

Johnson / Steffensen

INTERMEDIATE ALGEBRA

Second Edition



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Second Edition

L. Murphy Johnson
Arnold R. Steffensen
Northern Arizona University

 HarperCollins *Publishers*

For the Student

To help you study and understand the course material, a *Solutions and Study Guide*, by Joseph Mutter, is available from your college bookstore. This book provides complete, step-by-step solutions to half of the odd-numbered text exercises and to all Chapter Review and Chapter Test exercises, as well as detailed Chapter Summaries. Supplemental exercises, organized by chapter, are also included.

To Barbara, Barbara, Becky, Cindy, and Pam

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Preface

Intermediate Algebra, Second Edition is designed for college students who have had one year of high school algebra (or its equivalent) or who require a review before taking further courses in mathematics, science, business, or computer science.

The text is organized for maximum instructional flexibility; core and peripheral topics are included so that instructors can better meet their individual course needs. Chapter 1 provides a review of fundamental algebraic concepts which some classes may cover quickly or skip altogether. In Chapter 2, linear equations and inequalities are discussed with emphasis on practical applications. Detailed explanations of all types of polynomial factoring are provided in Chapter 3 before rational expressions, radicals, and quadratic equations are considered in Chapters 4 through 6. Because of their importance, graphing and functions are given detailed treatment in Chapter 7 rather than later as in many texts. Chapters 8 and 9 continue the topics of graphing and functions as systems of equations and inequalities and logarithmic and exponential functions are thoroughly discussed. For added flexibility, material on the conic sections and sequences and series is presented in Chapters 10 and 11.

FEATURES

The text is **written informally**; explanations are carefully worded to ensure student comprehension. Full-page **chapter introductions**, with photographs, provide an overview of upcoming material and demonstrate its relevance by including applications solved later in the chapters. **Second color** is used pedagogically throughout the text to highlight important steps. Key definitions, rules, and procedures are set off in colored boxes for increased emphasis. Labeled for easy reference, **figures and graphs** employ color to clarify the concepts presented. **Cautions** warn students of common mistakes and special problems, while **Notes** provide additional explanations or other pertinent information.

Examples

The text contains over **700 carefully selected examples** with detailed step-by-step solutions and helpful side annotations in color.

Exercises

There are over **5600 exercises** in the text. The exercise sets are carefully graded and begin with paired routine problems that gradually build to more challenging extension problems and numerous applications. Answers to odd-numbered section exercises are

included at the back of the book. To assist instructors in making assignments, a list of section exercises, classified according to difficulty, is provided in the *Instructor's Guide*.

The text includes over **1400 review exercises**. A set of **For Review exercises** is included at the end of most exercise sets to provide students with continuous review of previously covered material or special preparation for the next section. A collection of **Chapter Review exercises** and a **Practice Test** conclude each chapter. Answers to all For Review, Chapter Review, and Chapter Test exercises are included at the back of the book.

Applications

To demonstrate the usefulness and practicality of mathematics, applications are integrated throughout the text with a special strategy for solving word problems given in Section 2.3. Over **650 relevant applied problems** from such diverse areas as business, engineering, physics, chemistry, and agriculture are included in the chapter introductions, examples, and exercises. For easy reference, an index of these applications is given at the back of the book. Answers at the back of the book to some application problems also include variable assignments and initial set-up equations to help students having difficulty.

Calculators

It is assumed that most students have a hand-held calculator; therefore, calculator use is integrated where appropriate throughout the text. Step-by-step key entries for algebraic calculators are illustrated. Calculator exercises are not specifically marked, however, since students need to learn when to use and when not to use a calculator.

SUPPLEMENTS An expanded supplemental package is available for use with *Intermediate Algebra, Second Edition*.

For the Instructor

The *Instructor's Guide* contains a series of ready-to-duplicate tests including a Placement Test, four different but equivalent tests for each chapter and two final examinations. Over 1000 additional problems, grouped by section, are also included for use as inclass examples or on quizzes and tests. Answers are provided for all test items and to the even-numbered text exercises. A list of the text exercises, classified according to difficulty, is given as an aid in making student assignments. Additional instructional material on linear interpolation and solving linear systems of equations by matrix methods is also provided in ready-to-duplicate form for instructors wishing to cover these topics.

The **Instructor's Solutions Manual** contains complete solutions to the even-numbered text exercises as well as solutions to those odd-numbered exercises not included in the student's *Solutions and Study Guide*.

More than 70 **overhead transparencies** featuring key figures from the text are provided for classroom lectures and presentations.

A computerized testing system and tutorial software package in both IBM and Apple are available. These items feature ease-of-use and are gratis to instructors who adopt the text.

