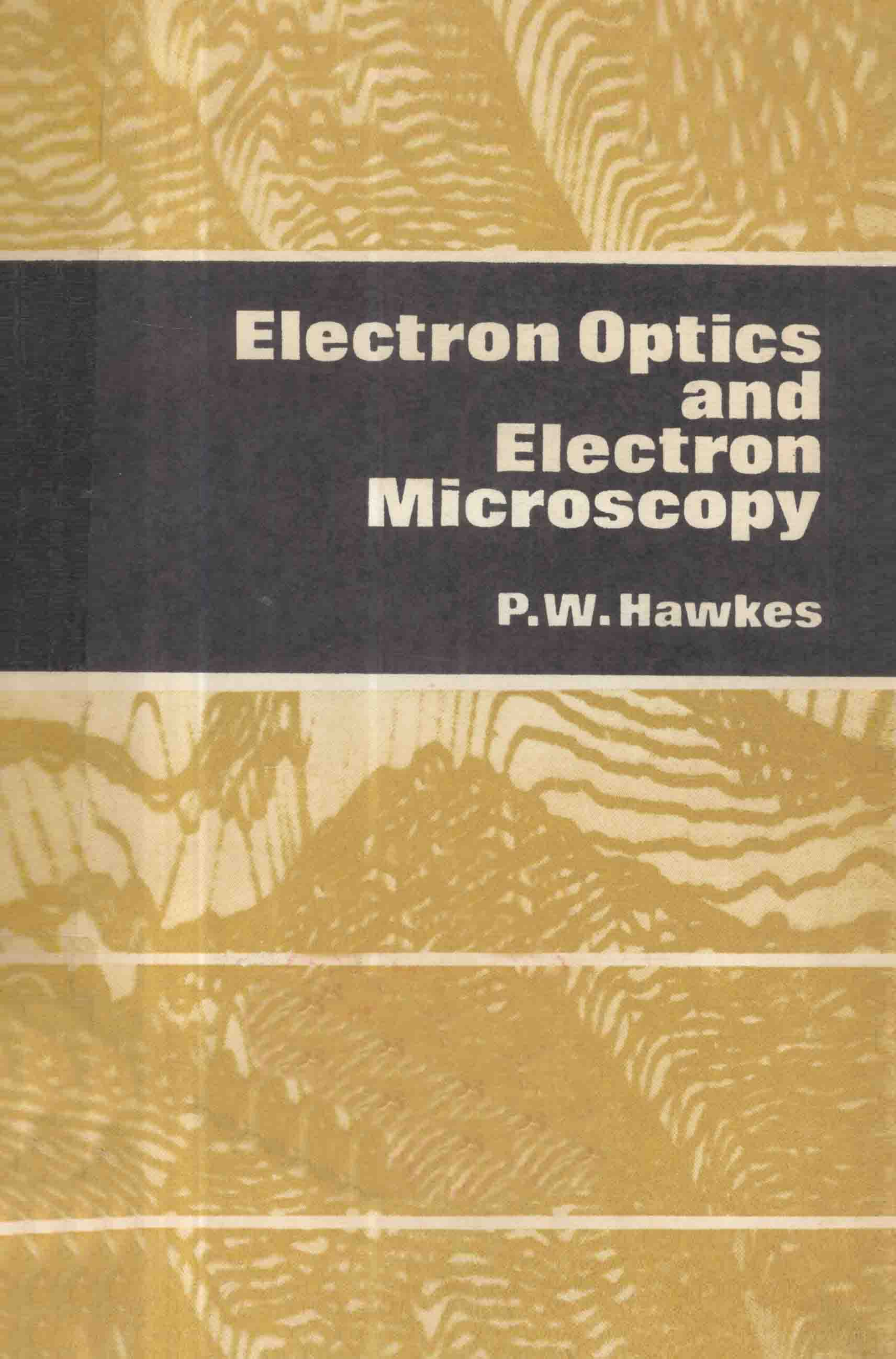

Electron Optics and Electron Microscopy

P. W. Hawkes

ELECTRON OPTICS AND ELECTRON MICROSCOPY

P. W. HAWKES
Churchill College and the
Cavendish Laboratory, Cambridge

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Preface

This introductory account of electron optics and the various types of electron microscopes is designed to be simple enough to be understood by the beginner, though it is by no means confined to the elementary aspects of the subject. Moreover, it is intended to be an up-to-date account: many topics of waning interest are not mentioned and others that have not yet found their way into the standard texts are covered here.

