# QUANTUM GRAVITY

## AN OXFORD SYMPOSIUM

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# Preface

One of the paradoxes of modern physics is that its two main theories - relativity and quantum mechanics - have never been properly reconciled. It is true that the union of special relativity and quantum mechanics has led to some spectacular successes, both qualitative and quantitative (for instance, the Dirac equation, the existence of antiparticles, the connexion between spin and statistics, the TCP theorem and the detailed agreement of quantum electrodynamics with experiment). Nevertheless, because of its divergences, the relativistic quantum theory of realistic interacting fields is not yet satisfactory, even in those cases when the theory is renormalisable.

When we go further, and attempt to reconcile general relativity with quantum mechanics, the situation becomes considerably more serious. These two theories are conceptually and structurally very different, and this has led to the development of many apparently disparate approaches to their reconciliation. It has even led some physicists to doubt whether such reconciliation is needed at all (what God hath not joined no physicist can, to adapt a remark of Pauli's).

However, we firmly believe that not only is there a problem waiting here to be solved, but that its eventual solution will represent a major advance in physical understanding. Several considerations underly our belief. In particular, there is the intuition that the whole of physics should be described by a single fundamental theory. Quantum physics and gravity are, after all, both constituents of one and the same physical world. The very fact that they are at present described by theories so different from one another is an indication of how much is potentially to be learned by attempting to bring the two together. It is clear that there is much in the workings of Nature that is at present only remotely gleaned. A possible route to obtaining the new insights that are required would seem to be the encompassing of known phenomena of quantum physics and gravity into one coherent scheme.

One possible link-up is suggested by the success of recent unified gauge theories in particle physics, general relativity being commonly regarded as the gauge theory par excellence. Yet again, there is the possibility, as has many times been suggested, that gravity might supply the short-distance cut-off which could remove the divergences in quantum field theory and thereby assign (finite) numerical values to the renormalization constants (for example by providing a relation between the fine structure constant and the gravitational constant).

At the other end of the scale is the fact that progress in astrophysics and cosmology is now being held up by the absence of an adequate quantum theory of gravity. We have in mind the situation in the early superdense stages of the universe, and at the end point of catastrophic stellar collapse, when classical general relativity implies that physical space-time singularities must occur, singularities whose structure (and perhaps also whose very existence) would be decisively influenced by quantum effects. We have in mind also the black hole explosions envisaged by Stephen Hawking. In their case a possible astrophysical phenomenon of importance needs for its study an understanding at least of quantum effects (such as pair production) in a classical curved background spacetime, and perhaps also of the full quantum theory of gravity.

For these reasons, and because of the recent progress that has been made in understanding quantum gravity, we decided to organise a Symposium to help take stock of the present position, and to suggest guidelines for future research. This Symposium was held at the Rutherford Laboratory on February 15-16 1974, and we are very grateful to the Director, Dr. Godfrey Stafford, and also to Professor Douglas Allen, for providing such excellent facilities, and for ensuring the smooth-running of all the arrangements.

We were pleasantly surprised by the numbers attending what might have seemed an esoteric meeting - well over a hundred physicists took part. The formal sessions were mainly devoted to survey talks, with one or two individual topics added, and a substantial time was allowed for discussion.

It had not been our original intention to publish any proceedings of the Symposium. However, many of the participants emphasised to us that the existing survey literature on recent developments is rather sparse, and that a permanent record of the Symposium might be found widely useful. The speakers readily agreed to provide written versions of their talks, and the Oxford University Press to publish them. Accordingly we present now the state of the art in quantum gravity, as our Symposium saw it in early 1974. May its publication stimulate further theoretical work which soon renders it obsolete, and may it help also to foster closer links with both astrophysics and cosmology.

C. J. Isham
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D. W. Sciama

C O N T E N T S

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* This is an amended version of an article which is being published in Communications in Mathematical Physics, 1975.

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I

# AN INTRODUCTION TO QUANTUM GRAVITY

C.J. Isham*

## 0. PREFACE

The purpose of my talk at the Oxford Conference was to provide a general introduction to some of the ideas and methods of quantum gravity as a precursor to the rather technical lectures which followed. This is reflected in these lecture notes which are concerned mainly with broad attitudes rather than with specific, up to date, technical tools. The scheme of the paper is as follows. The first section is a short introduction which emphasises the dual particle/field interpretation of conventional quantum field theory. The latter interpretation is used extensively in quantum gravity and, because of its relative unfamiliarity, is the subject of repeated discussion throughout these notes. The next two sections deal with the problem of defining a quantised field on an unquantised gravitational background. There has recently been considerable investigation on this topic (which is a preliminary to quantum gravity proper) and it promises to be of some relevance to astrophysical problems involving gravitational collapse (see the chapter by S. Hawking). The fourth section is concerned with covariant quantisation (see the chapter by M. Duff) while in the next two sections canonical quantisation is discussed in some technical detail since this was not the subject of any other specific lecture at the conference. The final section considers the currently popular quantum model/quantum cosmology approach to quantising the gravitational field, although again since a lecture was devoted to this topic (see the chapter by

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M.MacCallum) the treatment here is concerned with the general ideas rather than with specific details.

