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FOREWORD

This publication contains the papers presented in the field of High Pressure Technology at the annual ASME Pressure Vessels and Piping Conference held in Seattle, Washington, July 23–27, 2000. The papers cover a broad spectrum of topics within this technology including design, analysis, manufacturing, and various high-pressure applications. They are organized in the following five sections:

- Tubing and Fitting Design and Analysis
- Vessels and Tubing for the LDPE Industry
- Vessel Fatigue and Fracture Mechanics Analysis
- Vessel Design and Analysis
- Applications and Shielding

On behalf of the High Pressure Technology Committee, the editor would like to acknowledge the technical contributions of each author and of those who reviewed and commented on the papers. Additionally, the editor would like to express appreciation of the extraordinary expenditure of volunteer time by the authors and other contributors, as well as recognizing that the completion of this publication would not have been possible without the support of the various contributors' affiliates.

As a final note, an integral part of the ASME Pressure Vessels and Piping Division Conference is the Student Paper Competition. This competition is sponsored by the Senate of the PVP Division which is comprised of recent past Chairmen of the Division. To be accepted for publication, the student papers are refereed to established PVP Division publication review standards. The student papers are presented in a special technical session, and an outstanding paper is selected by a panel of judges based on the quality of the prepared paper, relevance to the pressure vessels and piping industry, and the presentation of the paper by the student author. A student paper is included in this volume and can be found at the back of the volume.

Sigurd C. Mordre
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AUTOFRETTAGE OF OPEN END TUBES (1) - OVERVIEW, PRESSURE CALCULATION AND STRESS PROFILES

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ABSTRACT

Autofrettage is used to introduce advantageous residual stresses into pressure vessels. The Bauschinger effect can produce less compressive residual hoop stresses near the bore than are predicted by 'ideal' autofrettage solutions.

A recently developed numerical analysis procedure is adopted and extended. The ratio of autofrettage pressure (numerical) / ideal autofrettage pressure (Tresca criterion & plane stress) is calculated. This is shown to be within 0.5% of available analytically-defined bounds for 0% and 100% overstrain for the case where the numerically calculated pressure relates to Von Mises criterion & plane stress. For practical geometries, the ratio varies between unity and 1.1547 ($2/\sqrt{3}$). This indicates that the frequently adopted pragmatic value of 1.15 may produce significant discrepancies.

The more practically relevant case of 'open end conditions' in which autofrettage pressure is based upon Von Mises & engineering plane strain (constant axial strain with zero net axial force) is examined in detail. The ratio in this case again varies between unity and 1.1547 but exhibits very significant variations from the plane stress case when the diameter ratio of the tube exceeds 1.8. These results are bounded by the 0% bound referred to above and, at 100%, by available numerical and experimental results. A simple numerical fit allows all these results to be replicated to within 0.5%. The true plane strain pressure ratio bound is examined and shown to be inappropriate in modeling engineering plane strain. A limited number of residual hoop and axial stress profiles is presented.

Part 2 of this paper employs these results and procedure in determining a large range of hoop residual stresses, comparing with current code recommendations and proposing an enhanced design procedure.

NOMENCLATURE

a, b, c, d, r Radii defined in Figure 1
bore Bore value
EPS Engineering plane strain
n Percentage overstrain
p Autofrettage pressure
T Tresca criterion
TPS True plane strain
$T\sigma$ Tresca criterion, plane stress
VM Von Mises' criterion
$VM\sigma$ Von Mises' criterion, plane stress
Y Uniaxial yield stress
α Factor defined in eqn (12)
σ_θ Residual hoop stress after autofrettage
$p\rceil^{T\sigma}_{100\%}$ Notation example : pressure for 100% overstrain with Tresca, plane stress condition

INTRODUCTION

Autofrettage is used to introduce advantageous residual stresses into pressure vessels and to enhance their fatigue lifetimes. For many years workers have acknowledged the probable influence of the Bauschinger effect, Bauschinger (1881), which serves to reduce the yield strength in compression as a result of prior tensile plastic overload. Chakrabarty (1987) provides some review of the microstructural causes.

The reduction of compressive yield strength within the yielded zone of an autofrettaged tube is of importance because, on removal of the autofrettage pressure, the region near the bore experiences high values of compressive hoop stress, approaching the magnitude of the tensile yield strength of the material, if the unloading is totally elastic. If the combination of stresses exceeds some yield criterion the tube will re-yield from the bore thus losing much of the potential benefit of autofrettage.

The purpose of Parts 1 and 2 of this paper is to employ an existing elastic-plastic numerical procedure, Parker et al (1999), to produce a wide range of residual stress predictions covering tube diameter ratios up to 3.0 and all possible levels of overstrain from 0% to 100%. Overstrain is defined as the proportion of the wall thickness of the tube which behaves plastically during the initial application of autofrettage pressure or bore interference. The formulation for the numerical procedure generally follows that proposed in Jahed and Dubey (1997) which is extremely flexible, is appropriate for use on standard spreadsheets and is well-suited to the numerous iterative procedures required. The formulation was further developed, together with a review of previous work, in Parker et al (1999), and is not repeated herein.

The following geometrical definitions apply, see Figure 1. : Tube inner radius, a; tube outer radius, b; radius of plastic zone at peak of autofrettage cycle, c; maximum radius of reversed plasticity, d; general radius location, r.

The materials considered are steels which conform with the descriptions contained within Milligan et al (1966) upon which the uniaxial stress-strain behavior in tension and subsequent compression is based. The materials reported in Milligan et al (1966) do not exceed a yield strength of 1100MPa; there is also good reason to believe that this behavior also extends to martensitic steels having a yield strength of 1200 MPa, Troiano (1998).

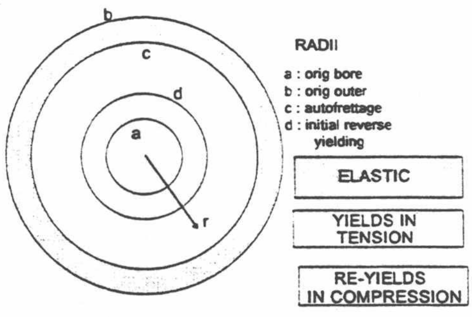

Figure 1 : Tube Geometry

Figure 2, taken from Parker et al (1999), shows typical residual stress profiles based upon a Tresca criterion plane stress analysis arising from autofrettage of a tube of radius ratio 2.0. The 'ideal' profile for elastic - perfectly plastic behavior without Bauschinger effect is included for comparison. Noteworthy effects include:

- A large reduction in bore hoop stresses as a result of Bauschinger effect
- The Bauschinger effect penetrates much deeper into the tube than previous attempts at modeling typical gun steels have suggested: approximately 22% and 30% of wall thickness for overstrains of 60% and 100% respectively for a tube of radius ratio 2.0. Previous work has suggested depths of around 16%.
- A minimum value of hoop stress at the bore associated with a 'saturation' value of 2% plastic strain. This is a direct result of the constant Bauschinger Effect Factor (BEF) values observed by Milligan et al (1966) for plastic strain > 2%
- Very limited benefit (in terms of increased compressive hoop stresses in the near-bore region) as a result of overstrain above 60%.
- Disadvantages in autofrettage above 60% because of the significant increase in tensile residual hoop stress at the outside diameter.

