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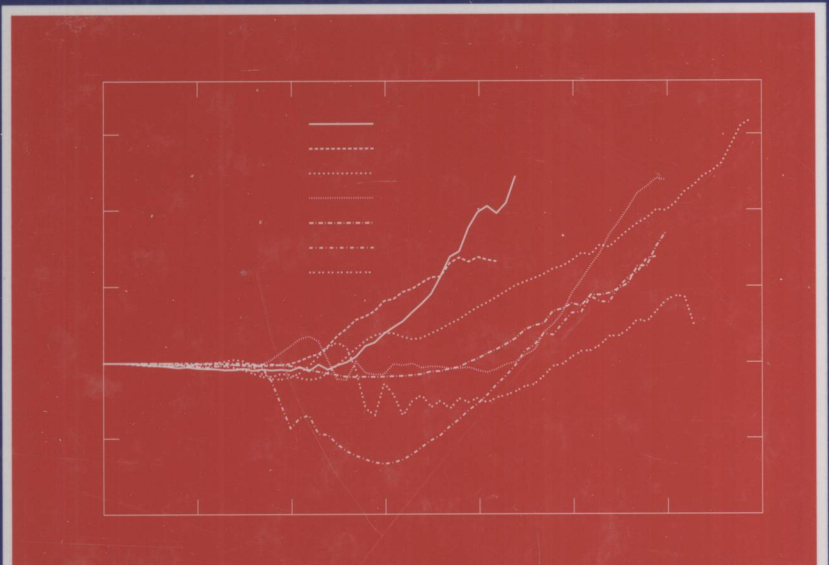
Fractal-Based Point Processes



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WILEY SERIES IN PROBABILITY AND STATISTICS

Fractal-Based Point Processes

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WILEY SERIES IN PROBABILITY AND STATISTICS

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Preface

Fractals and Point Processes

Fractals are objects that possess a form of self-scaling; a part of the whole can be made to recreate the whole by shifting and stretching. Many objects are self-scaling only in a statistical sense, meaning that a part of the whole can be made to recreate the whole in the likeness of their probability distributions, rather than as exact replicas. Examples of random fractals include the length of a segment of coastline, the variation of water flow in the river Nile, and the human heart rate.

Point processes are mathematical representations of random phenomena whose individual events are largely identical and occur principally at discrete times and locations. Examples include the arrival of cars at a tollbooth, the release of neurotransmitter molecules at a biological synapse, and the sequence of human heartbeats.

Fractals began to find their way into the scientific literature some 50 years ago. For point processes this took place perhaps 100 years ago, although both concepts developed far earlier. These two fields of study have grown side-by-side, reflecting their increasing importance in the natural and technological worlds. However, the domains in which point processes and fractals both play a role have received scant attention. It is the intersection of these two fields that forms the topic of this treatise.

Fractal-based point processes exhibit both the scaling properties of fractals and the discrete character of random point processes. These constructs are useful for representing a wide variety of diverse phenomena in the physical and biological sciences, from information-packet arrivals on a computer network to action-potential occurrences in a neural preparation.

Scope

The presentation begins with several concrete examples of fractals and point processes, without devoting undue attention to mathematical detail (*Chapter 1*). A brief introduction to fractals and chaos follows (*Chapter 2*). We then define point processes and consider a collection of measures useful in characterizing them (*Chapter 3*). This is followed by a number of salient examples of point processes (*Chapter 4*). With the concepts of fractals and point processes in hand, we proceed to integrate them (*Chapter 5*). Mathematical formulations for several important fractal-based point-process families are then set forth (*Chapters 6–10*). An exposition detailing how various operations modify such processes follows (*Chapter 11*). We then proceed to examine analysis and estimation techniques suitable for these processes (*Chapter 12*). Finally, we examine computer network traffic (*Chapter 13*), an important application used to illustrate the various approaches and models set forth in earlier chapters.

To facilitate the smooth flow of material, lengthy *Derivations* are relegated to *Appendix A*. *Problem Solutions* appear in *Appendix B*. For convenience, *Appendix C* contains a *List of Symbols*. A comprehensive *Bibliography* is provided.

Approach

We have been inspired by Feller's venerable and enduring *Introduction to Probability Theory and Its Applications* (1968; 1971) and Cox and Isham's concise but superb *Point Processes* (1980).

We provide an integrated exposition of fractal-based point processes, from definitions and measures to analysis and estimation. The material is set forth in a self-contained manner. We approach the topic from a practical and informal perspective — and with a distinct engineering bent. Chapters 3, 4, and 11 can serve as a comprehensive stand-alone introduction to point processes.

A number of important applications are examined in detail with the help of a canonical set of point processes drawn from biological signals and computer network traffic. This set includes action-potential sequences recorded from the retina, lateral geniculate nucleus, striate cortex, descending contralateral movement detector, and cochlea; as well as vesicular exocytosis and human-heartbeat sequences. We revisit these data sets throughout our presentation.