## 1. INTRODUCTION

The problem of quantising the gravitational field has exercised the minds of a number of people over the last forty years and will doubtless continue to do so for the next forty[1][2][3][4][5][6]. The importance and interest of this subject of study, which is reflected in the very considerable increase in attention which it has received during the last decade, derive from a number of different sources. General relativity and quantum theory are without doubt two of the greatest intellectual achievements of this century. This is in itself sufficient to guarantee a continued interest in the problem of unifying them; an interest which is heightened by consideration of the very special role played by general relativity within the framework of classical (viz. non-quantum) physics. In any conventional field theory the space-time structure is fixed and the field propagates in time on this background. In general relativity however the kinematical and dynamical aspects of the theory are tightly interlaced through the medium of the gravitational field, which, on the one hand, specifies the geometrical properties of space-time, and on the other fulfills the classical task of a field by propagating a physical force. Conventional quantum theory, however, is formulated on a rigidly fixed space-time background, Euclidean three-space in the case of non relativistic quantum mechanics and Minkowskian space-time in the case of relativistic quantum field theory. From this viewpoint it can be expected that any attempt to unify general relativity and quantum mechanics will inevitably lead to technical and conceptual problems. One of the main motivations for studying quantum gravity has always been that the resolution of these problems will lead to a fundamentally new insight into physics.

It is not a priori clear precisely what would be regarded as a quantisation of general relativity. The mathematical structure of the classical theory contains a number of features any of which might perhaps be expected to become subject to quantum laws. The primordial concept is that of a point set whose mathematical points are to be related in some way with physical space-time events. This set is then equipped with a topology and then with a differentiable structure which makes it into a four-dimensional manifold. Finally a metric tensor is constructed on this manifold in such a way as to satisfy the Einstein equations. One might attempt to introduce quantisation at any one of these levels. In practice most of the work which has been done takes the easiest route and fixes everything but the metric. Thus a differentiable manifold is specified once and for all and the metric tensor is regarded as an operator defined on this space. (Actually if canonical quantisation is being used then the relevant manifold may be three, rather than four, dimensional). This is clearly the attitude to quantisation which is closest to that prevalent in conventional quantum field theories. Nevertheless when one considers the role played by the lightcone structure in these theories it is clear that already a major difference has emerged - the lightcone structure of general relativity is indisputably dynamical and not part of the fixed background.

However, the opinion is frequently voiced that the quantisation procedure should take place at a more fundamental level. Two of the principal advocates of this line have been Professors J. Wheeler and R. Penrose. Wheeler has for many years emphasised the need to quantise the topological as well as the metric structure of space-time and, with his recent thoughts on the role played by formal logic in quantum gravity, has taken the quantisation level right back to the basic elements of mathematics. Similarly Penrose has frequently argued that space-time itself, rather than just the metric field, should be intimately linked with quantum theory. It was this point of view which partly motivated his combinatorial spin network theory[7] as well as his recent work on twistors[8]. Most people would agree that a deeper look at the problem of quantum gravity at this type of very basic level is probably mandatory if any really major advance is to be achieved. However, it is also important to understand how far conventional quantisation (by which is meant metric field quantisation) can be pushed. In particular, it is essential to distinguish carefully between those problems which are peculiar to quantum gravity and those which are shared by all quantum field theories. Hand in glove with this must go an appreciation of the practical applications of this type of quantisation and their implications for realistic physical systems. In this article I shall concentrate mainly on the metric quantisation schemes and refer the reader to the bibliography for material on some of the other aspects of quantum gravity.

Many different approaches to quantising the gravitational field have evolved since the subject was first considered in the early 1930's. These tend to be classified under two headings, 'covariant' (§4) and 'canonical' (§5, §6). These titles can, from a technical standpoint, be a little misleading but since they are widely used they will be retained here. Canonical quantisation itself will be split up into 'true' canonical quantisation (§5) and superspace-based quantisation (§6). There is a tendency, at least among particle physicists, to suppose that the whole of quantum gravity can be neatly accommodated by the notion of the graviton. This helicity two, massless particle is then thought of as interacting with itself in a way which is more or less conventional although it leads to a theory which is probably highly nonrenormalisable. This is the principle concept which arises from the covariant quantisation scheme but it leads to a rather restricted view of quantum gravity and indeed of quantum field theory in general.