For the Student The **Solutions and Study Guide** contains complete, step-by-step solutions to half of the odd-numbered text exercises and to all Chapter Review and Chapter Test exercises as well as detailed Chapter Summaries. Supplemental exercises, organized by chapter, are also included.

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L. Murphy Johnson

Arnold R. Steffensen

To the Student

During the past several years we have taught intermediate algebra to more than 1500 students and have heard the following comments many times. “I’ve always been afraid of math and have avoided it as much as possible. Now my major requires algebra and I’m petrified.” “I don’t like math, but it’s required to graduate.” “I can’t do word problems!” If you have ever made a similar statement, now is the time to think positively and start down the path toward success in mathematics. Don’t worry about this course as a whole. The material in the text is presented in a way that lets you take one small step at a time. As you begin, keep in mind these guidelines that are both necessary and helpful.

GENERAL GUIDELINES

1. Mastering algebra requires motivation and dedication. Just as an athlete does not improve without commitment to his or her goal, an algebra student must be prepared to work hard and spend time studying. Make it a point to keep up with your assignments. If you need extra help, *ask* for it right away.
2. Algebra is not learned simply by watching, listening, or reading; *it is learned by doing*. Use your pencil and practice. When your thoughts are organized and written in a neat and orderly way, you have taken a giant step toward success. Be complete and write out all details. The following are samples of two students’ work on a word problem. Can you tell which one was more successful in the course?

Student A

Let t = time the woman walks
 $7-t$ = time she rides
 $4.5t$ = distance she walks
 $2.5(7-t)$ = distance she rides
 $4.5t = 2.5(7-t)$
 $4.5t = 17.5 - 2.5t$
 $7t = 17.5$
 $t = 2.5$
She walks for 2.5 hr.

Student F

3. If use of a calculator is permitted in your course, become familiar with its features by consulting your owner's manual. When you are working problems that involve computing with decimals or approximating irrational numbers, a calculator can be a time-saving device. On the other hand, you should not be so dependent on a calculator that you use it to perform simple calculations that might be done mentally. For example, it would be ridiculous to use a calculator to solve an equation such as $2x = 8$, while it would be helpful to solve $1.12x = 7343.84$. It is important that you learn when to use and when not to use your calculator.

SPECIFIC GUIDELINES

1. As you begin to study each section, look through the material for a preview of what is coming.
2. Return to the beginning of the section and start reading slowly; study the explanations carefully. Pay particular attention to the definitions, rules, and procedures in the colored boxes.
3. Read through each example and make sure that you understand every step. The side comments in color will help you if something is not quite clear.
4. Periodically you will see a CAUTION or NOTE. The CAUTION warns you of common mistakes and special problems to avoid. The NOTE provides pertinent information or additional explanations.
5. After you have completed the material in the body of the section, check your understanding of the skills and practice, practice, practice! Work all of the exercises assigned by your instructor. Answers to the odd-numbered exercises are at the back of the text. Complete worked-out solutions to every other odd-numbered problem are also included in the *Solutions and Study Guide*.
6. To help retain what you've learned, be sure to work the exercises marked *For Review*, located at the end of most exercise sets. These problems provide continuous review of previously covered material or special preparation for the next section. Answers to all *For Review* problems are at the back of the text.
7. After you have completed a chapter, review each section. As an additional review aid, detailed Chapter Summaries and supplemental exercises (with answers) are provided in the *Solutions and Study Guide*. Then work the Chapter Review exercises in the text. They provide additional practice before you take the Chapter Test. Answers to all Chapter Review and Chapter Test problems are at the back of the book. Step-by-step solutions are provided in the *Solutions and Study Guide*.

If you follow these suggestions and work closely with your instructor, you will greatly improve your chances for success in algebra.

Best of luck, and remember that you can do it!

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To the Student *x*

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Review of Fundamental Concepts

The study of intermediate algebra allows us to develop new concepts and reinforce material learned in previous courses. As our knowledge increases, we can also consider a greater variety of applications. The following is an example of the type of problem that we can solve using the review material in this chapter. Its solution is in Example 4 of Section 1.5.

CONSTRUCTION



A type of finishing material is priced at \$5.25 per square foot. How much must be paid for enough of this material to cover an area shaped like a trapezoid with bases 10.5 ft and 12.5 ft and height 8.0 ft?

We begin this chapter by discussing the set of real numbers, which will be used throughout the text. We review the properties of these numbers and discuss powers, roots, and order of operations. Next, algebraic expressions are reviewed, before considering integer exponents.