The results presented in Parker et al (1999) all relate to Tresca's yield criterion under plane stress conditions and are limited principally to $b/a = 2$. One objective of this work is to cover the range $1.1 \le b/a \le 3.0$ and to include the more practically relevant case of Von Mises' yield criterion combined with Engineering Plane Strain (EPS) conditions, i.e. constant axial strain with zero net axial force sometimes referred to as 'open end conditions'. However, in order to simplify the presentation of results for such a large range of geometries and overstrain levels, in Part 2 of this paper, Parker (2000), only bore hoop stresses are presented. Bore hoop stress values are of overriding importance because it is this value which dominates fatigue crack growth calculations and which is used to determine pressure for re-yielding (Parker et al, 1999). A further objective, examined at length in Part 2 of this paper, is to relate EPS results over a wide range of geometries to the current ASME pressure vessel code, ASME (1997), and thereby make some proposals to further extend its validity and accuracy.

In order to achieve these objectives it is necessary to critically examine certain common assumptions. The first of these is the frequent use of a multiplying factor of 1.15 ($2/\sqrt{3}$) in order to determine residual stress profiles based upon Von Mises' criterion from those obtained using Tresca's criterion. The second, separate, assumption is the use of a somewhat similar multiplying factor in determining the required bore pressure to achieve a given percentage overstrain. This factor is used to scale the autofrettage pressure determined via a Tresca plane stress analysis. For various reasons the combined effect of any error in these factors can far exceed the intuitive expectation of a maximum effect of 15%.

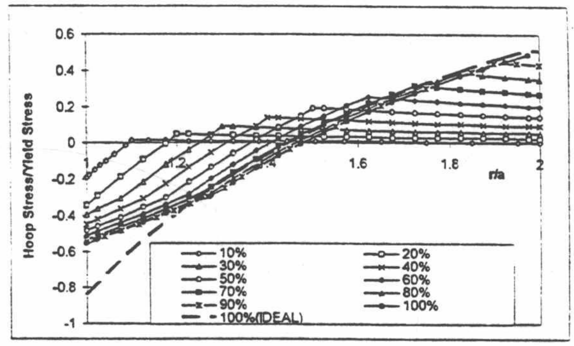

Figure 2 : Total Hoop Residual Stress Profile for b/a=2 with Various Percentage Overstrains. Tresca and Plane Stress Conditions Assumed (after Parker et al (1999))

ANALYSIS PROCEDURE

The residual compressive hoop stress within the plastically deformed region of an autofrettaged tube determined via a Tresca plane stress analysis without Bauschinger effect is well known, Chakrabarty (1987), and is given by:

$$\sigma_\theta \rceil^{T\sigma} = -p\rceil^{T\sigma} + Y[1 + \ln(r/a)] - [p\rceil^{T\sigma} a^2(b^2 - a^2)].[1 + b^2/r^2] \quad (1)$$

where $c/a \le 2.22$, Y is the uniaxial yield stress for the material and the autofrettage pressure (Tresca, plane stress), $p\rceil^{T\sigma}$ is given by:

$$p\rceil^{T\sigma} = Y[\ln(c/a) + (b^2 - c^2)/2b^2] \quad (2)$$

Thus the onset of yielding at the bore occurs when c=a :

$$p\rceil^{T\sigma}_{0\%} = Y(b^2 - a^2)/2b^2 \quad (3)$$

The equivalent pressure for the case of Von Mises and plane stress, $p\rceil^{VM\sigma}_{0\%}$, Chakrabarty (1987), may be obtained from:

$$p\rceil_{0\%}^{VM\sigma}/p\rceil_{0\%}^{T\sigma} = (2/\sqrt{3})/\sqrt{1 + 1/\{3(b/a)^4\}} \quad (4)$$

100% overstrain (c=b), for Tresca, plane stress requires a pressure:

$$p\rceil_{100\%}^{T\sigma} = Y\ln(b/a) \quad (5)$$

Substituting eqn (2) into eqn (1):

$$\sigma_\theta\rceil^{T\sigma}/Y = (c^2 + b^2)/2b^2 + \ln(r/c) - [a^2/(b^2 - a^2)].[1 + b^2/r^2].[(b^2 - c^2)/2b^2 + \ln(c/a)] \quad (6)$$

the value of hoop stress at the bore is obtained by setting r = a in eqn (6) to give

$$\sigma_\theta\rceil_{bore}^{T\sigma}/Y = [(c^2 - a^2) - 2b^2\ln(c/a)]/[b^2 - a^2] \quad (7)$$

Hill (1967) reviews approximate methods of correcting eqn (7) to simulate Von Mises' criterion in modeling the autofrettage process. Hill concludes with the now familiar finding that by substituting $2Y/\sqrt{3}$ (generally represented as 1.15Y) for Y in eqns (6) and (7) the errors in residual stress prediction are less than 2%. The implication is that for a given percentage overstrain a simple scaling of the Tresca residual stress predictions by 1.15 will produce the desired Von Mises' prediction. Note that Hill's analysis implicitly assumes true plane strain conditions (TPS), i.e. zero axial strain. This will be of importance is understanding upcoming results relating to EPS.

The question of modification of autofrettage pressure to account for Von Mises' criterion with open-end conditions has been addressed by several workers. Davidson et al (1963) obtained experimental values of pressure at 100% overstrain in the range $1.6 \le b/a \le 2.4$, Marcal (1965) employed a stiffness method and determined pressure for 100% overstrain in the range $1.5 \le b/a \le 4.0$, together with hoop strains at the outer surface for the complete range of possible overstrain pressures. Davidson and Kendall (1970) proposed an empirical pressure value of $1.08Y\ln(b/a)$ for the case of 100% overstrain, with an associated maximum error of 2%. The current ASME pressure vessel code, ASME (1997), uses a fixed scaling factor of $1.15p\rceil^{T\sigma}$, but limits code validity to a maximum of 40% overstrain.