The particle interpretation, with its corresponding set of particle-based observables, of a quantum field theory, which the notion of a graviton epitomises, may not always be the most appropriate one. There is in fact an important alternative physical interpretation of what is essentially the <u>same</u> mathematics, even in the case of an ordinary flat-space quantum field theory. As this alternative view is the one which is most commonly used in quantum gravity (mainly in the canonical approaches) it is worth discussing it here, at least in a heuristic manner. For the sake of simplicity consider a free neutral scalar field $\phi(x)$ in ordinary flat Minkowski space-time. The conventional quantisation of this system using Fock space, with the corresponding particle interpretation, is well known (see §2 for more details). On the one hand it can be obtained by quantising the scalar field $\phi(x)$ per se and looking for a suitable representation (in the Schrödinger picture say) of the canonical commutation relations

$$[\hat{\phi}(\underline{x}),\ \hat{\pi}(\underline{y})] = i\hbar\,\delta^{(3)}(\underline{x} - \underline{y})\ . \tag{1.1}$$

On the other hand one can begin with one-particle states, two-particle states etc. described in terms of ordinary quantum mechanics and construct a large state-space which accommodates them all, namely Fock space. 'Annihilation' and 'creation' operators can then be defined which connect together these various finite particle subspaces and from which a quantum field $\hat{\phi}(x)$ can be reconstructed. However, it is interesting to ask how this simple problem of quantising a free field looks from the viewpoint of conventional quantum mechanics. If a classical system has a Euclidean configuration space Q with global cartesian coordinates $q_1 \ldots q_n$ corresponding to n degrees of freedom, then the basic problem of quantum theory (in the Schrödinger picture) is to find a representation of the canonical commutation relations

$$\begin{aligned} [\hat{q}_i\ ,\ \hat{p}_j] &= i\hbar\,\delta_{ij} \qquad\qquad i,j = 1\ldots n \\ [\hat{q}_i\ ,\ \hat{q}_j] &= 0 \\ [\hat{p}_i\ ,\ \hat{p}_j] &= 0 \end{aligned} \tag{1.2}$$

with self-adjoint operators on a Hilbert space of states. Then the dynamical equation

$$H(\hat{q}_1, \hat{q}_2 \ldots \hat{q}_n\ ;\ \hat{p}_1, \hat{p}_2 \ldots \hat{p}_n)\ \psi_t = i\hbar\,\frac{\partial \psi_t}{\partial t} \tag{1.3}$$

must be solved for the time evolution of the state vector $\psi_t$ in terms of the quantised Hamiltonian operator H.

By virtue of the Stone-Von Neumann theorem, the unique solution (up to unitary transformations) is that in which the state space is the set of all complex valued functions of Q which are square integrable with respect to the Lebesgue measure $dq_1\, dq_2 \ldots dq_n$. The operators $\hat{q}_i$, $\hat{p}_j$ are then represented by

$$(\hat{q}_i\ \psi)\ (q_1 \ldots q_n) = q_i\ \psi(q_1 \ldots q_n) \tag{1.4}$$

$$(\hat{p}_j\ \psi)\ (q_1 \ldots q_n) = -\, i\, \hbar\, \frac{\partial \psi}{\partial q_j}\ (q_1 \ldots q_n) \tag{1.5}$$

and any other representation of eqn (1.2) (or more precisely of the exponentiated Weyl form) is unitarily equivalent to this one. The wave function has the interpretation that if B is any Borel set in $\mathbb{R}^n$ then

$$p_B = \int_B |\psi(q_1 \ldots q_n)|^2\ dq_1 \ldots dq_n \tag{1.6}$$

is the probability that if the system is in the state $\psi$ and a measurement is made on the system of the values of $q_1 \ldots q_n$ (i.e. of the <u>classical</u> <u>configuration</u> of the system) then they lie in B. Now a classical <u>field</u> theory can be regarded as a classical mechanical system with <u>infinitely</u> many degrees of freedom. Essentially, an orthonormal basis set of functions on $\mathbb{R}^3$, $\{e_i(\underline{x})\}$ say, is chosen (typically with properties in relation to the Hamiltonian which simplify the dynamical evolution problem) and the fields are expanded as

$$\phi(\underline{x},t) = \sum_{i=1}^{\infty}\ q_i(t)\ e_i(\underline{x}) \tag{1.7}$$