1.1 THE SET OF REAL NUMBERS

Sets and Elements

In this chapter we develop basic terminology used throughout our study of algebra. Two important terms are *set* and *element*. A **set** is a collection of objects called **elements**. The elements of a set can be listed within braces, $\{ \}$, and the symbol $\in$ denotes the phrase “is an element of.” For example,

$$c \in \{a, b, c, d\}$$

means that c is an element of the set $\{a, b, c, d\}$. The symbol $\notin$ means “is *not* an element of.” Capital letters are used to denote sets. In

$$A = \{\text{Troy, Eric, Mike, Jeff, Andy}\},$$

the letter A represents the set of five people. Another way of describing the set A is to give the common characteristic of its elements.

$$A = \{\text{the starting five on the State University basketball team}\}$$

Sets of Numbers

Most sets we deal with are sets of *numbers*, concepts named by symbols called **numerals**. The most basic set of numbers is the set of **natural** or **counting numbers**, N .

$$N = \{1, 2, 3, 4, 5, \dots\} \quad \text{Natural (counting) numbers}$$

The three-dot notation ($\dots$) indicates that the pattern continues. Sets that have no limit or no last element, such as N , are called **infinite sets**, while sets whose elements can be counted, such as A above, are called **finite sets**. One special set is the **empty** or **null set** which has no elements. The empty set is denoted by the symbol $\emptyset$.

When the number 0 is included with the set of natural numbers, we have the set of **whole numbers**.

$$W = \{0, 1, 2, 3, 4, \dots\} \quad \text{Whole numbers}$$

When the negatives of all the natural numbers are included with the whole numbers, we have the set of **integers**.

$$I = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\} \quad \text{Integers}$$

We often talk about the **negative integers**, $\{-1, -2, -3, \dots\}$, and the **positive integers**, $\{1, 2, 3, \dots\}$.

The concept of a *variable* can be defined using sets. A **variable** is a symbol such as x that represents an unspecified element of a set containing more than one number. The set that provides values for the variable is called the **domain** or **replacement set** for the

variable. A **constant** is a symbol denoting an element in a set with exactly one element. For example, if x represents an element of $\{4\}$, then x , or 4, is a constant.

Using variables, we can describe sets with **set-builder notation**. For example,

$$\{x|x \text{ is a natural number between 2 and 7}\},$$

is read “the set of all x such that x is a natural number between 2 and 7,” and describes the set

$$\{3, 4, 5, 6\}$$

in set-builder notation.

The set of numbers formed by taking quotients of integers (division by zero is excluded) is called the set of **rational numbers**. In set-builder notation, the rational numbers, denoted by Q , are given by

$$Q = \{x|x = \frac{a}{b}, \text{ where } a \text{ and } b \text{ are integers with } b \neq 0\}.$$

Some of the rational numbers are

$$\frac{2}{3}, \quad \frac{-1}{5}, \quad \frac{7}{-4}, \quad \frac{4}{1}, \quad \frac{-5}{1}, \text{ and } \frac{0}{1}.$$

Subsets

Notice that every integer can be expressed as a quotient of itself and 1, such as 4, -5 , and 0 above, so that the set of integers is included in the set of rational numbers. This is an example of an important relationship between sets. If every element of set A is also an element of set B , A is called a **subset** of B . The symbol $\subset$ represents the phrase “is a subset of” so we write $A \subset B$. The symbol $\not\subset$ represents the phrase “is not a subset of.”

Example 1

Consider the sets $A = \{x|x \text{ is a natural number between 1 and 5}\}$, $B = \{1, 2, 3, 4, 5\}$, $C = \{3, 4, 5, 6\}$, and $D = \{2, 3, 4\}$.

- (a) $2 \in A$ Since 2 is a natural number between 1 and 5, 2 is an element of A .
- (b) $6 \notin B$ The number 6 is not an element of B .
- (c) $A \subset B$ Since every natural number between 1 and 5, that is, every element of A , is also an element of B , A is a subset of B .
- (d) $C \not\subset B$ Since 6 is an element of C but 6 is not an element of B , C is not a subset of B .
- (e) $A \subset D$ and $D \subset A$ Every element of each set is also an element of the other. When this occurs, we say that the sets are **equal** and write $A = D$. Notice that the same set is described in two different ways, by set-builder notation and by listing the elements.
- (f) $A \subset A$ Clearly every element of A is also an element of A .
- (g) $\emptyset \subset A$ Every element of $\emptyset$ (there are none) can also be thought of as an element of A . ■

Every rational number can be expressed as a *fraction* or as a *decimal*. For example, $\frac{3}{8}$ can be written as 0.375, the result obtained by dividing 3 by 8. Likewise, $\frac{3}{11}$ can be expressed as the decimal 0.2727 The decimal 0.375 is a **terminating decimal** since the sequence of digits comes to an end. The decimal 0.2727 . . . is an infinitely **repeating decimal** since a block of digits, in this case 27, repeats indefinitely. We usually write such numerals by placing a bar over the repeating block. For example,

$$\frac{3}{11} = 0.\overline{27} \quad \text{and} \quad \frac{1}{3} = 0.\overline{3}.$$

In general, every rational number can be written as either a terminating decimal or an infinitely repeating decimal.