A rational approach to the numerical procedure, which is lengthy and often involves multiple iterations, requires that analyses are undertaken in a specific sequence, namely:

In current paper:

Step 1. For each tube geometry iteratively determine pressure to achieve a given percentage overstrain. This is repeated for the case of Von Mises', plane stress, $p\rceil^{VM\sigma}$, and Von Mises, EPS, $p\rceil^{VMEPS}$. Both sets of results are normalized with Tresca, plane stress, $p\rceil^{T\sigma}$. The first set is used to validate numerical results by comparison with analytical bounds, the second set is used as the basis of a proposed design procedure.

Step 2. Determine some simple numerical fit to the ratios determined in Step 1 for use by designers.

Step 3. Employ the autofrettage pressures determined in Step 1 for the Von Mises, EPS case in determining a limited range of autofrettage residual stress profiles. These results to cover hoop and axial stress and encompass both Bauschinger affected and non-Bauschinger affected situations.

In part 2 of this paper:

Step 4. Determine numerous bore hoop stress values for the Von Mises, EPS case, normalized with $\sigma_\theta\rceil_{bore}^{T\sigma}$ from eqn (4).

Step 5. Determine some simple numerical fit to the ratios determined in Step 4 for use by designers.

Step 6. Use procedures employed in Steps 1-5 to assess accuracy of ASME code.

Step 7. Propose a procedure which will improve accuracy and extend code validity beyond current 40% overstrain limit.

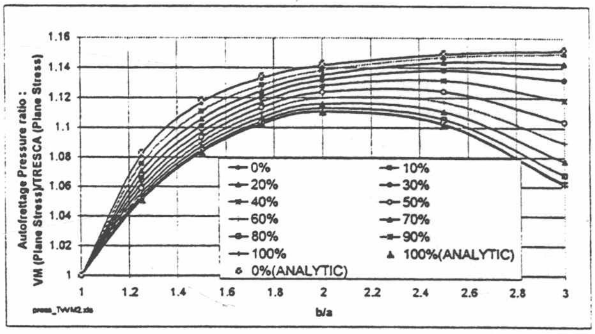

Figure 3 : Bore Pressure for Given Percentage Overstrain, Von Mises and Plane Stress Conditions Assumed

PRESSURE FOR GIVEN OVERSTRAIN LEVEL

Figure 3 shows numerical results of bore pressure for a given percentage overstrain based upon Von Mises' criterion, plane stress, $p\rceil^{VM\sigma}$, normalized with the equivalent Tresca, plane stress pressure $p\rceil^{T\sigma}$. Elastic-perfectly plastic behavior is assumed during loading. Two analytical bounds are also shown, the first is the onset of autofrettage as defined by eqn (3) whilst the second bound relates to 100% overstrain and was obtained iteratively from Weigle (1960). This equation, in current notation, is:

$$2\ln\left[\frac{(b/a)^2\sqrt{3\gamma}}{1+\sqrt{3\gamma-3}}\right] = \sqrt{3}\,\pi - 2\sqrt{3}\,\arctan\sqrt{\gamma - 1} \quad (8)$$

$$\text{where } \gamma = \tfrac{4}{3}\left[Y/(p\rceil_{100\%}^{VM\sigma})\right]^2 \quad (9)$$

$$\text{and, for a real solution, } (p\rceil_{100\%}^{VM\sigma})/Y \le 2/\sqrt{3} \quad (10)$$

The numerical results for the cases 0% and 100% overstrain are within 0.2% of the analytical bounds. The 100% bound is not valid beyond b/a=2.5 because of the restriction imposed by eqn (10). Figure 3 gives considerable confidence in the numerical procedure employed. The technique is now extended to the case of Von Mises, EPS.

The numerical procedure required to encompass EPS requires only one enhancement to the procedure employed thus far and described in Parker et al (1999). This involves an additional iterative stage in which a true plane strain (TPS) (i.e. zero axial strain) solution is obtained initially and total axial force in the tube determined. An appropriate constant strain is then applied to the tube and iteratively adjusted until EPS is achieved. This extended procedure requires two alternating sets of iterations. It was found that each EPS solution for a given geometry and autofrettage pressure required around 1000 iterations in total; however, since the selection of autofrettage pressure for a given overstrain is itself iterative this number must be factored by a further 10 or 20. Since the pressure-iteration procedure does not readily lend itself to complete automation the process is undeniably time consuming! The scheme does nonetheless provide a monotonic, repeatable, mesh-independent convergence.

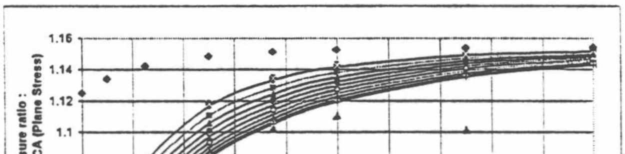

Figure 4 : Bore Pressure for Given Percentage Overstrain, Von Mises and Engineering Plane Strain (Open-End) Conditions Assumed

Figure 4 is in precisely the same format as Figure 3 but relates to Von Mises criterion, EPS solutions, $p\rceil^{VMEPS}$. In this case the previous bounds are clearly in evidence, with 0% forming an excellent upper bound but with significant deviation at $b/a > 1.5$, as might be anticipated, from the 100%, Von Mises, plane stress bound. The analytic TPS bound for 0% overstrain, Chakrabarty (1987), is also included. This leads to an important observation - the use of a fixed pressure ratio of $2/\sqrt{3}$ may be justifiable in the TPS case but is clearly inappropriate in the EPS case.