Many scientists concerned with electron microscopy have been kind enough to provide drawings and photographs for this book and I take this opportunity of thanking most sincerely all those individuals and firms who have sent such material.

The book was read critically in manuscript by Mr. A. J. Kahn, Senior Physics Master of Manchester Grammar School, who made numerous suggestions for improving and clarifying the text, all of which I have gratefully adopted.

Units

The international system of units (SI) is used throughout this book. The list below is based on the 1971 report of the Symbols Committee of the Royal Society, *Quantities, Units and Symbols*. The ångström (= 100 pm) is occasionally retained in the text, to avoid such clumsy phrases as ' a few tenths of a nanometre' but SI prefixes are always used for numerical values. Pressures are given in pascals but, since only the torr and the millimetre of mercury are to be found in the literature, the approximate value in torrs is also given. (It is rarely necessary to know vacuum pressures accurately, so the crude conversion rule 1 Torr = 100 Pa has been adopted.)

Basic SI units

length	metre (m)
mass	kilogramme (kg)
time	second (s)
electric current	ampère (A)
thermodynamic temperature	kelvin (K)

Supplementary units

plane angle	radian (rad)
solid angle	steradian (sr)

Derived SI units with special names

energy	joule (J)	capacitance	farad (F)
force	newton (N)	magnetic flux	weber (Wb)
power	watt (W)	inductance	henry (H)
charge	coulomb (C)	magnetic flux density	tesla (T)
potential difference	volt (V)	frequency	hertz (Hz)
resistance	ohm (Ω)	pressure	pascal (Pa)

SI prefixes

10^{12}	T	10^{-12}	p
10^{9}	G	10^{-9}	n
10^{6}	M	10^{-6}	μ
10^{3}	k	10^{-3}	m

Other units

ångström (Å) $= 10^{-10}$ m $= 100$ pm
electronvolt (eV) $\simeq 1{\cdot}6021 \times 10^{-19}$ J
torr (Torr) $= 101325/760$ Pa
millimetre of mercury (mmHg) $= 13{\cdot}5951 \times 980{\cdot}665 \times 10^{-2}$ Pa
(The torr and the millimetre of mercury are very nearly equal)

List of Symbols

(The page numbers are those on which symbols are first employed; symbols used only transitorily are not always included.)

A	relative atomic mass (atomic weight), 94
$A(x,y)$	aperture transparency function, 140
a	half-width of Glaser's bell-shaped field, 47
$\bar{a}(x,y)$	amplitude transmission function of thespecimen (see $t(x,y)$), 127
$a(x,y)$	$1-\bar{a}(x,y)$, 127
B	illumination, 169
$B(z)$	magnetic induction on the optic axis, 32
B_0	maximum value of $B(z)$, 58
B_p	parallel field between the pole-pieces far from the axis, 58
$\mathbf{B}$	magnetic induction, 27
C_s	spherical aberration coefficient, 66
C_∞	value of C_s for infinite magnification, 166
C_4,C_3,C_2,C_1 and C_0	coefficients of magnification in the polynomial form of C_s, 71
C_c	chromatic aberration coefficient, 68
C_M	chromatic aberration of magnification coefficient, 68
C_θ	anisotropic chromatic aberration coefficient, 68
D	bore radius of symmetric magnetic lenses, 54
	isotropic distortion coefficient, 67
	optical density of developed emulsion, 102
D_1,D_2	bore radii of unsymmetric magnetic lenses, 54
D_1,D_0	coefficients of magnification in the polynomial form of D, 71
d	anisotropic distortion coefficient, 67
$\mathbf{E}$	electric field, 27
e	absolute value of the charge on the electron, 3, 27
f	focal length, 44
	atomic scattering or form factor, 93
$f(z)$	$B(z)/B_0$, 58
$G(z)$	solution of paraxial trajectory equation satisfying $\lim\limits_{z\to-\infty} G(z)=1$, 39
$\bar{G}(z)$	solution of paraxial trajectory equation satisfying $\lim\limits_{z\to\infty} \bar{G}(z)=1$, 43