Other applications are drawn from a diverse collection of topics, including $1/f$ noise events in electronic devices and systems, trapping in amorphous semiconductors, semiconductor high-energy particle detectors, diffusion processes, error clustering in telephone networks, digital generation of $1/f^{\alpha}$ noise, photon statistics of Čerenkov radiation, power-law mass distributions, molecular evolution, and the statistics of earthquake occurrences.

Audience

Our exposition is addressed principally to students and researchers in the mathematical, physical, biological, psychological, social, and medical sciences who seek to understand, explain, and make use of the ever-growing roster of phenomena that are found to exhibit fractal and point-process characteristics. The reader is assumed to have a strong mathematical background and a solid grasp of probability theory. While not required, a rudimentary knowledge of fractals and a familiarity with point processes will prove useful.

This book will likely find use as a text for graduate-level courses in fields as diverse as statistics, electrical engineering, neuroscience, computer science, physics, and psychology. An extensive set of solved problems accompanies each chapter.

Website and Supplementary Material

Supplementary materials related to the practical aspects of data analysis and simulation are linked from the book's website. Errata are posted and readers are encouraged to contribute to the compilation. Kindly visit http://www.wiley.com/statistics/ and scroll down to the icon labeled "Download Software and Supplements for Wiley Math & Statistics Titles." Then find the entry "Lowen and Teich." Alternatively, you may directly access the authors' websites at http://cordelia.mclean.org/~lowen/ and http://people.bu.edu/teich/.

Photo Credits

We express our appreciation to the many organizations that have provided assistance in connection with our efforts to assemble the photographs used at the beginnings of each chapter: Penck (courtesy of Bildarchiv der Österreichischen Nationalbibliothek, Vienna); Richardson (courtesy of Olaf K. F. Richardson); Cantor and Poincaré (courtesy of the Aldebaran Group for Astrophysics, Prague); Poisson, Yule, Pareto, Hurst, and Erlang [from Heyde & Seneta (2001), courtesy of Chris Heyde, Eugene Seneta, and Springer-Verlag]; Lapicque (courtesy of the National Library of Medicine); Cox (courtesy of Sir David R. Cox); Fourier (courtesy of John Wiley & Sons); Haar (courtesy of Akadémiai Kiadó, Budapest); Kolmogorov (courtesy of A. N. Shiryaev); Van Ness (courtesy of John W. Van Ness); Mandelbrot (courtesy of Benoit B. Mandelbrot); Gauss (S. Bendixen portrait, 1828); Lévy and Feller [from Reid (1982), courtesy of Ingram Olkin, Constance Reid, and Springer-Verlag]; Schottky (from the Schottky family album); Rice (courtesy of the IEEE History Center, Rutgers University); Neyman [from Reid (1982), courtesy of Constance Reid and Springer-Verlag]; Bartlett (courtesy of Walter Bird and Godfrey Argent); Bernoulli [frontispiece from Fleckenstein (1969), courtesy of Birkhäuser-Verlag]; Allan (courtesy of David W. Allan); Palm (courtesy of Jan Karlqvist, from the Olle Karlqvist family album). The photographs of Lowen and Teich were provided courtesy of Jeff Thiebauth and Boston University, respectively.

We are indebted to a number of individuals who assisted us in our attempts to secure various photographs. These include Tedros Tsegaye, who helped us obtain a photograph of Conny Palm; Steven Rockey, Mathematics Librarian at the Cornell University Mathematics Library, who tracked down a photograph of Alfréd Haar in

a collection of Haar's works (Szőkefalvi-nagy, 1959); and Jan van der Spiegel and Nader Engheta at the University of Pennsylvania, who valiantly attempted to secure a photograph of Gleason Willis Kenrick from the University archives.

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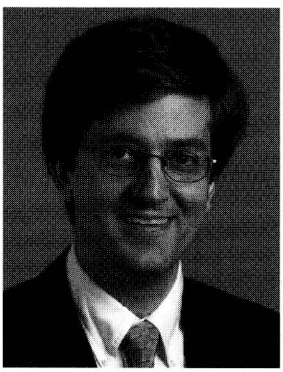

Steven Bradley Lowen received the B.S. degree in electrical engineering from Yale University in 1984, *Magna cum Laude* and with distinction in the major. He was elected to Tau Beta Pi that same year. Following two years with the Hewlett–Packard Company he entered Columbia University, from which he received the M.S. and Ph.D. degrees in 1988 and 1992, respectively, both in electrical engineering. Lowen was awarded the Columbia University Armstrong Memorial Prize in 1988 and in 1990 he was the recipient of a Joint Services Electronics Program Fellowship in the Columbia Radiation Laboratory.

He began his research career by examining fractal patterns in the sequences of action potentials traveling along auditory nerve fibers. Recognizing that efforts to understand these fractal processes were hampered by the lack of a solid theoretical framework, he set out to develop the relevant mathematical models. This effort led to the development of alternating fractal renewal processes and fractal shot noise, as well as point-process versions thereof. In connection with this effort he also investigated fractal renewal point processes and several other fractal-based processes. This body of work served as the foundation for his Ph.D. thesis, entitled *Fractal Point Processes* (Lowen, 1992), as well as the basis of a number of journal articles and the core of several chapters in this book.