The following expression provides a fit which is generally within 0.5% over the entire range of results:

$$\frac{p\rceil^{VMEPS}}{p\rceil^{T\sigma}} = \frac{(2/\sqrt{3})}{\sqrt{1+1/\{3(b/a)^{\alpha}\}}} \qquad (11)$$

where $\alpha = 4 - 2.3n$ and (12)

$$n = (c-a)/(b-a)\ , \quad (c-a)/(b-a) \le 70\% \qquad (13)$$

$$n = 70\%\ , \quad (c-a)/(b-a) > 70\% \qquad (14)$$

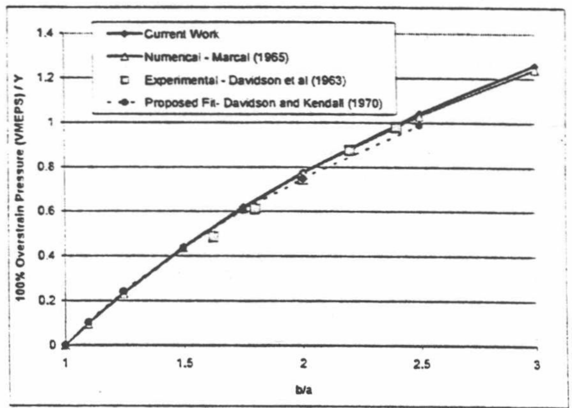

Figure 5 : Bore Pressure for 100% Overstrain - Comparison with Other Work. Von Mises and Engineering Plane Strain Conditions

The results for 100% overstrain with open ends may also be compared with two other sources. Figure 5 shows pressure for 100% overstrain normalized with yield stress. The results of Marcal (1965) who used a numerical procedure are shown together with the averaged experimental results of Davidson et al (1963) and the empirical fit proposed by Davidson and Kendall (1970). The current work is within 1% of Marcal (1965) and within 4% of the experimental results. The experimental results fall generally below the numerical.

Whilst it is not the purpose of this paper to provide strain values, by way of further confirmation it is noted that the outside surface hoop strains, covering the full range of partial autofrettage pressure and diameter ratios reported in Figure 1 of Marcal (1965), are replicated to within 0.5% by the current method.

RESIDUAL STRESS PROFILES AFTER PRESSURE REMOVAL

The pressure ratios presented in the previous section relate only to the application of the autofrettage pressure at the bore. They provide the pressure necessary to achieve a given percentage overstrain. When the bore pressure defined in Figure 4 is removed residual stresses are 'locked in' to the tube. It is during this unloading phase that the Bauschinger effect may manifest itself.

Figure 6 shows typical residual hoop stress profiles $\sigma_\theta\rceil^{VMEPS}$, for the case of b/a=2 for the full range of possible overstrain. The profile relating to 100% overstrain without Bauschinger effect is shown as a heavy broken line, the remainder of the results include Bauschinger effect. Qualitatively these results are very similar to those for Tresca, plane stress shown in Figure 2 and the observations listed as bullet points in the introduction are unchanged. However there is some increase in magnitude of residual hoop stress between Tresca, plane stress and Von Mises, EPS. Examples of percentage increase in compressive bore hoop stress are 10.3% (20% overstrain), 11.4% (40% overstrain), 10.7% (60% overstrain), 9.4% (80% overstrain), 8.5% (100% overstrain).

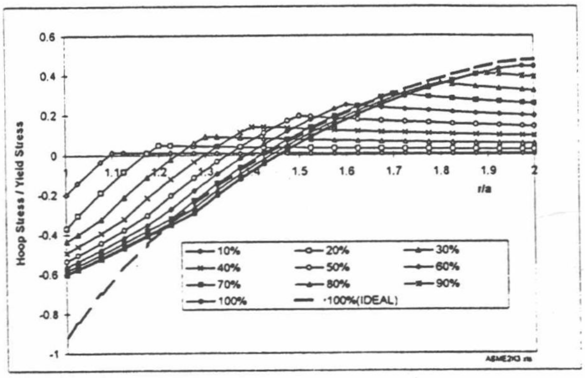

Figure 6 : Hoop Residual Stress Profile for b/a=2 with Various Percentage Overstrains. Von Mises and Engineering Plane Strain (EPS) Conditions Assumed

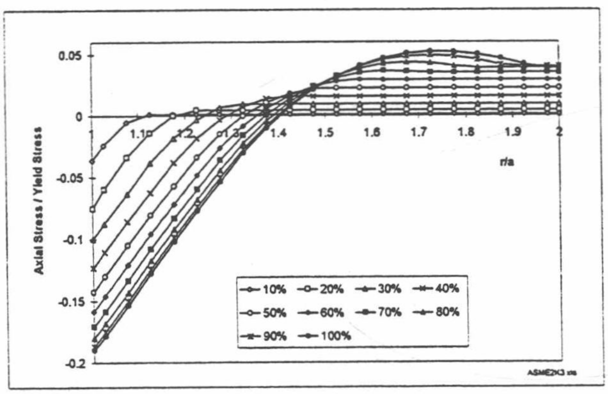

Figure 7 : Axial Residual Stress Profile for b/a=2 with Various Percentage Overstrains. Von Mises and Engineering Plane Strain (EPS) Conditions Assumed

The associated values of axial stress are presented in Figure 7. The approximate rule-of-thumb: *bore hoop stress x Poisson's ratio = bore axial stress* is observed. Because there is such a large number of b/a and overstrain combinations under investigation the results in Part 2, Parker (2000), focus upon bore hoop stress values.

SUMMARY & CONCLUSIONS

This work extended an existing numerical procedure to calculate a wide range of autofrettage pressures and a limited number of hoop and axial residual stress fields for tubes under open-end (engineering plane strain) conditions using Von Mises criterion.

A design curve with numerical fit is proposed which allows the open-end pressure results to be replicated to within 0.5%.

The practice of using an autofrettage design pressure, for a given overstrain, of 1.15 times the ideal pressure from a Tresca criterion, plane stress analysis is shown to be inappropriate.

The true plane strain pressure ratio bound is examined and shown to be inappropriate in modeling engineering plane strain.

A limited number of residual stress profiles is presented for Bauschinger and non-Bauschinger affected tubes. These confirm earlier observations relating to hoop stress and provide additional profiles for axial residual stress.

ACKNOWLEDGMENT

The author gratefully acknowledges helpful discussions with Mr. David P. Kendall.

REFERENCES

ASME Pressure Vessel and Piping Design Code, 1997, "Design Using Autofrettage", Division 3, Section 8, Article KD-5, pp. 71-73.

Bauschinger, J., 1881, "Ueber die Veranderung der Elasticitatagrenze und dea Elasticitatamoduls verschiadener Metalle", Zivilingenieur, Vol 27, columns 289-348.

Chakrabarty, J., 1987, "Theory of Plasticity", McGraw Hill.

Davidson, T. E., Barton, C.S., Reiner, A.N. and Kendall, D. P., 1963, "Overstrain of High Strength Open-End Cylinders of Intermediate Diameter Ratio", Proceedings of First International Congress on Experimental Mechanics, Pergamon Press, Oxford.

Davidson, T. E., and Kendall, D. P., 1970, "Design of High Pressure Containers", contained in 'Mechanical Behavior of Metals Under Pressure', pp. 54-77, Elsevier, H. Pugh, Ed.

Hill, R., "The Mathematical Theory of Plasticity," Oxford University Press, 1967.

