

FIFTH EDITION

# Engineering Mathematics

John Bird



# Engineering Mathematics

*Fifth edition*

John Bird BSc(Hons), CEng, CSci, CMath, FIET, MIEE,  
FIIIE, FIMA, FCollT

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# Engineering Mathematics

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*In memory of Elizabeth*

# Preface

**Engineering Mathematics 5th Edition** covers a wide range of syllabus requirements. In particular, the book is most suitable for the latest **National Certificate and Diploma courses and City & Guilds syllabuses in Engineering**.

This text will provide a foundation in mathematical principles, which will enable students to solve mathematical, scientific and associated engineering principles. In addition, the material will provide engineering applications and mathematical principles necessary for advancement onto a range of Incorporated Engineer degree profiles. It is widely recognised that a students' ability to use mathematics is a key element in determining subsequent success. First year undergraduates who need some remedial mathematics will also find this book meets their needs.

In **Engineering Mathematics 5th Edition**, new material is included on inequalities, differentiation of parametric equations, implicit and logarithmic functions and an introduction to differential equations. Because of restraints on extent, chapters on linear correlation, linear regression and sampling and estimation theories have been removed. However, **these three chapters are available to all via the internet**.

A new feature of this fifth edition is that a **free Internet download** is available of a **sample of solutions** (some 1250) of the 1750 further problems contained in the book – see below.

Another new feature is a **free Internet download (available for lecturers only)** of all **500 illustrations** contained in the text – see below.

Throughout the text theory is introduced in each chapter by a simple outline of essential definitions, formulae, laws and procedures. The theory is kept to a minimum, for problem solving is extensively used to establish and exemplify the theory. It is intended that readers will gain real understanding through seeing problems solved and then through solving similar problems themselves.

For clarity, the text is divided into **eleven topic areas**, these being: number and algebra, mensuration, trigonometry, graphs, vectors, complex numbers,

statistics, differential calculus, integral calculus, further number and algebra and differential equations.

This new edition covers, in particular, the following syllabuses:

- (i) **Mathematics for Technicians**, the core unit for **National Certificate/Diploma** courses in Engineering, to include all or part of the following chapters:
  1. **Algebraic methods:** 2, 5, 11, 13, 14, 28, 30 (1, 4, 8, 9 and 10 for revision)
  2. **Trigonometric methods and areas and volumes:** 18–20, 22–25, 33, 34
  3. **Statistical methods:** 37, 38
  4. **Elementary calculus:** 42, 48, 55
- (ii) **Further Mathematics for Technicians**, the optional unit for **National Certificate/Diploma** courses in Engineering, to include all or part of the following chapters:
  1. **Advanced graphical techniques:** 29–31
  2. **Algebraic techniques:** 15, 35, 38
  2. **Trigonometry:** 23, 26, 27, 34
  3. **Calculus:** 42–44, 48, 55–56
- (iii) **The mathematical contents of Electrical and Electronic Principles units of the City & Guilds Level 3 Certificate in Engineering (2800)**.
- (iv) Any **introductory/access/foundation course** involving **Engineering Mathematics at University, Colleges of Further and Higher education and in schools**.

Each topic considered in the text is presented in a way that assumes in the reader little previous knowledge of that topic.

**Engineering Mathematics 5th Edition** provides a follow-up to **Basic Engineering Mathematics** and a lead into **Higher Engineering Mathematics 5th Edition**.

This textbook contains over **1000 worked problems**, followed by some **1750 further problems** (all with

answers). The further problems are contained within some **220 Exercises**; each Exercise follows on directly from the relevant section of work, every two or three pages. In addition, the text contains **238 multiple-choice questions**. Where at all possible, the problems mirror practical situations found in engineering and science. **500 line diagrams** enhance the understanding of the theory.

At regular intervals throughout the text are some **18 Revision tests** to check understanding. For example, Revision test 1 covers material contained in Chapters 1 to 4, Revision test 2 covers the material in Chapters 5 to 8, and so on. These Revision Tests do not have answers given since it is envisaged that lecturers could set the tests for students to attempt as part of their course

structure. Lecturers may obtain a complimentary set of solutions of the Revision Tests in an **Instructor's Manual** available from the publishers via the internet – see below.

A list of **Essential Formulae** is included in the Instructor's Manual for convenience of reference. **Learning by Example** is at the heart of **Engineering Mathematics 5th Edition**.

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## Free web downloads

### Additional material on statistics

Chapters on Linear correlation, Linear regression and Sampling and estimation theories are available for free to students and lecturers at <http://books.elsevier.com/companions/9780750685559>

In addition, a suite of support material is available to lecturers only from Elsevier's textbook website.

