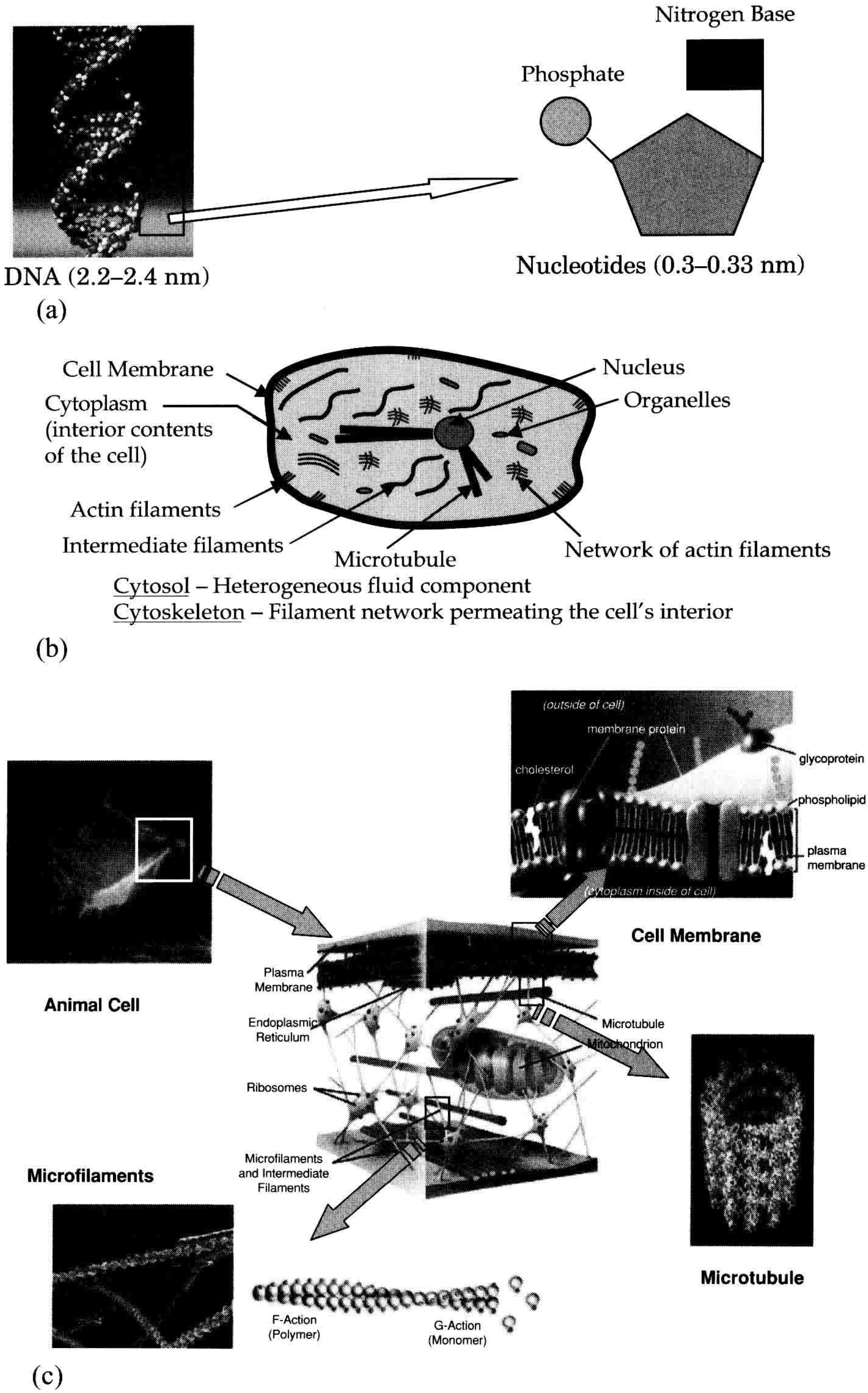

Figure 1.1.2 (a) Hierarchical nature of a chromosome. (b) Structure of a generalized cell. (c) Macromolecular structure in a cell.

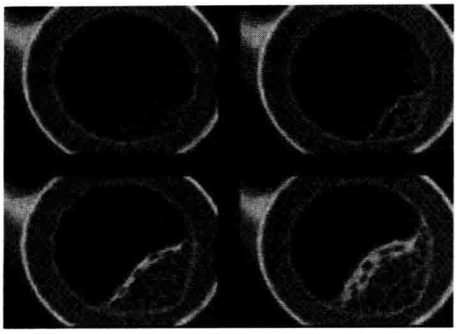

Figure 1.1.3 Progressive damage of artery due to deposition of particles in the arterial wall.

level must be taken to the macrosopic scale (i.e., a scale that a human eye can see) to be able to study its behavior. Central to this study is the assumption that the discrete nature of matter can be overlooked, provided the length scales of interest are large compared to the length scales of discrete molecular structure. Thus, matter at sufficiently large length scales can be treated as a *continuum* in which all physical quantities of interest, including density, are continuously differentiable.