$g(z)$	solution of paraxial trajectory equation satisfying $g(z_0)=1$, $g'(z_0)=0$, 35
$H(z)$	solution of paraxial trajectory equation satisfying $\lim\limits_{z\to-\infty} H(z)=z-z_0$, 39
H_c	critical field of a superconductor, 110
h	Planck's constant, 3
$h(z)$	solution of paraxial trajectory equation satisfying $h(z_0)=0$, $h'(z_0)=1$, 36
I	see NI
i	image, when used as suffix, 2, 36 current, 85
j	current density, 85, 137
K	transfer function, 141
k	Boltzmann's constant, 85
k^2	$\eta^2a^2B^2{}_0/4\Phi$ or $\eta^2a^2B^2{}_0/4V$, 48
L	camera length, 101
M	magnification, 36
m_0	rest mass of the electron, 32
N_A	Avogadro's number, 94
NI	number of ampère-turns, 57
n	refractive index, 1, 128
o	object, when used as suffix, 1, 35
p	spatial frequency variable in Fourier transforms, 141
$\bar{p}$	$(C_s\lambda^3)^{1/4}p$, 146
q	spatial frequency variable in Fourier transforms, 141
$\bar{q}$	$(C_s\lambda^3)^{1/4}q$, 146
R	alternative symbol for brightness, 85
r	radial coordinate, 27
S	gap in magnetic lenses, 54
T	filament temperature, 85
T_c	critical temperature of a superconductor, 110
t	total scanning time, 169
$t(x,y)$	transmission function of the specimen, $t(x,y)=\bar{a}(x,y)\exp i\phi(x,y)$, 127
$u(\phi)$	$x(\phi)\sin\phi$, 48

V	$\Phi(1+\varepsilon\Phi)$ relativistically corrected electrostatic potential on the axis or accelerating voltage, 31, 57
v	velocity, 3, 27
X	$xV^{1/4}$ or $x\Phi^{1/4}$, 34
x	(rotating) cartesian coordinate, 33
Y	$yV^{1/4}$ or $y\Phi^{1/4}$, 34
y	(rotating) cartesian coordinate, 33
Z	atomic number, 94, 174
z	coordinate along the optic axis, 27
z_F	coordinate of a focus, 42
z_P	coordinate of a principal plane, 43
α	dx/dz, 35
β	brightness, 85
γ	dy/dz, 35
	phase shift caused by C_s and Δ, 141
	contrast of photographic emulsion, 102
Δ	defocus, 141
$\bar{\Delta}$	reduced defocus, $\bar{\Delta}=(C_s\lambda)^{-1/2}\Delta$, 146
ε	$e/2m_0c^2 \simeq 1\ \mathrm{MV}^{-1}$, 31
ε_0	$8.854\ \mathrm{pF\ m}^{-1}$
η	$(e/2m_0)^{1/2} \simeq 3\times10^5\ \mathrm{C}^{1/2}\ \mathrm{kg}^{-1/2}$, 32
θ	$(\alpha^2+\gamma^2)^{1/2}$, 66
	image rotation, 33
Λ	spacing, $\Lambda=f\lambda/(x^2{}_a+y^2{}_a)^{1/2}=(p^2+q^2)^{-1/2}$, 143
$\bar{\Lambda}$	$(\bar{p}^2+\bar{q}^2)^{-1/2}=(C_s\lambda^3)^{-1/4}\Lambda$, 146
λ	wavelength, 1, 3
μ_0	$4\pi\times10^{-7}\ \mathrm{H\ m}^{-1}$,
μ_1, μ_0	coefficients of magnification in the polynomial form of C_M, 71
$\bar{\rho}$	$(\bar{p}^2+\bar{q}^2)^{1/2}$, 147
σ	$1+2\varepsilon\Phi$, 31
	scattering cross-section, 93
Σ	differential scattering cross-section, 94
τ	mass-thickness, 93
ϕ	$z=a\cot\phi$, 48
$\phi(x,y)$	phase transmission function of the specimen (see $t(x,y)$), 127