After receiving the Ph.D. degree, Dr. Lowen continued his research at Columbia as an Associate Research Scientist. He then joined Boston University as a Senior Research Associate in the Department of Electrical and Computer Engineering in 1996. He was elected to Sigma Xi in 1994.

With a collection of models for fractal-based point processes in hand, Lowen focused on establishing appropriate methods for their analysis and synthesis. This work quantified the performance of fractal estimators for point processes and highlighted the practical realities of generating realizations for these processes. He also studied the interaction between dead time (refractoriness) and fractal behavior in point processes.

Concurrently, working with various collaborators, he returned to examining applications for point processes with fractal characteristics by adapting the mathematical framework he developed to a number of biomedical point processes. He demonstrated that suitably modified fractal-based point processes serve to properly characterize action-potential sequences on auditory nerve fibers. He then turned his attention to signaling in the visual system by identifying fractal models that could describe the neural firing patterns of individual cells in this system, as well as collections of such cells, and detailing how the fractal patterns affect information transmission in this network. Using a similar approach, he also examined human heartbeat patterns and investigated how different measures of these fractal data sets could serve as markers of the cardiovascular health of the subject. Finally, he explored neurotransmitter secretion at the neuromuscular junction, and developed a suitable model showing that it, too, exhibits fractal characteristics.

Dr. Lowen also applied his fractal models to physical phenomena. These included charge transport in amorphous semiconductors and noise in infrared CCD cameras; he developed multidimensional versions of his fractal-based point processes for the latter. He also devoted substantial efforts to the modeling, synthesis, and analysis of computer network traffic.

In 1999 Dr. Lowen joined McLean Hospital and the Harvard Medical School, where he is currently Assistant Professor of Psychiatry. He has brought his fractal expertise to bear on attention-deficit and hyperactivity disorder, and the analysis of data collected with functional magnetic resonance imaging (fMRI). He is currently investigating fractal and other aspects of these applications, as well as carrying out research on drug abuse.

Dr. Lowen has authored or co-authored some 30 refereed journal articles as well as a collection of book chapters and proceedings papers. He holds a number of patents, and serves as a reviewer for several technical journals and funding agencies. Over the course of his career, he has supervised three graduate students.

Malvin Carl Teich received the S.B. degree in physics from MIT in 1961, the M.S. degree in electrical engineering from Stanford University in 1962, and the Ph.D. degree from Cornell University in 1966. His bachelor's thesis comprised a determination of the total neutron cross-section of palladium metal while his doctoral dissertation reported the first observation of the two-photon photoelectric effect in metallic sodium. His first professional affiliation was with MIT's Lincoln Laboratory in Lexington, Massachusetts, where he demonstrated that heterodyne detection could be achieved in the middle-infrared region of the electromagnetic spectrum.

Teich joined the faculty at Columbia University in 1967, where he served as a member of the Electrical Engineering Department (as Chairman from 1978 to 1980), the Applied Physics Department, the Columbia Radiation Laboratory, and the Fowler Memorial Laboratory for Auditory Biophysics. Extending his work on heterodyning, he recognized that the interaction could be understood in terms of the absorption of individual polychromatic photons, and demonstrated the possibility of implementing the process in a multiphoton configuration. He developed the concept of nonlinear heterodyne detection — useful for canceling phase or frequency noise in an optical system.

During his tenure at Columbia, he also carried out extensive work in point processes, with particular application to photon statistics, the generation of squeezed light, and noise in fiber-optic amplifiers and avalanche photodiodes. Among his achievements is a description of luminescence light in terms of a photon cluster point process. This perspective led him to suggest that detector dead time could be used advantageously to reduce the variability of this process and thereby luminescence noise. This approach was incorporated in the design of the star-scanner guidance system for the Galileo spacecraft, which was subjected to high radio- and beta-luminescence background noise as a result of bombardment by copious Jovian gamma- and beta-ray emissions. In the domain of quantum optics he developed the concept of pump-fluctuation control in which the variability of a pump point process comprising a beam of electrons is reduced by making use of self-excitation in the form of Coulomb repulsion. Using a space-charge-limited version of the Franck–Hertz experiment in mercury vapor he demonstrated the validity of this concept by generating the first source of unconditionally sub-Poisson light. His work on fiber-optic amplifiers led to an understanding of the properties of the photon point process that emerges from the laser amplifier and thereby of the performance characteristics of these devices.

Teich's interest in point processes in the neurosciences was fostered by a chance encounter in 1974 with William J. McGill, then Professor of Psychology and President of Columbia University. This, in turn, led to a long-standing collaboration with Shyam M. Khanna, Director of the Fowler Memorial Laboratory for Auditory Biophysics and Professor in the Department of Otolaryngology at the Columbia College of Physicians & Surgeons. Together, Teich and Khanna carried out animal experi-