Jahed, H. and Dubey, R. N., 1997, "An Axisymmetric Method of Elastic-Plastic Analysis Capable of Predicting Residual Stress Field", Trans ASME, J Pressure Vessel Technology, Vol 119, pp. 264-273.

Marcal, P.V., 1965, "A Note on the Elastic-Plastic Thick Cylinder with Internal Pressure in the Open and Closed-End Condition", Int J Mech sci, Vol 7, pp. 841-845.

Milligan, R. V., Koo, W. H. and Davidson, T. E., 1966, "The Bauschinger Effect in a High Strength Steel", Trans ASME, D, pp. 480-488.

Parker, A. P., Underwood, J. H. and Kendall, D. P., 1999, "Bauschinger Effect Design Procedures for Autofrettaged Tubes Including Material Removal and Sachs' Method", Trans ASME, J Pressure Vessel Technology, Vol 121, pp. 430-437.

Parker, A.P., 2000, "Autofrettage of Open-End Tubes (2) - Bore Hoop Stresses, Code Comparison and Design Recommendations", presented at ASME Pressure Vessels and Piping Conference, Seattle.

Troiano, E., 1998, Private Communication.

Weigle, R.E., 1960, "Elastic-Plastic Analysis of a Cylindrical Tube", Watervliet Arsenal Technical Report WVTRR-6007

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AUTOFRETTAGE OF OPEN END TUBES (2) - BORE HOOP STRESSES, CODE COMPARISON AND DESIGN RECOMMENDATIONS

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ABSTRACT

Autofrettage is used to introduce advantageous residual stresses into pressure vessels. The Bauschinger effect can produce less compressive residual hoop stresses near the bore than are predicted by 'ideal' autofrettage solutions.

In Part 1 of this paper a recently developed numerical analysis procedure was adopted and extended to determine autofrettage pressures for the case of 'open end conditions' in which autofrettage pressure is based upon Von Mises & engineering plane strain (constant axial strain with zero net axial force).

Within this paper the above pressures are used to determine residual hoop stress values for tube diameter ratios from 1.1 to 3.0 for the full range of percentage overstrain levels. These comparisons indicate that Bauschinger effect is evident when the ratio autofrettage radius/bore radius exceeds 1.2, irrespective of diameter ratio. To assist designers the important values of residual hoop stress at the bore are summarized in a composite plot and a numerical fit is provided.

The procedure is also used to assess the accuracy of the current ASME code. The code is shown to be generally and modestly conservative. A design procedure is proposed which appears capable of extending code validity beyond 40% overstrain (the limit of the current code) and of eliminating the small non-conservatism at very low overstrain.

NOMENCLATURE

a, b, c, d, r	Radii defined in Figure 1
bore	Bore value
EPS	Engineering plane strain
n, m	Percentage overstrain, eqns (10), (11) and (15)
p	Autofrettage pressure
R	Factor defined in eqn (13)
T	Tresca criterion
TPS	True plane strain
$T\sigma$	Tresca criterion, plane stress
VM	Von Mises' criterion
$VM\sigma$	Von Mises' criterion, plane stress
Y	Uniaxial yield stress
α	Factor defined in eqn (9)
σ_θ	Residual hoop stress after autofrettage

INTRODUCTION

Autofrettage is used to introduce advantageous residual stresses into pressure vessels and to enhance their fatigue lifetimes. For many years workers have acknowledged the probable influence of the Bauschinger effect, Bauschinger (1881), which serves to reduce the yield strength in compression as a result of prior tensile plastic overload. Chakrabarty (1987) provides some review of the microstructural causes.

The reduction of compressive yield strength within the yielded zone of an autofrettaged tube is of importance because, on removal of the autofrettage pressure, the region near the bore experiences high values of compressive hoop stress, approaching the magnitude of the tensile yield strength of the material, if the unloading is totally elastic. If the combination of stresses exceeds some yield criterion the tube will re-yield from the bore thus losing much of the potential benefit of autofrettage.

The purpose of Parts 1 and 2 of this paper is to employ an existing elastic-plastic numerical procedure, Parker et al (1999), to produce a wide range of residual stress predictions covering tube diameter ratios up to 3.0 and all possible levels of overstrain from 0% to 100%. Overstrain is defined as the proportion of the wall thickness of the tube which behaves plastically during the initial application of autofrettage pressure or bore interference. The formulation for the numerical procedure generally follows that proposed in Jahed and Dubey (1997). The formulation was further developed, together with a review of previous work, in Parker et al (1999), and is not repeated herein.

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The materials considered are steels which conform with the descriptions contained within Milligan et al (1966) upon which the uniaxial stress-strain behavior in tension and subsequent compression is based. The materials reported in Milligan et al (1966) do not exceed a yield strength of 1100MPa; there is also good reason to believe that this behavior also extends to martensitic steels having a yield strength of 1200 MPa, Troiano (1998)

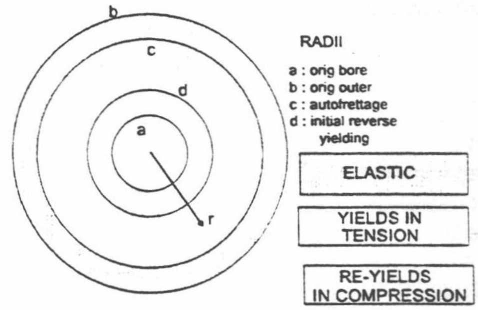

Figure 1 : Tube Geometry

ANALYSIS PROCEDURE

The residual compressive hoop stress within the plastically deformed region of an autofrettaged tube determined via a Tresca & plane stress analysis without Bauschinger effect is well known, Chakrabarty (1987), and is given by:

$$\sigma_\theta\rceil^{T\sigma} = -p\rceil^{T\sigma} + Y[1+\ln(r/a)] \\ -[p\rceil^{T\sigma} a^2(b^2-a^2)].[1+b^2/r^2] \quad (1)$$

where $c/a \le 2.22$, Y is the uniaxial yield stress for the material and the autofrettage pressure (Tresca, plane stress), $p\rceil^{T\sigma}$ is given by:

$$p\rceil^{T\sigma} = Y[\ln(c/a) + (b^2-c^2)/2b^2] \quad (2)$$

Thus the onset of yielding at the bore occurs when c=a :

$$p\rceil^{T\sigma}_{0\%} = Y(b^2-a^2)/2b^2 \quad (3)$$

The equivalent pressure for the case of Von Mises and plane stress, $p\rceil^{VM\sigma}_{0\%}$, Chakrabarty (1987), may be obtained from:

$$p\rceil^{VM\sigma}_{0\%}/p\rceil^{T\sigma}_{0\%} = (2/\sqrt{3})/\sqrt{1+1/\{3(b/a)^4\}} \quad (4)$$