$\phi(x,y,z)$	electrostatic potential, 28
$\Phi(z)$	potential distribution on the optic axis, 29
Φ	accelerating voltage, 3
χ_2,χ_1,χ_0	coefficients of magnification in the polynomial form of C_c, 71
$\psi(x,y,z)$	magnetostatic potential, $\boldsymbol{B}=-\mathrm{grad}\,\psi$, 59 wave function, 127
ω^2	$1+k^2$, 48
$\sim$	symbol used to indicate the Fourier transform of a function: $\tilde{f}(p,q)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp\{-2\pi\mathrm{i}(px+qy)\}\,\mathrm{d}x\,\mathrm{d}y$, 129

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CHAPTER 1

the limitations of the light microscope and the development of the electron microscope

1.1. *Resolving Power*

THE purpose of a light microscope is to form a visible image of structures, the fine detail of which cannot be discerned with the naked eye. This visible image must be magnified and must be sufficiently contrasty for the eye to be able to see the details against the background. Moreover, it must reproduce the features of the structure faithfully. In the simplest situation, light is shone through a specimen placed on the specimen stage of the microscope; the variations in the opacity of the specimen weaken the corresponding parts of the light beam by differing amounts and the lenses of the microscope then form a magnified image of the specimen that can be seen with the eye directly. Such a specimen affects the amplitude of the incident light and is hence known as an *amplitude object*. We shall later meet another, very important, type of object known as a *phase object*.

It may seem, at first sight, that there is no limit to the smallness of the objects that can be seen with a microscope, provided that the magnification is high enough. We can certainly increase the magnification indefinitely but, beyond a certain point, it is found that no further detail is seen with a light microscope, however high the magnification. This limit cannot be explained by geometrical optics, but when the fact that light propagation is a wave phenomenon is taken into consideration, the existence of such a limit can be readily demonstrated. The existence of this limit was first explained by the great optician Ernst Abbe who, in collaboration with Carl Zeiss, was engaged in the design of microscopes of very high quality. Despite the importance of his result, Abbe was depressed: 'It is poor comfort,' he wrote, 'to hope that human ingenuity will find ways and means of overcoming this limit.' Detailed calculations show that the smallest structure of which a faithful image can be produced is of the order of $k\lambda/A$, where λ is the wavelength of the illumination, k is a constant lying between 0·6 and 0·8, and A is the numerical aperture of the objective. (The numerical aperture is defined by $A = n_o \sin \theta_o$, where n_o is the refractive index of the medium between the specimen and the objective lens and θ_o is the semi-angle subtended at the specimen by the objective lens, or that part of the objective lens through which light rays that eventually contribute to the image pass.) An approximate form of this result may be obtained by the following argument, in which we assume that the illumination is incoherent, that is, that the phase of the light leaving some point in the object plane does not bear any fixed relation to the phase of the light from any other point. (The reasoning in the other extreme case, coherent illumination, and in the intermediate situation, partially coherent illumination, is more elaborate and leads to the same result, except that a different value is obtained for the constant k; it is for this reason that we have given the range within which k falls but have not attached a specific value to it.) Figure 1.1 represents a simple magnifying system in which the points P_o and Q_o in the specimen plane are imaged at P_i and Q_i in the plane conjugate to the specimen plane. The largest angle at which rays can reach the image point P_i, θ_i, is determined by the radius R of the exit pupil and the distance, D, of the latter from the image plane: $\theta_i \approx R/D$. The effect of diffraction is to spread the light at P_i over a broad region, but most of the intensity is concentrated within a circular disc of radius $1{\cdot}22\lambda_i D/2R$, where λ_i is the wavelength in image space; if n_i is the refractive index of the medium, $\lambda_i = \lambda/n_i$, where λ is the vacuum wavelength. The image point Q_i will likewise be spread over a disc, and the two are said to be resolved if the centre of the disc surrounding Q_i falls on the perimeter of the main disc around P_i. At the limit, therefore, $\overline{P_iQ_i} = 0{\cdot}61\lambda D/n_iR = 0{\cdot}61\lambda/n_i\theta_i$. If the magnification is M, $\overline{P_oQ_o} = \overline{P_iQ_i}/M$ and it can be shown that $n_i\theta_i = n_o\theta_o/M$ so that