### Solutions manual

Within the text are some 1750 further problems arranged within 220 Exercises. A sample of over 1250 worked solutions has been prepared for lecturers.

### Instructor's manual

This manual provides full worked solutions and mark scheme for all 18 Revision Tests in this book.

### Illustrations

Lecturers can also download electronic files for all illustrations in this fifth edition.

To access the lecturer support material, please go to <http://textbooks.elsevier.com> and search for the book. On the book web page, you will see a link to the Instructor Manual on the right. If you do not have an account for the textbook website already, you will need to register and request access to the book's subject area. If you already have an account but do not have access to the right subject area, please follow the 'Request Access to this Subject Area' link at the top of the subject area homepage.

(iv) Any introductory, access/qualification course involving Engineering Mathematics at University, College of Further and Higher Education and in schools

Each topic is supported by worked examples and exercises in the order to progressively develop the student's knowledge of the topic.

Engineering Mathematics 5th Edition provides a follow-up to Basic Engineering Mathematics and is ideal for Higher Engineering Students 12-14th Edition.

The textbook contains over 1800 worked problems, followed by some 1750 further problems (all with



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## Number and Algebra

### fractions, decimals and percentages

#### 1.1 Fractions

When 7 is divided by 3, it may be written as  $\frac{7}{3}$  or  $2\frac{1}{3}$  or  $2\frac{1}{3}$ .  $\frac{7}{3}$  is called a fraction. The number above the line is called the numerator and the number below the line is called the denominator.

When the value of the numerator is less than the value of the denominator, the fraction is called a proper fraction. Thus  $\frac{1}{3}$  is a proper fraction. When the value of the numerator is greater than the denominator, the fraction is called an improper fraction. Thus  $\frac{4}{3}$  is an improper fraction and can also be expressed as a mixed number, which is an integer and a proper fraction. Thus the improper fraction  $\frac{4}{3}$  is equal to the mixed number  $1\frac{1}{3}$ .

When a fraction is simplified by dividing the numerator and denominator by the same number, the process is called cancelling. Cancelling by 0 is not permitted.

Alternatively

$$\begin{aligned} \frac{7}{3} &= \frac{6+1}{3} && \text{Step (1) Step (2)} \\ &= \frac{6}{3} + \frac{1}{3} && \text{Step (3)} \\ &= 2 + \frac{1}{3} && \text{Step (4)} \\ &= 2\frac{1}{3} && \text{Step (5)} \end{aligned}$$

- Step 1: The value of the numerator is greater than the denominator.
- Step 2: The numerator is written as the sum of a multiple of the denominator and a remainder.
- Step 3: For the fraction  $\frac{7}{3}$ , 6 is written as  $2 \times 3$ .
- Step 4: For the fraction  $\frac{7}{3}$ , 6 is written as  $2 \times 3$  and 1 is the remainder, so  $2 + \frac{1}{3}$ .
- Step 5: For the fraction  $\frac{7}{3}$ , 6 is written as  $2 \times 3$  and 1 is the remainder, so  $2 + \frac{1}{3}$ .

Thus  $\frac{7}{3} = \frac{6+1}{3} = \frac{6}{3} + \frac{1}{3}$  is obtained previously.

#### Problem 1: Find $2\frac{1}{3} + 1\frac{2}{3}$ .

One method is to add the mixed numbers into integers and their fractional parts. Thus

$$\begin{aligned} 2\frac{1}{3} + 1\frac{2}{3} &= (2 + \frac{1}{3}) + (1 + \frac{2}{3}) \\ &= 2 + 1 + \frac{1}{3} + \frac{2}{3} \\ &= 3 + \frac{1}{3} + \frac{2}{3} = 3 + \frac{1+2}{3} \\ &= 3 + \frac{3}{3} = 3 + 1 = 4 \end{aligned}$$

Another method is to express the mixed numbers as improper fractions.

#### Problem 2: Find $2\frac{1}{3} + 1\frac{2}{3}$ .

The lowest common multiples (i.e. L.C.M.) of the two denominators is 3. (i.e. 21)

Express each fraction as to the same denominator (i.e. 21) gives

$$\begin{aligned} \frac{7}{3} + \frac{4}{3} &= \frac{7 \times 7}{3 \times 7} + \frac{4 \times 7}{3 \times 7} = \frac{49}{21} + \frac{28}{21} \\ &= \frac{77}{21} = 3\frac{14}{21} \end{aligned}$$