100% overstrain (c=b), with Tresca, plane stress requires a pressure:

$$p\rceil^{T\sigma}_{100\%} = Y\ln(b/a) \quad (5)$$

Substituting eqn (2) into eqn (1):

$$\sigma_\theta\rceil^{T\sigma}/Y = (c^2+b^2)/2b^2 + \ln(r/c) \\ -[a^2/(b^2-a^2)].[1+b^2/r^2]. \\ [(b^2-c^2)/2b^2 + \ln(c/a)] \quad (6)$$

the value of hoop stress at the bore is obtained by setting r = a in eqn (6) to give :

$$\sigma_\theta\rceil^{T\sigma}_{bore}/Y = \\ [(c^2-a^2) - 2b^2\ln(c/a)]/[b^2-a^2] \quad (7)$$

Hill (1967) reviews approximate methods of correcting eqn (7) to simulate Von Mises' criterion in modeling the autofrettage process. Hill concludes with the now familiar finding that by substituting $2Y/\sqrt{3}$ (generally represented as 1.15Y) for Y in eqns (6) and (7) the errors in residual stress prediction are less than 2%. The implication is that for a given percentage overstrain a simple scaling of the Tresca residual stress predictions by 1.15 will produce the desired Von Mises' prediction. Note that Hill's analysis implicitly assumes true plane strain conditions (TPS), i.e. zero axial strain. This will be of importance in understanding upcoming results relating to EPS.

The question of modification of autofrettage pressure to account for Von Mises' criterion is addressed by Davidson and Kendall (1970) who propose an empirical pressure value of $1.08Y\ln(b/a)$ for the case of 100% overstrain, with an associated maximum error of 2%. These effects are reviewed in more detail in Part 1 of this paper, Parker (2000A). The current ASME pressure vessel code, ASME (1997), uses a fixed scaling factor of 1.15, but limits code validity to a maximum of 40% overstrain when calculationsd involve atofrettage pressure.

A rational approach to the numerical procedure, which is lengthy and often involves multiple iterations, requires that analyses are undertaken in a specific sequence, namely:

In Part 1 of this paper, Parker (2000A):

Step 1. For each tube geometry iteratively determine pressure to achieve a given percentage overstrain. This is repeated for the case of Von Mises', plane stress, $p\rceil^{VM\sigma}$, and Von Mises', EPS, $p\rceil^{VMEPS}$. Both sets of results are normalized with Tresca, plane stress, $p\rceil^{T\sigma}$. The first set is used to validate numerical results by comparison with analytical bounds, the second set is used as the basis of a proposed design procedure and (in part 2 of this paper) to make some assessment of the autofrettage pressure calculation procedure within the ASME code.

Step 2. Determine a numerical fit to the ratios determined in Step 1 for use by designers.

Step 3. Employ the autofrettage pressures determined in Step 1 for the Von Mises, EPS case in determining a limited range of autofrettage residual stress profiles. These results to cover hoop and axial stress and encompass both Bauschinger affected and non-Bauschinger affected situations.

In current paper:

Step 4. Determine numerous bore hoop stress values for the Von Mises, EPS case, normalized with $\sigma_\theta\rceil^{T\sigma}_{bore}$ from eqn (4).

Step 5. Determine some simple numerical fit to the ratios determined in Step 4 for use by designers.

Step 6. Use procedures employed in Steps 1-5 to assess accuracy of ASME code.

Step 7. Propose a procedure which will improve accuracy and extend code validity with pressure beyond current 40% overstrain limit.

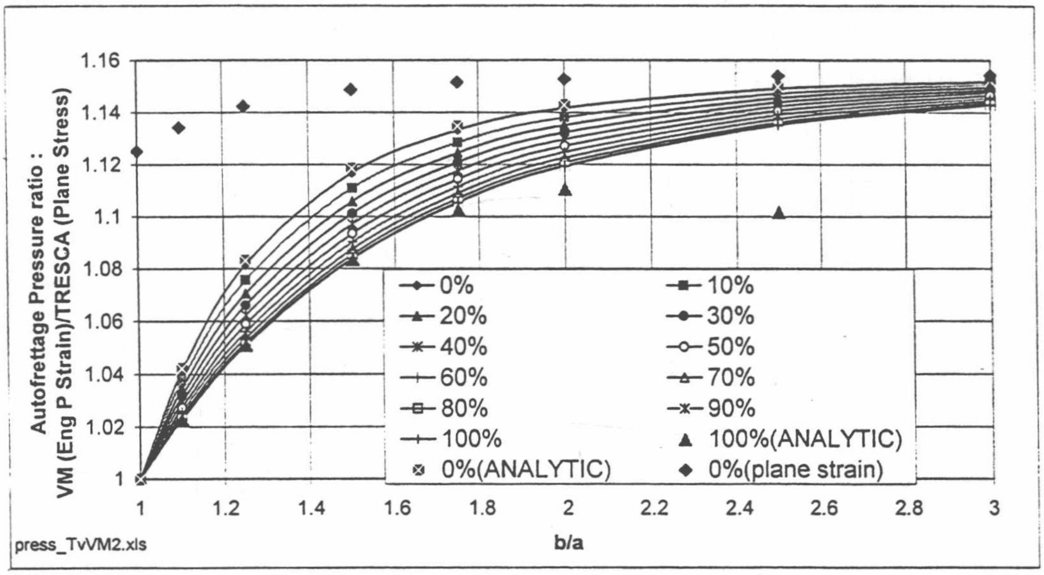

Figure 2 : Bore Pressure for Given Percentage Overstrain, Von Mises and Engineering Plane Strain Conditions Assumed