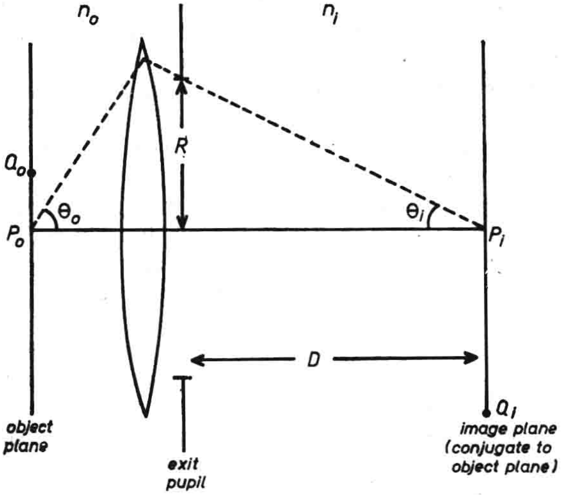

Fig. 1.1 A simple magnifying system.

$$P_oQ_o = 0{\cdot}61 \frac{\lambda}{n_o\theta_o}.$$

A careful examination of the approximations shows that θ_o in this formula is a first approximation to sin θ_o.

It is commonly said that the microscope has a limit of resolution of approximately $k\lambda/A$, therefore, but this gives a somewhat misleading impression, since the cut-off is not sharp: the resemblance between the image and the object is close for structures much larger than this limit and poor or non-existent for structures smaller than it but, close to the limit, the relationship is very complicated.

It is obvious from the expression for the limit of resolution that only two means of improving the resolution are open to us: we can either decrease the wavelength λ or increase the numerical aperture, A. With a light microscope, we are limited to a resolution of the order of 200 nm, since the shortest visible wavelength is about 400 nm and the highest numerical aperture is about 1·4. Although types of electromagnetic radiation with shorter wavelengths are of course known—X-rays have wavelengths of the order of ångströms and those of gamma-rays are still shorter—they are not directly suitable for microscopy because they cannot be focused. In order to find suitable radiation of short wavelength, we must leave the electromagnetic spectrum and enter the domain of particles. In the mid 1920's, Louis de Broglie suggested that a wavelength should be associated with material particles, and a wealth of experiments have since vindicated his proposal. The wavelength is given by the formula $\lambda = h/p$, where h is Planck's constant and p is the momentum of the particles. We shall be dealing with charged particles, and in particular electrons, and since charged particles are accelerated to a high speed by allowing them to pass through a potential difference, Φ, it is convenient to replace the momentum, p, by an expression involving the quantity Φ. If a particle of charge $-e$ passes through a region in which the potential changes from 0 to Φ, then the principle of conservation of energy tells us that

$$e\Phi = \tfrac{1}{2}mv^2 = \frac{p^2}{2m},$$

where v is the velocity of the particle and m its mass. We select the zero of potential in such a way that the particle is stationary when $\Phi = 0$. Hence

$$p = (2me\Phi)^{1/2}$$

and so

$$\lambda = \frac{h}{(2me\Phi)^{1/2}}. \tag{1.1}$$

For electrons,

$$\lambda \approx \frac{1{\cdot}2}{\Phi^{1/2}} \tag{1.2}$$

when λ is measured in nm and Φ in volts. If $\Phi = 90$ kV, therefore,