The subject of *mechanics* deals with the study of motion and forces in solids, liquids, and gases and the deformation or flow of these materials. In such a study, we make the simplifying assumption, for analytic purposes, that the matter is distributed continuously, without gaps or empty spaces (i.e., we disregard the molecular structure of matter). Such a hypothetical continuous matter is termed a *continuum*. In essence, in a continuum all quantities such as the density, displacements, velocities, stresses, and so on vary continuously so that their spatial derivatives exist and are continuous. The continuum assumption allows us to shrink an arbitrary volume of material to a point, in much the same way as we take the limit in defining a derivative, so that we can define quantities of interest at a point. For example, the density (mass per unit volume) of a material at a point is defined as the ratio of the mass Δm of the material to a small volume ΔV surrounding the point in the limit that ΔV becomes a value ϵ^3, where ϵ is small compared with the mean distance between molecules,

$$\rho = \lim_{\Delta V \to \epsilon^3} \frac{\Delta m}{\Delta V}. \tag{1.1.1}$$

In fact, we take the limit $\epsilon \to 0$. A mathematical study of mechanics of such an idealized continuum is called *continuum mechanics*.

Engineers and scientists undertake the study of continuous systems to understand their behavior under "working conditions," so that the systems can be designed to function properly and produced economically. For example, if we were to repair or replace a damaged artery in a human body, we must understand the function of the original artery and the conditions that lead to its damage. An artery carries blood from the heart to different parts of the body. Conditions like high blood pressure and increases in cholesterol content in the blood may lead to deposition of particles in the arterial wall, as shown in Figure 1.1.3. With time, accumulation of these particles in the arterial wall hardens and constricts the passage, leading to cardiovascular diseases. A possible remedy for such diseases is to repair or replace the damaged portion of the artery. This in turn requires an understanding of the deformation and stresses caused in the arterial wall by the

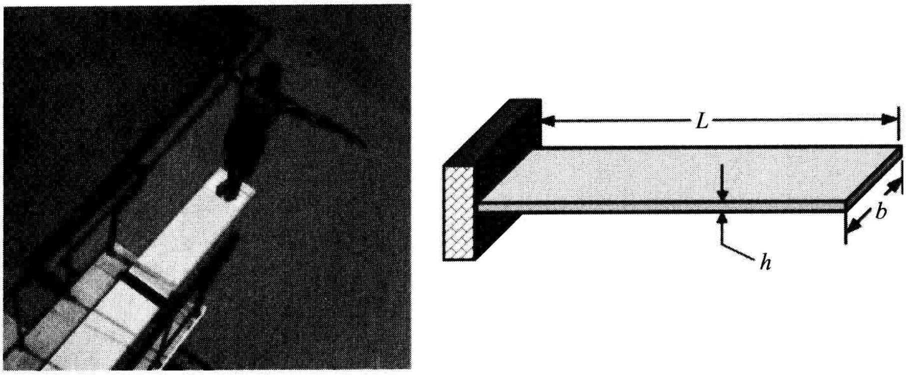

Figure 1.1.4 A diving board fixed at the left end and free at the right end.

flow of blood. The understanding is then used to design the vascular prosthesis (i.e., an artificial artery).

The primary objectives of this book are (1) to study the conservation principles in mechanics of continua and formulate the equations that describe the motion and mechanical behavior of materials, and (2) to present the applications of these equations to simple problems associated with flows of fluids, conduction of heat, and deformation of solid bodies. Although the first of these objectives is an important topic, the reason for the formulation of the equations is to gain a quantitative understanding of the behavior of an engineering system. This quantitative understanding is useful in the design and manufacture of better products. Typical examples of engineering problems sufficiently simple to cover in this course are described in the following. At this stage of discussion, it is sufficient to rely on the reader's intuitive understanding of concepts.

PROBLEM 1 (MECHANICAL STRUCTURE)

We wish to design a diving board that must enable the swimmer to gain enough momentum for the swimming exercise. The diving board is fixed at one end and free at the other end (see Figure 1.1.4). The board is initially straight and horizontal, and of length L and uniform cross section $A = bh$.

The design process consists of selecting the material with Young's modulus E and cross-sectional dimensions b and h such that the board carries the weight W of the swimmer. The design criteria are that the stresses developed do not exceed the allowable stress and the deflection of the free end does not exceed a pre-specified value δ. A preliminary design of such systems is often based on mechanics of materials equations. The final design involves the use of more sophisticated equations, such as the three-dimensional elasticity equations. The equations of elementary beam theory may be used to find a relation between the deflection δ of the free end in terms of the length L, cross-sectional dimensions b and h, Young's modulus E, and weight W:

$$\delta = \frac{4WL^3}{Ebh^3}. \tag{1.1.2}$$

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