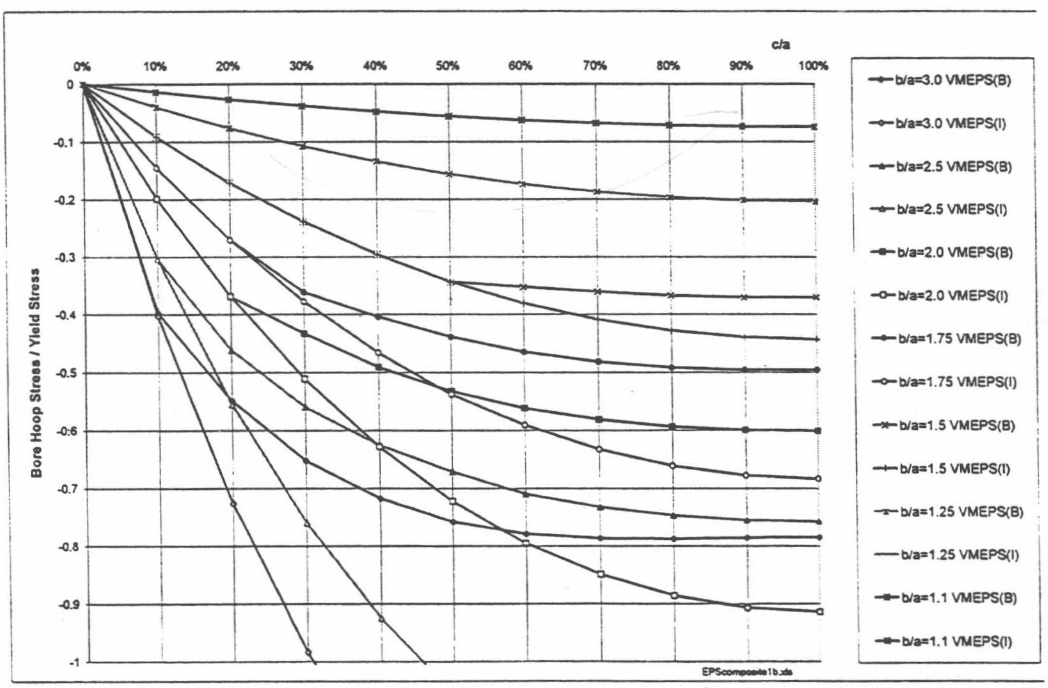

Figure 3 : Bore Hoop Stress Values as a Function of Percentage Overstrain for a Range of Autofrettaged Tube Geometries. (I) indicates ideal solution, (B) indicates results incorporating Bauschinger effect

PRESSURE FOR GIVEN OVERSTRAIN LEVEL

Figure 2, taken from Part 1 of this paper, Parker (2000A) shows the pressures required with open-end conditions to achieve various levels of percentage overstrain. An accurate fit to these numerical data was obtained in Part 1 of this paper, namely:

$$\frac{p\rceil^{VMEPS}}{p\rceil^{T\sigma}} = \frac{(2/\sqrt{3})}{\sqrt{1+1/\{3(b/a)^{\alpha}\}}} \qquad (8)$$

$$\alpha = 4 - 2.3n \qquad \text{and} \qquad (9)$$

$$n = (c-a)/(b-a)\ ,\ (c-a)/(b-a) \le 70\% \qquad (10)$$

$$n = 70\%\ \ ,\ (c-a)/(b-a) > 70\% \qquad (11)$$

RESIDUAL BORE HOOP STRESSES

Figure 3 shows a composite plot of bore hoop stress values. One set is predicted from an ideal, Von Mises, EPS, elastic-perfectly plastic analysis without Bauschinger effect (annotated 'I') and the other set from a similar analysis which includes Bauschinger effect (annotated 'B'). In all cases bore hoop stress values are normalized with yield stress and are plotted as a function of percentage overstrain

A popular approximation to the Bauschinger effect is that compressive hoop stress at the bore is capped at 70% of yield when the ideal value exceeds this level. Figure 3 indicates that this assumption may be significantly non-conservative. For example, an overstrain of 27% for b/a = 2.5 would be capped at 70% of yield whereas its value is only 53% of yield. Such an overestimate could produce orders of magnitude shift in fatigue lifetime calculations from pre-existing defects in cyclically pressurized cylinders.

It has been noted, Parker and Underwood (1998), that percentage plastic strain during initial autofrettage pressurization is of crucial importance. Because this is a strong function of c/a and relatively insensitive to b/a it is frequently more physically significant to plot hoop stresses as a function of c/a rather than percentage overstrain. Figure 4 shows the same data as Figure 3 but plotted in this alternative format.

Figure 4 indicates a consistent 'cut-off' at c/a = 1.2 below which the results follow the 'ideal' curve without Bauschinger effect and above which they exhibit an increasing loss of compressive yield strength arising from the Bauschinger effect. The cut-off is clear for all results with $b/a \ge 1.5$. Whilst the effect was more subtle it is also exhibited in the numerical results for $b/a = 1.25$. The Bauschinger effect is absent for $b/a \le 1.2$.

POSSIBLE DESIGN PROCEDURE

The data in Figures 3 and 4 may be presented in a useful format which leads to a possible design procedure. Figure 5 shows the same data normalized with $\sigma_\theta\rceil_{bore}^{T\sigma}$, eqn (7). Two features emerge:

(a) There is an upper bound (shown as a heavy line) which defines residual stress for those cases in which Bauschinger effect is absent. All ideal curves shown in Figures 3 and 4 fall on a single curve. Deviation from the curve is less than 1% over the full range of overstrain and b/a ratios considered.

(b) When Bauschinger effect is present the residual stress variation is a near-linear function of b/a with constant slope for all overstrain levels.

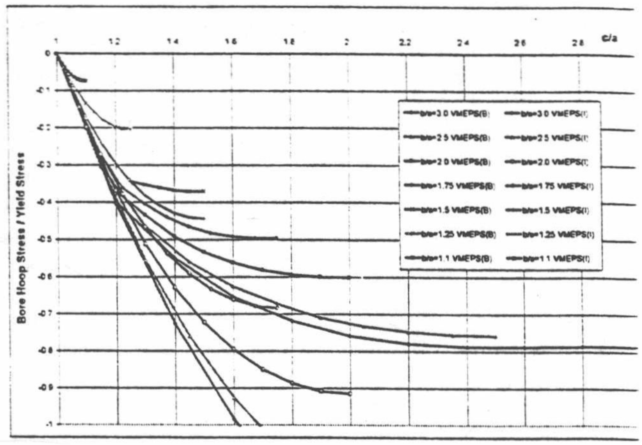

Figure 4 : Bore Hoop Stress Values as a Function of c/a for a Range of Autofrettaged Tube Geometries. (I) indicates ideal solution, (B) indicates results incorporating Bauschinger effect

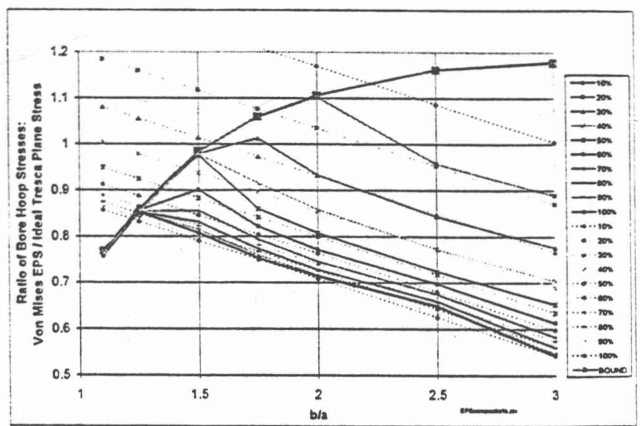

Figure 5 : Ratio of Bore Hoop Stresses, Von Mises EPS / Ideal Tresca Plane Stress. Broken Lines Indicate Linear Fit to Bauschinger-Affected Results.