Numbers which are not rational, that is, which *cannot* be expressed as a quotient of two integers, are called **irrational numbers**. An irrational number cannot be written as either a terminating decimal or an infinitely repeating decimal. One of the best known irrational numbers is π , the ratio of the circumference of any circle to its diameter. Irrational numbers can be approximated by rational numbers; for example, π is often approximated by 3.14.

The set of **real numbers**, denoted by R , consists of the rational numbers together with the irrational numbers. Numbers associated with most square roots of natural numbers, such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, and $\sqrt{8}$, are irrational numbers, but some, such as $\sqrt{1} = 1$, $\sqrt{4} = 2$, $\sqrt{9} = 3$, and $\sqrt{16} = 4$, are rational. The relationships among the types of numbers we have discussed are displayed in Figure 1.1.

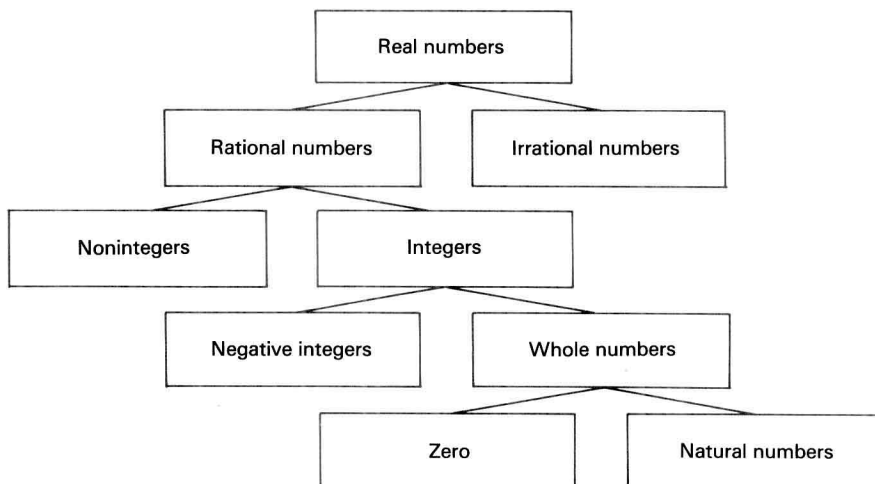


Figure 1.1 Sets of Numbers

Number Line

An excellent means of displaying numbers and showing some of their properties is a **number line**, as shown in Figure 1.2. We draw a line; arrows on each end indicate that it can be extended infinitely far in both directions. We then select a starting point or

origin and a unit of length (any unit will do). The origin is labeled zero and unit lengths in both directions are marked off. Points to the right of zero are identified with the positive numbers, while points to the left of zero correspond to negative numbers.

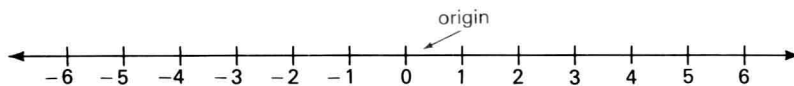


Figure 1.2 Number Line

Every real number corresponds to exactly one point on a number line, and every point on a number line corresponds to exactly one real number. We plot a number on a line, and the point is called the **graph** of the number. The number line in Figure 1.3 has several numbers plotted on it.

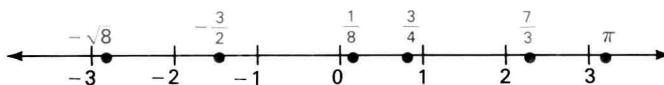


Figure 1.3 Points on a Number Line

Equality

Two numbers a and b that correspond to the same point on a number line are equal ($a = b$). For example,

$$\frac{4}{2} = 2, \quad 1.5 = \frac{3}{2}, \quad \text{and} \quad \frac{3}{11} = 0.\overline{27}.$$

Numbers that are not equal (written $a \neq b$) correspond to different points.

Axioms of equality are properties that are assumed to be true without verification. These properties, which are listed below, are used for solving equations and simplifying algebraic expressions.

Axioms of Equality

Let a , b , and c be real numbers.

Reflexive Law $a = a$

Symmetric Law If $a = b$, then $b = a$.

Transitive Law If $a = b$ and $b = c$, then $a = c$.

Substitution Law If $a = b$, then either may replace the other in any statement without affecting the truth of the statement.

- Example 2**
- (a) $4 = 4$, $-3 = -3$, $\frac{1}{2} = \frac{1}{2}$ Reflexive law
- (b) If $x = 8$, then $8 = x$. Symmetric law