Figure 5 could form the basis of a single, accurate design curve whereby autofrettage pressure for required overstrain is obtained using eqn (8) and residual stress is obtained via Figure 5. The procedure involves entering with b/a, moving vertically to appropriate percentage overstrain or bounding curve, whichever is encountered first, and reading off bore hoop stress.

The upper bound of Figure 5 may be approximated as follows (maximum error 0.5%):

$$\sigma_\theta \Big]_{bore}^{VMEPS} / \sigma_\theta \Big]_{bore}^{T\sigma} = 0.0791(b/a)^3 - 0.6502(b/a)^2 + 1.8141(b/a) - 0.5484 \qquad (12)$$

Because reversed yielding will occur even in the absence of Bauschinger effect when c/a > 2.22, eqn (12) is limited to $c/a \le 2.22$. The linear sections which incorporate the Bauschinger effect are approximated by:

$$\sigma_\theta \Big]_{bore}^{VMEPS} / \sigma_\theta \Big]_{bore}^{T\sigma} = R - 0.7086 \, .(1.0296m^3 - 2.7994m^2 + 2.6631m - 0.89) \qquad (13)$$

where: $$R = 1.0388 - 0.1651(b/a) \qquad (14)$$

and $$m = (c-a)/(b-a) \qquad (15)$$

This fit is shown for comparison as straight, dotted lines in Figure 5. Overall the fit is conservative with maximum errors of 5%. In the ranges of most practical application, $1.75 \le b/a \le 3.0$, $30\% \le (c/a)/(b-a) \le 80\%$, maximum error, again conservative, is generally less than 2%.

The overall design procedure (i.e. pressure calculation via eqn (8) and stress calculation via eqn (12) and eqn (13)) was compared with the original data presented in Figure 3. Maximum difference is generally 1.5%; this is considered adequate for a simple design procedure.

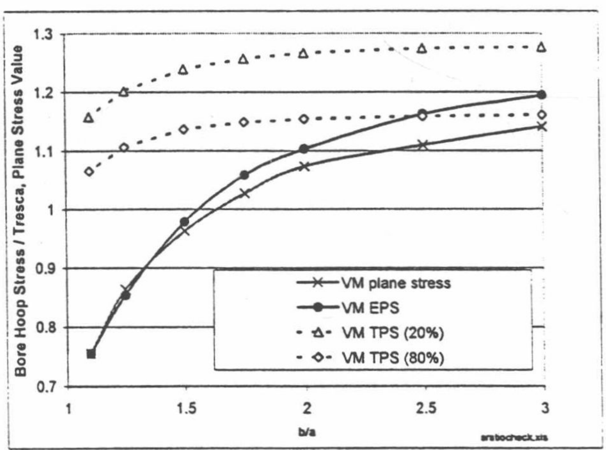

Figure 6 : Bore Hoop Stresses, Von Mises EPS, Tresca EPS and Von Mises TPS each Normalized with Tresca Plane Stress

The upper bound, defined by eqn (12) is now examined in more detail in order to assess existing models. Figure 6 shows the bound relating to Von Mises, EPS based upon eqn (12), together with the equivalent, numerically determined, bound for Von Mises, plane stress; the latter appears to exhibit a limit of $2/\sqrt{3}$ with increasing b/a.

In order to examine the Hill (1967) model referred to in the earlier 'Analysis Procedure' section the equivalent, numerically determined, solutions for the case of Von Mises TPS are presented for overstrains of 20% and 80%. The maximum difference between these results and the $2/\sqrt{3}$ value proposed by Hill varies between +11% (conservative) and -7% (non-conservative). It is also apparent, perhaps surprisingly, that Von Mises plane stress is a better (and consistently conservative) approximation to Von Mises EPS than is Von Mises TPS.

ASME CODE COMPARISONS

Autofrettage, including Bauschinger effect, is covered by Article KD5 of the ASME Pressure Vessel Code, ASME (1997), which fully defines the procedure for calculating residual stresses. Points of significance in the code are:

a. Autofrettage pressure is defined as $1.15p\big]^{T\sigma}$ with $p\big]^{T\sigma}$ defined in eqn (2).

b. Residual bore hoop stress in the absence of Bauschinger effect is given by $1.15\sigma_\theta\big]_{bore}^{T\sigma}$ with $\sigma_\theta\big]_{bore}^{T\sigma}$ defined in eqn (7).

c. Section KD-522.2 contains details of correction for Bauschinger effect

d. The code limits the bore pressure calculation to a maximum of 40% overstrain.

Both (a) and (b) above are at odds with the conclusions arising from numerical solutions presented herein. However this does not necessarily invalidate the code since the correction procedure serves to modify residual stress calculations. In order to properly compare the code with the numerical procedure presented herein it is necessary to follow code procedure precisely.

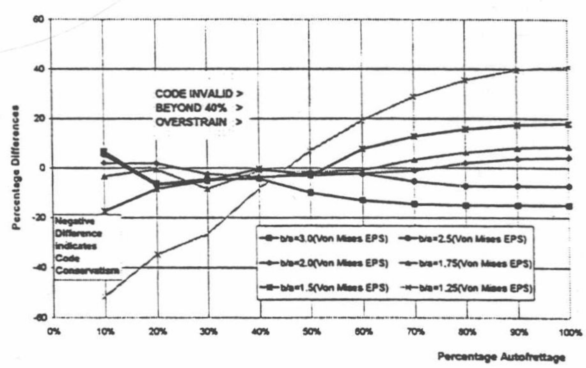

Figure 7 : Percentage Difference between ASME Code and Von Mises EPS Numerical Model, Negative Values Indicate Code Conservatism. Note - Code Validity Limited to 40% Overstrain

Figure 7 shows the result of a detailed comparison of code with the Von Mises EPS numerical procedure. The results cover the range $1.25 \le b/a \le 3.0$. The case $b/a = 1.1$ is omitted since use of the code